EXPERIMENT – 1A

Aim of the experiment: To determine the frequency of AC mains using resonance produced by Lorentz force on a stretched wire.

Apparatus required: A thin metal (conducting) wire on a sonometer with knife edges and a measuring scale attached to it, horse-shoe magnet, weights with hanger, an ac step down transformer, a rheostat, connecting wires.

Principle: The velocity of a wave on a stretched string is $= \sqrt{T/\mu}$
and its natural frequency, that also depends on its length is given by,

$$\nu_0 = \frac{n}{2l}\sqrt{\frac{T}{\mu}} \tag{1}$$

where,
$T\ (= mg)$ is the tension in the wire is the
μ is the mass per unit length of the wire
l is the length of the wire between the two wedges
n is the mode of vibration ($n = 1$ for fundamental or first harmonic, $n = 2$ for first overtone or second harmonic)

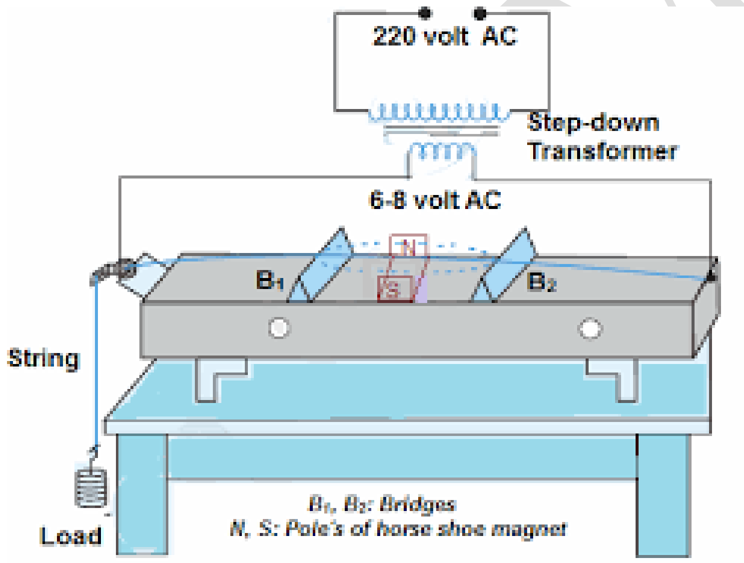


Fig-1. The diagram showing the experimental arrangement of a current carrying wire under tension, on a sonometer box subject to a steady magnetic field.

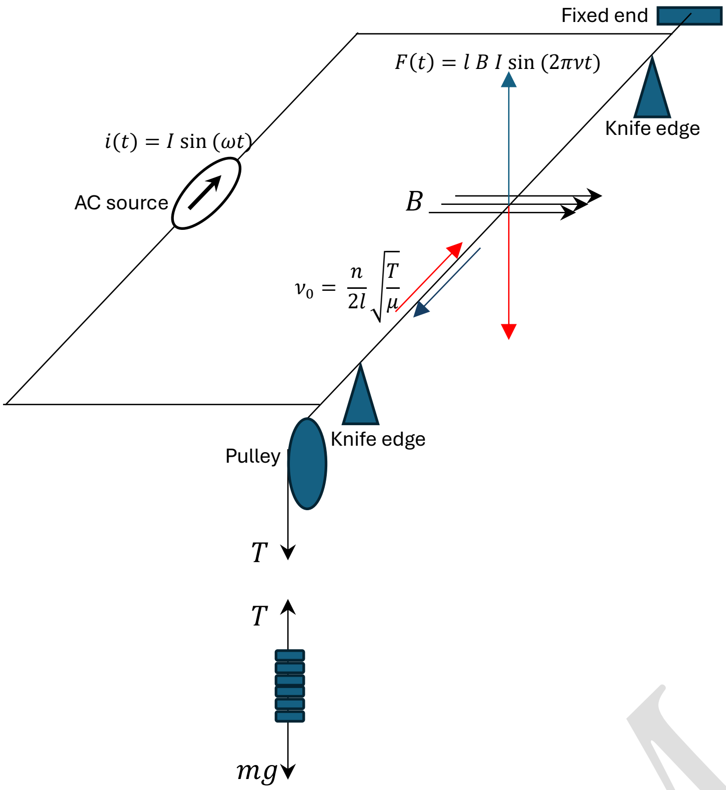


Fig-2. The schematic diagram of an alternating current carrying conductor between two knife edges, subject to tension and steady magnetic field at right angles for the determination of frequency of the current.

