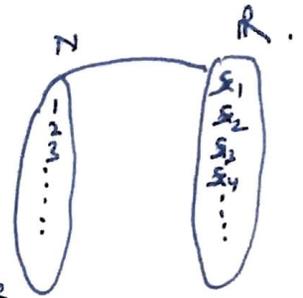


Module II -1-

Sequences (Real)

Definition A function whose domain is the set N of natural numbers and range a set of real numbers is called a real sequence.

Symbolically, $S: N \rightarrow R$.



Since the domain for a sequence is always N , we specify a sequence by the values $s_n, n \in N$. Then, we can write a sequence in the form

$$\{s_n\}_{n \in N} \quad \text{or} \quad \{s_n\}_{n=1}^{\infty} \quad \text{or} \quad \{s_1, s_2, s_3, \dots, s_n, \dots\}$$

\downarrow \downarrow \downarrow \downarrow
1st element 2nd element 3rd element n-th element

- We treat m -th term & n -th terms s_m & s_n as distinct terms even as ~~$s_m = s_n$~~ $s_m = s_n$.
- The terms of a sequence are arranged in a definite order as 1st, 2nd, 3rd, ..., n -th, ... & the terms occurring different positions are treated as distinct terms even they have the same values.
- The number of terms in a sequence is always infinite.
- In other words, we define a sequence as an ordered set of real numbers whose members can be put in a one-one correspondence with the set of natural numbers.
- However, a sequence may have only a finite number of distinct terms.

Some Examples

1. $s_n = (-1)^n$ $\xrightarrow{\text{sequence}}$ $\{(-1)^n\}_{n \in N}$. only two elements 1, -1, both distinct.

2. $s_n = \frac{1}{n}$ $\xrightarrow{\text{consider the sequence}}$ $\{\frac{1}{n}\}_{n \in N}$. infinite no. of elements, all are distinct.

3. $s_n = (1 + \frac{1}{n})^n$, 4. $s_n = 1 + (-1)^n$ 5. $s_n = 1$ 6. $s_n = \frac{(-1)^{n-1}}{n!}$
 $n \in N$ $n \in N$ $n \in N$ $n \in N$.

Range of a sequence :

The range of a sequence is the set consisting of all distinct elements of a sequence, without repetition or without regard to the position of a term.

Thus, the set (range) may be a finite or an infinite set.

Bounds of a sequence :

A sequence $\{S_n\}$ is said to be bounded above if there exists a real number K such that

$$S_n \leq K \quad \forall n \in \mathbb{N}.$$

A sequence $\{S_n\}$ is said to be bounded below if there exists a real number k such that

$$S_n \geq k \quad \forall n \in \mathbb{N}.$$

A sequence is said to be bounded if it is both bounded above and bounded below. So, obviously, we see that the ranges are bounded.

Convergence of Sequences

Defⁿ: A sequence $\{S_n\}$ is said to converge to a real number l (sometimes, we say l to be a limit of the sequence) if for each $\epsilon > 0$, there exists a positive integer m (depending on ϵ) such that

$$|S_n - l| < \epsilon \quad \text{for all } n \geq m.$$

We denote the convergence/limit of the sequence as

$$S_n \rightarrow l \text{ as } n \rightarrow \infty \quad \text{or} \quad \lim_{n \rightarrow \infty} S_n = l.$$

what is the meaning of the above definition: ???

→ From some stage onwards, the differences between s_n and l can be made less than any preassigned number ϵ , however small.

That is, given any positive real ϵ , no matter how small it is, there exists a +ve integer m (finite value) such that the terms after m / m -th term onwards, that is, $s_m, s_{m+1}, s_{m+2}, \dots$ remains arbitrary close to l .

We say l to be the limit point of the sequence.

→ For any $\epsilon > 0$, at the most a finite number of terms (depending on the choice of ϵ) of the sequence can lie outside $(l-\epsilon, l+\epsilon)$, that is, there is at the most a finite number of n 's for which

$$s_n \leq l-\epsilon \text{ and } s_n \geq l+\epsilon.$$

→ Since $l-\epsilon < s_n < l+\epsilon$ for all $n \geq m$, then we have the property that $s_n < l+\epsilon$ for infinite number of terms, we have the following observations:

infinite number of terms $\in (l-\epsilon, l+\epsilon)$.