

Thus, the successive reverse implications are immediate consequences of the above discussion.  $\rightarrow$  (iii) so (ii)

Now, let  $Y = K$ . If  $f$  is continuous, then since  $\{0\}$  is closed in  $K$ ,  $f^{-1}(\{0\}) = Z(f)$  is closed in  $X$ .

Conversely, assume that  $Z(f)$  is closed in  $X$ . If  $f$  is not continuous at 0, there would exist  $x_n \rightarrow 0$  in  $X$  such that  $|f(x_n)| \geq \delta$  for some  $\delta > 0$ , and  $n=1, 2, \dots$ . Let  $a \in X$  with  $f(a)=1$ , and  $\textcircled{B}$

$$z_n = a - \frac{x_n}{f(x_n)}.$$

Then  $f(z_n) = f(a) - \frac{f(x_n)}{x_n} = 1 - 1 = 0$ , so that  ~~$z_n \in Z(f)$~~   
 $z_n \in Z(f)$ , but  $z_n \rightarrow a - 0 = a \notin Z(f)$ , which is a contradiction to the closedness (assumption) of  $Z(f)$ .  
 Thus, the conditions (ii) and (vi) are equivalent.

Definition:  
 Let  $X$  and  $Y$  be nbs over  $k$ . They are said to be linearly homeomorphic if  $\exists$  a linear map  $f: X \rightarrow Y$  which is homeomorphism onto;  $f$  is an isometry into then  $X$  and  $Y$  are said to be linearly isometric.

$$\left. \begin{array}{l} f: X \rightarrow Y \text{ one-one, onto, cont.} \\ f^{-1}: Y \rightarrow X \text{ cont.} \end{array} \right\} f \text{ is homeomorphism.}$$

$X, Y \rightarrow$  homeomorphic.

$Y = X \rightarrow$  equivalent.

distance preserving

$$\|x-y\|$$

$$= \|f(x)-f(y)\|$$

On this case,  
 we say the  
 nbs  $X$  and  $Y$   
 are isometric.

Theorem: