EXPERIMENT – 2

Aim: To determine the resistance per unit length of a Carey-Foster Bridge wire and hence determine the resistivity of a wire sample provided.

Apparatus: Carey-Foster bridge, Leclanché cell or low voltage power source, two resistance boxes, standard 1 Ω resistance, wire sample (the resistance of which is to be determined), copper strip (to serve as a zero-ohm resistance), dc galvanometer, jockey, connecting wires

Principle: The Carey-Foster bridge is a modified form of a meter bridge wherein we have two gaps on either side instead of one. So, in all there are four gaps.

The basic idea behind the use of Carey-Foster bridge is to avoid or nullify the effect of the difference in end resistances that exist between the two sides of a meter-bridge. Usually, this small difference in resistance is immaterial provided the resistance to be determined is of several ohms. However, it becomes extremely relevant and important that the effect of end resistances be nullified when the resistance to be determined is small (usually a couple of ohms)

In order to achieve this, we employ a method in which we use two additional resistances $1\ \Omega$ and $0\ \Omega$, who's sides are swapped during the course of the experiment.

We wire a circuit as shown in the following diagram.

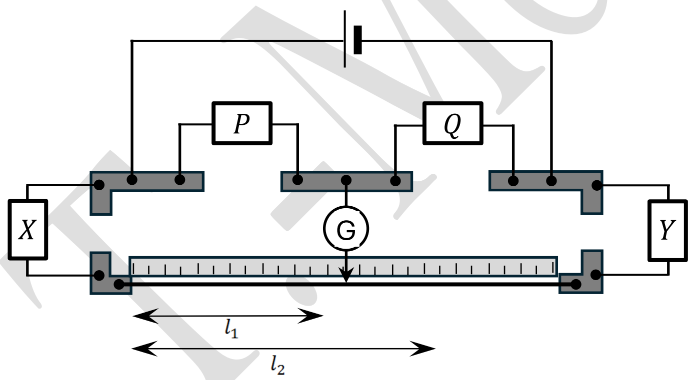

Fig-1. Circuit diagram of a Carey Foster bridge for the determination of low resistance.

For this arrangement we write the balanced Wheatstone's bridge relation assuming resistance X to be in the left gap and Y in the right,

$$\frac{P}{Q} = \frac{X + \sigma(l_1 + \alpha)}{Y + \sigma(100 - l_1 + \beta)} \tag{1}$$

where,

α – is the end resistance on one side of the bridge (left-side, say) and
β – is the end resistance on the other side of the bridge (right-side)

Here, σ is not conductivity, its rather the resistance per unit length of the Carey Foster bridge wire and its unit is $\Omega\ cm^{-1}$.

Again, we write the balanced Wheatstone's bridge relation for the two resistances X and Y are swapped i.e., resistance Y is in the left gap and X in the right, this time.

$$\frac{P}{Q} = \frac{Y + \sigma(l_2 + \alpha)}{X + \sigma(100 - l_2 + \beta)} \tag{2}$$

Adding one to both sides of the above two equations in order to make the right-hand side numerators of the two equations equal, we get,

$$\frac{P}{Q} + 1 = \frac{X + Y + \sigma(100 + \alpha + \beta)}{Y + \sigma(100 - l_1 + \beta)} \tag{3}$$

$$\frac{P}{Q} + 1 = \frac{Y + X + \sigma(100 + \alpha + \beta)}{X + \sigma(100 - l_2 + \beta)} \tag{4}$$

Equating right hand sides of (3) and (4),

$$\frac{X + Y + \sigma(100 + \alpha + \beta)}{Y + \sigma(100 - l_1 + \beta)} = \frac{Y + X + \sigma(100 + \alpha + \beta)}{X + \sigma(100 - l_2 + \beta)} \tag{5}$$

Here it's easy to see that the numerators cancel out and hence the denominators must be equal.

Equating the denominators we get,

$$X - Y = \sigma(l_2 - l_1) \tag{6}$$

For the first part of the experiment, we must use a standard 1 Ω resistance in place of X and a copper strip of 0 Ω resistance in place of Y.

