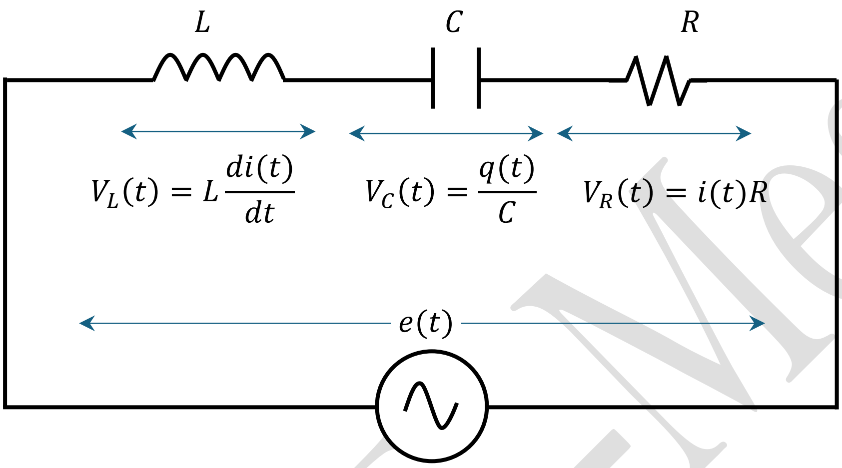
EXPERIMENT – 9

Aim of the experiment: To study the frequency response a series LCR (Inductor-Capacitor-Resistor) circuit.

Apparatus required: Oscilloscope (CRO/DSO) with probe, Function generator, Inductor, Capacitor, decade resistance box, a 56 Ω $\frac{1}{4}W$ resistor, connecting wires with BNC and crocodile clip terminations,

Theory: An LCR circuit is essentially a frequency sensitive circuit, that means it responds differently to time dependent voltages of different frequencies. The current through any series LCR circuit depends on the frequency of the applied signal, provided the voltage across the combination is maintained.



The net impedance offered by the circuit to the flow of time dependent current may be written as,

$$Z = \sqrt{R^2 + X^2}$$

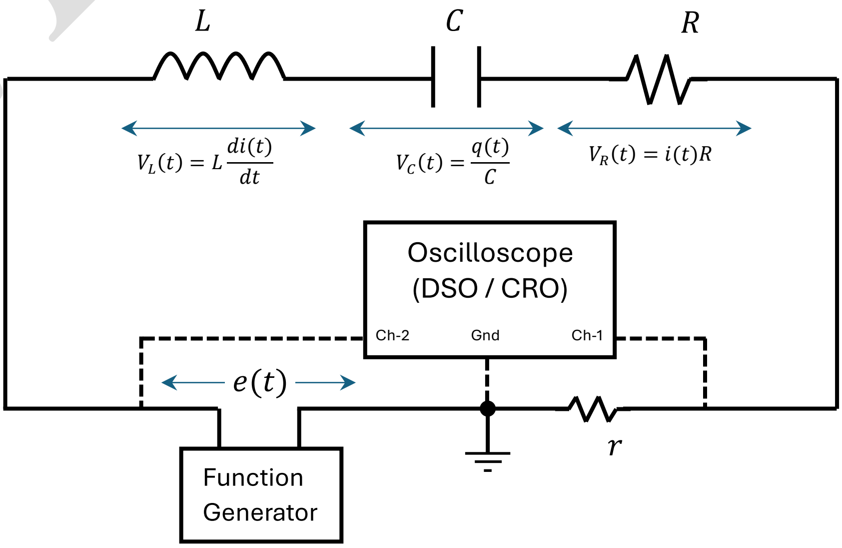
where, R is its dc resistance and X is the reactance of the circuit (frequency dependent resistance to the passage of ac, offered by L or C or their combination).

or, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

where, X_L is the inductive reactance and X_C is the capacitive reactance.

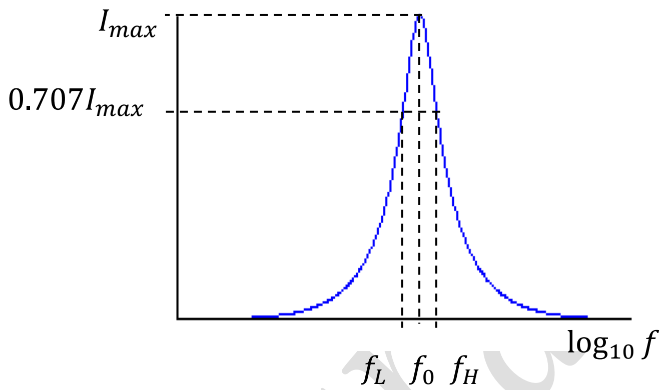
or, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

where, $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$



A phase angle also gets introduced between the applied voltage and the current through the circuit, that is given by, $\emptyset = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

Therefore, the amplitude of current in the circuit may be given by



$$I = \frac{E}{Z}$$

where, E is the voltage amplitude

substituting the value of Z in the above equation we have,

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

If the frequency ω of applied voltage, matches the natural frequency ω_0, of the circuit then the inductive reactance and the capacitive reactance equals each other i.e., $\omega_0 L = 1/\omega_0 C$ and the current in the circuit is solely decided by the value of R, i.e., $I = \frac{E}{R}$

The frequency at which the inductive reactance equals the capacitive reactance is the natural frequency or the resonant frequency of the circuit. The resonant frequency of an LCR circuit depends upon the values of L and C by the relation

$$\omega_0{}^2 = \frac{1}{LC}$$

or, $f_0 = \frac{1}{2\pi\sqrt{LC}}$

The sharpness of resonance for a particular value of L and C depends upon the value of R and is computed from the plot of I versus f by the relation

$$Q = \frac{f_0}{\Delta f} = \frac{f_0}{f_H - f_L}$$

Now the 'Quality factor' or the 'Q factor' is a dimensionless parameter that describes how under-damped an oscillator or resonator is. Higher values of Q indicate lower rate of energy loss relative to the energy stored in the oscillator.

Procedure and Precautions:

Note: A function generator produces voltage as a function of time for example a sinusoidal waveform, a square waveform or a triangular waveform, wherein one can select the function and adjust its frequency and voltage amplitude; and a cathode ray oscilloscope (CRO) or a digital storage oscilloscope (DSO) displays time dependent signals or waveforms on its screen and allows measurements on them.

1. Before making any connections (except mains power cords) switch on the oscilloscope and ensure that the sensitivity of the oscilloscope is set at a low value, for example 5V/div.
2. Switch on the function generator and adjust its voltage amplitude to a level ≤ 5V and let it remain constant throughout the experiment.

3. Also, make sure you are using a compatible probe with the CRO / DSO and function generator. Please note that the probes may appear similar but are not interchangeable.
4. Press the function generator in sinusoidal signal mode.
5. Connect the L, C, R and a 56 Ω resistor in series and the two extreme ends to the function generator output through a BNC (as shown in figure).
6. Ensure that the 56 Ω resistor is connected to the function generator output ground.
7. Connect the ground terminal of the oscilloscope probe to the ground of function generator, and the other terminal of the probe in such a way that the oscilloscope gets connected just across the 56 Ω resistor as shown in the preceding circuit diagram.
8. Set R=0 from the resistance box.
9. Adjust the frequency output of the function generator to 10Hz.
10. Record the voltage amplitude of the signal as shown by the oscilloscope.
11. Go on incrementing the frequency logarithmically and record your observation, i.e., repeat step-9 till you reach 1MHz.
12. Next increase R to 200 Ω and repeat steps 8, 9 &10.
13. Current (in ampere) can be obtained by dividing the voltage amplitude by the resistor value (56 Ω in our case).

Observations:

L = 10 mH; C = 0.01 μF

Table-1: Readings of current against frequency of the voltage signal applied across the series LCR circuit for the study of frequency response.

Frequency υ (Hz)	R = 0 Ω		R = 200 Ω	
	I (mA)	Φ (deg)	I (mA)	Φ (deg)
100				
200				
..				
..				
900				
1k				
2k				
..				
..				
9k				
10k				
20k				
..				
..				
90k				
100k				

Table-2: Observed and calculated values of resonant frequency, lower & upper cut-off frequencies, bandwidth and the quality factor of the LCR combination.

	f_0 @ I_{pk} (a)	f_L @ $I_{pk}/\sqrt{2}$	f_H @ $I_{pk}/\sqrt{2}$	$BW = f_H - f_L$ (b)	Q (a/b)
$R = 0\ \Omega$					
$R = 200\ \Omega$					

Plot:

1. Plot the current versus frequency for the two resistor values (i.e., $0\ \Omega$ and $200\ \Omega$) on a semi-log graph sheet neatly and mark the resonant frequency (f_0), the peak amplitude I_{max} and $\frac{I_{max}}{\sqrt{2}}$ for each plot. Sketch a dotted horizontal line corresponding to $\frac{I_{max}}{\sqrt{2}}$ and note that it cuts the plot at two points. Drop perpendiculars from these points on the x axis and mark the lower cutoff f_L and upper cutoff frequency f_H points. Note that the gap between the cutoff frequencies widens as the value of R increases.

Calculations:

The resonant frequency from the graph:

$f_0 =$ ________kHz;

Calculating quality factor (for different values of R)

$$Q = \frac{f_0}{f_H - f_L}$$

Result & discussion

Compare with theoretically calculated and experimentally determined values of resonant frequency and comment.

Comment on the quality factor as well, for different values of R.

Viva questions:

1. What is resonance?
2. What is reactance and how is it different from resistance?
2. What is impedance and how is it different from reactance?
3. Compare the resonant frequency and quality factors for the three resistor values. Comment on the results obtained by you.
4. What is the utility of a series LCR circuit and what are its applications.