If a current i is made to pass through the wire and is also subjected to a steady magnetic field acting at right angles to the length of the wire, then the segment of the wire experiences a force (Lorentz force) in a direction perpendicular to its length and the magnetic field.
Let the current from an AC source be given by:

$$i(t) = I\,\sin\,(\omega_{ac} t) \qquad (2)$$

where, I be the peak current, ν_{ac} is the frequency and $\omega_{ac}\ (= 2\pi\nu_{ac})$, is the cyclic frequency of the current.

We know that the Lorentz force experienced by a current-element, $id\vec{l}$ in a magnetic field $\vec{B}$ is,

$$\vec{F} = id\vec{l} \times \vec{B}. \qquad (3)$$

If a wire of length l, carrying a sinusoidal current $i(t)$, is placed in a magnetic field that acts on it perpendicularly to its length, then the force experienced by it may be written as

$$F(t) = l\,B\,I\,sin\,(\omega_{ac} t) \qquad (4)$$

or,

$$F(t) = l\,B\,I\,sin\,(2\pi\nu_{ac} t). \qquad (5)$$

The segment of the wire within the magnetic field experiences this time varying force.
When the frequency of the force (on the wire segment within the pole pieces) matches the natural frequency of the wire (between the two knife edges) then it starts resonating. At resonance we can say that both the frequencies match. This fact, in turn, can be utilized to get the frequency of the applied current using this method.

Procedure and Precautions

1. *Make sure that the sonometer wire is straight, free of kinks. If not, straighten it before starting the experiment.*
2. Measure its weight on a sensitive digital balance and its length by placing it alongside the edge of a table. Calculate the value of mass per unit length (μ), to be used in the determination of frequency of ac.
3. Tie one end of the wire to a peg on the sonometer.

4. Allow the wire to pass over the knife edges and the pulley at the other extreme end of the sonometer. Make a loop at the end of the wire and suspend a weight hanger.
5. Now the wire is under tension. Bring a horse-shoe magnet near the center of the wire such that a small segment of the wire lies between the two poles of the magnet and is perpendicular to the field.
6. Take a power step-down transformer of nearly 6 to 12 V and connect it to ends of the wire through a rheostat of $20\ \Omega$. *Adjust the rheostat so that the wire doesn't get overheated (mild heating is normal, but even that would affect the results).*
7. Now adjust the positions of the two knife edges keeping the magnet nearly at the center, so that the wire is set into vibrations. Adjust carefully for maximum amplitude of vibrations. Under this condition the wire is said to be in resonance, i.e., the natural frequency of the length of wire between the knife edges matches with the Lorentz force frequency.
8. Record your observations on the table.
9. Increase the load on the hanger in steps of 50g and repeat the process to record your observations further. *Ensure that the load on the hanger doesn't keep oscillating.*
10. Once you have acquired all the data and tabulated the same in your observation table, go for calculations and get the results.

Observations

Mass per unit length of the metal wire $\mu = 0.015\ g\ cm^{-1}$

Table – 1: Readings for the resonant length of the wire against the tension that it is subject to, for the determination of the frequency of ac current flowing through it.

