

Lecture -1Module -1.

-1-

Sets, Equality of sets, Notations: $\forall, \exists, \Rightarrow \Leftrightarrow,$
 \wedge, \vee, \sim

\forall : The statement $x < y \wedge x \in S$ means x is less than y for all members of S , i.e., all members of S are less than y .

\exists : there exists

Ex: For $x^2 - 1 = 0$, \exists unique positive number $x = 1$ such that $x^2 = 1$, so $x^2 - 1 = 0$.

\Rightarrow : implies that

Ex: If P and Q are two statements, then $P \Rightarrow Q$ means that the statement P implies the statement Q .

Simply: $x = 5 \Rightarrow x^2 = 25$.

\wedge : and

\vee : or.

$P \wedge Q$: both statements P and Q hold.

Ex: $x^2 - 1 = 0 \Rightarrow x = 1 \text{ and } x = -1$
i.e. $x = 1 \wedge -1$.

So: $x \in \{-1, 1\}$.

$P \vee Q$: One of P or Q holds.

Ex: $x^2 \geq 25 \Rightarrow x = 5; 6, \dots$
any real number greater than or equal to 5.

\sim : negation:

P : Statement is true.

$\sim P$: ^{the} Statement is not true. $\therefore \text{not } P$.

So, $P \wedge \sim P$ is always false.

$P \vee \sim P$ is always true.

Subsets, union and intersection of sets,
Union and intersection of arbitrary family of sets,

$$A_i, i=1, 2, \dots, n \quad \bigcup_{i=1}^n A_i \quad \bigcap_{i=1}^n A_i.$$

Sometimes, we shall be dealing with the unions
and intersections of large (really large) class of
sets.

Let Λ be a set and $\{A_\lambda; \lambda \in \Lambda\}$ be an
entirely arbitrary class or family \mathcal{F}_c of sets
of which contains a set A_λ for each $\lambda \in \Lambda$. Then

$$\left\{ \begin{array}{l} \bigcup_{\lambda \in \Lambda} A_\lambda = \{x; x \in A_\lambda \text{ for at least one } \lambda \in \Lambda\} \\ \bigcap_{\lambda \in \Lambda} A_\lambda = \{x; x \in A_\lambda \text{ for every } \lambda \in \Lambda\} \end{array} \right.$$

defines the union and intersection of an
arbitrary family \mathcal{F}_c .

Here, $\Lambda \rightarrow$ Index set.

Universal set; Difference of sets; Complement of a set;
Difference of sets; Complement of a set;

\hookrightarrow am skipping the above.

Relation: $R \subset X \times Y$, $X, Y \rightarrow$ sets.
No rules.

functions

Let A and B be two sets and let there be a rule
which associates to each member x of A , a member y of B .

Such a rule or correspondence f under which
to each element x of the set A there corresponds
exactly one element y of B is called a mapping
or a function.

$f: A \rightarrow B$. ie f is a mapping from A to B .

$A \rightarrow$ Domain of f .

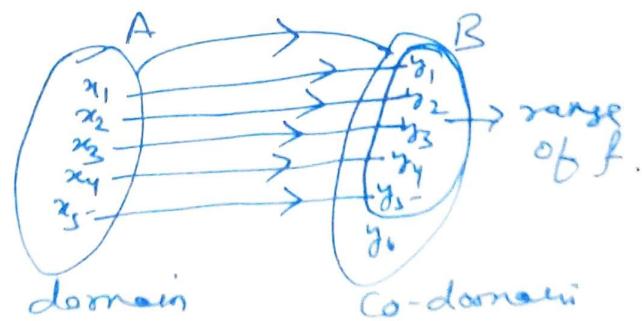
The set B contains all the elements which correspond to the elements of A and is called the co-domain of f .

\rightarrow The unique element of B which corresponds to an element x of A is called the image of x or the value of the function at x and is denoted by $f(x)$.

$x \rightarrow$ pre-image of $f(x)$ (preimage)

\rightarrow It may be observed that while every element of the domain (Ex. A set) finds its image in B , there may be some elements in B which are not the image of any element of A (domain set).

The set of all those elements of the co-domain set B which are the images of the elements of the domain A is called the range set of the function f .

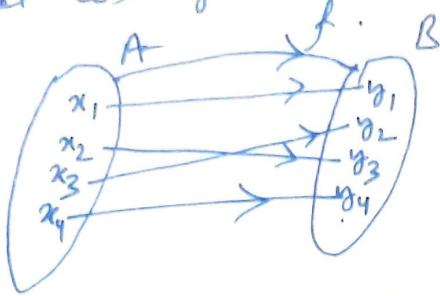


If $B = \text{Range of } f$, then we say that f is an onto function or f is a function from A onto B .

We call a function $f: A \rightarrow B$ be one-one if two different elements in A always have two different images under f , i.e.,

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \quad \forall x_1, x_2 \in A.$$

I am going to give some nice ways of studying one-one and onto; and we see what we get.



$x_1 \rightarrow y_1, x_2 \rightarrow y_3, x_3 \rightarrow y_2, x_4 \rightarrow y_4$.
 $x_1 \neq x_2 \Rightarrow y_1 \neq y_2 \dots$ If it follows, then
 f is one one.

For y_1 , \exists x_1 s.t $f(x_1) = y_1$.

For y_2 , \exists x_3 s.t $f(x_3) = y_2$

For y_3 , \exists x_2 s.t $f(x_2) = y_3$

For y_4 , \exists x_4 s.t $f(x_4) = y_4$.

} f is onto.

} If there is one ~~one~~ y_5 in B, for this there should exist one at only one ~~one~~ x_5 in A s.t $f(x_5) = y_5$. This also falls under onto property.

→ Oneone and onto. (no. of elements are equal, one element is related/mapped to exactly one element) $\Rightarrow y_4 \xrightarrow[\text{mapped}]{\text{inversely}} x_4$,

$y_3 \xrightarrow[\text{mapped}]{\text{inversely}} x_2$, $y_2 \xrightarrow[\text{mapped}]{\text{inversely}} x_3$,

$y_1 \xrightarrow[\text{mapped}]{\text{inversely}} x_1$. i.e. inverse exists.

Thus, if f is

oneone }
 onto } $\Rightarrow f^{-1}$ exists.