Then the above equation becomes

$$1 - 0 = \sigma(l_2 - l_1) \tag{7}$$

and the resistance per unit length may be calculated as follows:

$$\sigma = \frac{1}{(l_2 - l_1)}\ \Omega\, cm^{-1} \tag{8}$$

Once we know the *resistance per unit length* of the Carey Foster bridge wire, we may proceed for the second part of the experiment wherein a sample wire the resistance of which is to be determined is used in place of X. Once again, a copper strip of 0 Ω resistance is used in place of Y. The entire process is repeated once again to get the value R of the sample wire using the following relation.

$$R = \sigma(l_2' - l_1') \tag{9}$$

This time the balancing lengths are l_1' and l_2' that are necessarily different from l_1 and l_2

After getting the value of R using the above process, we may measure the length and cross-sectional area of the wire we are experimenting with and its resistivity may be easily determined using the relation,

$$\rho = \frac{RA}{l} = \frac{R\pi r^2}{l} \tag{10}$$

Procedure and Precautions:

1. Wire the circuit as shown in the diagram keeping $P \approx Q \approx 5\ \Omega$, the standard 1 Ω resistor in the left gap and the 0 Ω copper strip in the right. While doing so ensure that you connect all the wires on a particular strip at the same place on a single binding post. Do not rely on the metal strip for good connections as with efflux of time it might have gotten oxidized.
2. Connect a rheostat in case you are using a power supply instead of a Leclanché cell and adjust it to restrict the total current well below 100 mA, preferably 10 mA.

3. Record the balancing length in the column for l_1.
4. Swap the two resistors $1\ \Omega$ and $0\ \Omega$. Get a balance point and record it in the column for l_2.
5. We record both the lengths l_1 and l_2 from the same end of the bridge.
6. For one set of data the ratio of P and Q must be held constant.
7. We may choose to alter the values of P and Q for the next set of data. However, this is not necessary.
8. After grabbing at least five sets of data, we may take the average to obtain the value of resistance per unit length of the CF bridge wire. First part of the experiment is over and the value of resistance per unit length obtained will be utilized in the next part of the experiment.
9. In the second part of the experiment, we again keep $P \approx Q \approx 5\ \Omega$, and this time we use a resistor of unknown resistance R in place of X and a copper strip of $0\ \Omega$ resistance in place of Y. Proceeding in the same way we record the balancing length of wire as l_1'. Then swap the two resistors and record l_2'.
10. Once again, both the lengths l_1' and l_2' are recorded from the same end of the bridge, holding the ratio of P and Q the same.
11. Again, taking at least five sets of data, we may take the average to obtain the value of resistance of the wire sample of unknown resistance.
12. Measuring the length of the wire sample using a scale and its diameter using a screw gauge, we may get its resistivity using the relation $\rho = \frac{\pi r^2 R}{l}$.
13. We compare the resistivity from its standard value and report the result along with the standard error and percent error.

Observations:

Table-1: Readings for balancing lengths of CF bridge wire for determination of its resistance per unit length

S.No.	$P \approx Q$	Position of balance point		$(l_2 - l_1)$	$\sigma = \frac{1}{(l_2 - l_1)}$	$(\sigma_i - \bar{\sigma})^2$
		l_1	l_1			
	Ω	cm	cm	cm	$\Omega\ cm^{-1}$	
1.						
2.						
3.						
4.						
5.						
					$\frac{\sum_{i=1}^{n} \sigma_i}{n}$	$\sum_{i=1}^{n} (\sigma_i - \bar{\sigma})^2$

Table-2: Readings for balancing lengths of CF bridge wire for determination of resistance of a given wire.

S.No.	$P \approx Q$	Position of balance point		$(l'_2 - l'_1)$	$R = \sigma\,(l'_2 - l'_1)$	$(R_i - \bar{R})^2$
		l'_1	l'_2			
	Ω	cm	cm	cm	Ω	
1.						
2.						
3.						
4.						
5.						
					$\frac{\sum_{i=1}^{n} R_i}{n}$	$\sum_{i=1}^{n} (R_i - \bar{R})^2$

Calculation:

$$Avg\ value\ of\ resistance\ per\ unit\ length, \sigma = \frac{\sum_{i=1}^{n} \sigma_i}{n}$$

$$Standard\ error\ in\ resistance\ per\ unit\ length, \sigma = \sqrt{\frac{\sum_{i=1}^{n} (\sigma_i - \bar{\sigma})^2}{n(n-1)}}$$

$$Avg\ value\ of\ resistance, R = \frac{\sum_{i=1}^{n} R_i}{n}$$

$$Standard\ error\ in\ resistance, R = \sqrt{\frac{\sum_{i=1}^{n} (R_i - \bar{R})^2}{n(n-1)}}$$

Results and discussion:

The value of resistivity is $\rho =$ (________ $\pm$ ________) Ωm

and the percentage error in the determination is

$$\%\ error = \frac{|\rho_{std} - \rho|}{\rho_{std}} \times 100\ \%$$

The results obtained by the method is accurate to ____ % and the data has a fair degree of precision. This is an elegant method of determining resistances that are relatively quite low.