S No	m (g)	T (dyne)	$\sqrt{T}$ (dyne)$^{1/2}$	While moving the knife edges	Position of knife edges (cm)		length l (cm)	frequency ν_0 (Hz)	$(\nu_0 - \bar{\nu}_0)^2$
					1st	2nd			
1	50	49000	221.0	Apart	40.6	58.6	18.0	50.1	0.0348
2				Close	40.0	57.3	17.3	52.1	3.3896
3	100	98000	313.0	Apart	37.2	62.8	25.6	49.9	0.1565
4				Close	36.5	61.5	25.0	51.1	0.6433
5	150	147000	383.4	Apart	34.5	65.8	31.3	50.0	0.0920
6				Close	34.8	66.8	32.0	48.9	1.9514
7	200	196000	442.7	Apart	30.1	66.3	36.2	49.9	0.1479
8				Close	31.4	67.2	35.8	50.5	0.0300
9	250	245000	495.0	Apart	28.5	68.9	40.4	50.0	0.0840
10				Close	27.4	67.0	39.6	51.0	0.5190
11	300	294000	542.0	Apart	25.2	69.8	44.6	49.6	0.4621
12				Close	24.0	68.1	44.1	50.2	0.0137
								$\bar{\nu}_0 = 50.3$	$\sum_1^n (\nu_0 - \bar{\nu}_0)^2 = 7.5244$

Calculations

We calculate the frequency of ac applied across the wire using the equation:

$$\nu_0 = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$

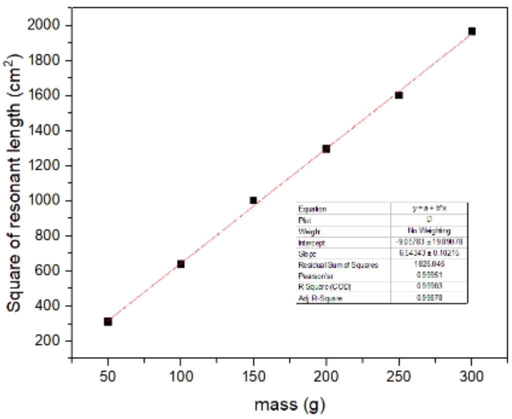
and the standard deviation and standard error using the following equations.

$$Standard\ deviation \quad \sigma = \sqrt{\frac{\sum_1^n (\nu_0 - \bar{\nu}_0)^2}{n-1}} = 0.83\ Hz$$

$$Standard\ error \quad \bar{\sigma} = \sqrt{\frac{\sum_1^n (\nu_0 - \bar{\nu}_0)^2}{n(n-1)}} = 0.24\ Hz$$

Graphical calculation:

1. We plot a graph of $\sqrt{T}$ vs l



Using the slope of the plot of $\sqrt{T}$ vs l in the equation (for graphical calculation):

$$\nu_0 = \frac{n}{2\sqrt{\mu}}\left(\frac{\sqrt{T}}{l}\right)$$

to obtain the frequency of ac, for $n = 1$ or 2, as the case may be.

Results and discussion

The frequency of ac mains is experimentally determined to be 50.3 Hz, using resonance in a stretched wire

$$\nu_0 = (50.3 \pm 0.2)\ Hz$$

$$\%\ error = \frac{\nu_{std} - \bar{\nu}_0}{\nu_{std}} \times 100\ \%$$

Percentage error = 0.6 %

The frequency is determined using the present method to a fair degree of accuracy and precision. The result being accurate suggests that the parameters involved therein, primarily, the masses (put on the hanger) and the mass per unit length are exactly known. We could get a quite good figure for standard error implies that the resonant length of the wire at various loads was determined quite meticulously and sufficient number of trials were attempted.

Conclusion

This is an elegant method of determining the frequency of domestic ac supply. Conversely, it is also possible to determine the mass per unit length of the metal wire with fair degree of accuracy provided the frequency of AC is known. This method is not just limited to the frequency of ac mains. Using this technique, it must be possible to determine frequencies within a fairly wide range. This method can also be effectively used to determine the linear density of any other wire of interest to us.

Questions for viva / quiz

1. What is the principle of this experiment?
2. Is the hollow sonometer box necessary for the experiment?
3. Can this experiment be carried out using a wire of a magnetic material? Scheme?
4. How would you estimate the maximum accuracy and precision of the arrangement?
5. What would you do to enhance the precision of the experiment?
6. How would you verify your result?
7. Is it a first order system or a second order system?
8. What is the velocity of wave on a stretched string?