COURSE INFORMATION SHEET

Course Code:	**ME24311**
Course Title:	**Robotics Engineering**
Pre-requisite(s):	**NIL**
Co- requisite(s):	**NIL**
Credits:	**4** (L:3 T:1 P:0)
Class schedule per week:	**4**
Class:	**B. Tech.**
Semester / Level:	**SIXTH**
Branch:	**Mechanical Engineering**
Name of Teacher:	

COURSE OBJECTIVES

This course envisions to impart to students to:

1.	Present a comprehensive and rigorous treatment of different robot types.
2.	Lay the mathematical background which is required to understand the mechanical design of different industrial robots used in modern industry.
3.	Develop an intuitive understanding of the limitations of various robots and its safe handling.
4.	Present real world engineering examples to demonstrate how a robot system is applied in engineering practice.

COURSE OUTCOMES (COs)

After the completion of this course, students will be able to:

CO1	Explain the fundamental concepts and mathematical foundations of Robotics Engineering, including degrees of freedom, transformation matrices, homogeneous coordinates, and Euler angle representation.
CO2	Utilize the Denavit-Hartenberg representation and various kinematic approaches to solve problems in forward and inverse kinematics of serial robots.
CO3	Evaluate the role of the Jacobian matrix in robot statics, velocity, and acceleration analysis, and solve gravity compensation problems in robotic systems.
CO4	Derive equations of motion for serial robots using Lagrange-Euler and Newton-Euler formulations, and assess different control methods for trajectory following in manipulators.
CO5	Design and implement control strategies for robotic systems, including feedback and closed-loop control, and integrate robotic systems into industrial applications such as PUMA and SCARA configurations.

SYLLABUS

MODULE	(NO. OF LECTURE HOURS)
Module – I Introduction to Robotics Engineering. Degrees of Freedom for Open and Closed loop systems, Serial robot kinematics: Transformation matrices and homogeneous coordinates, Composite rotation matrix, Rotation about an arbitrary axis, Euler angle representation. Links, Joints and their parameters, Denavit-Hartenberg representation, Forward kinematics.	**9**
Module – II Inverse kinematics of serial robot: Geometrical and Algebraic Approach. Velocity analysis: Jacobian matrix, Acceleration analysis. Role of Jacobian in robot Statics. Gravity compensation.	**9**
Module – III Dynamics of serial robots: Lagrange-Euler formulation, Newton Euler approach, Motion equations of a manipulator. Inverse and Forward dynamics approaches.	**9**
Module – IV Trajectory planning: Cartesian and Joint space trajectories, Cubic trajectory. Linear control of manipulator: Feedback and closed-loop control, Control of second order system, Trajectory following control, Modeling and control of a single joint.	**9**
Module – V Parallel robot structures, Inverse kinematics of parallel robots, Classical Industrial robot systems, PUMA, and SCARA configurations, Robotic system integration, Industrial applications of robotics.	**9**

TEXTBOOKS:

T1. Subir Kumar Saha, Introduction to Robotics, TMH, New Delhi, 2014.
T2. John J. Craig, Introduction to Robotics, Pearson Education, 2011.
T3. J. P. Marlett, Parallel Robots, Springer, 2006.

REFERENCE BOOKS:

R1. Dilip K. Pratihar, Fundamentals of Robotics, Narosa Publishing House, 2016.
R2. KS Fu, C. S. G Lee, R. Gonzalez, Robotics: Control, Sensing, Vision and Intelligence, McGraw-Hill Education, 1987.
R3. Bruno Siciliano and OussamaKhatib, Handbook of Robotics, Springer, 2016.
R4. Saeed B. Niku, An Introduction to Robotics Analysis, Systems, Applications, Prentice-Hall, 2001.

GAPS IN THE SYLLABUS (TO MEET INDUSTRY/PROFESSION REQUIREMENTS)

AI and machine learning integration in robotics, real-time operating systems, ROS (Robot Operating System), human-robot interaction.

POS MET THROUGH GAPS IN THE SYLLABUS: PO 1-5.

TOPICS BEYOND SYLLABUS/ADVANCED TOPICS/DESIGN

Design and analysis of hybrid robotic systems combining serial and parallel architectures for enhanced performance and flexibility.

POS MET THROUGH TOPICS BEYOND SYLLABUS/ADVANCED TOPICS/DESIGN: PO 1-5, PO 11-12.

COURSE OUTCOME (CO) ATTAINMENT ASSESSMENT TOOLS & EVALUATION PROCEDURE

DIRECT ASSESSMENT

Assessment Tool	% Contribution during CO Assessment
Progressive Evaluation	50
End Semester Examination	50

Continuous Internal Assessment	% Distribution
Mid Semester Examination	25
Quiz, Assignment	10 + 10
Teacher's Assessment	5

Assessment Components	CO1	CO2	CO3	CO4	CO5
Continuous Internal Assessment	√	√	√	√	√
Semester End Examination	√	√	√	√	√

INDIRECT ASSESSMENT

1. **Student Feedback on Course Outcome**

COURSE DELIVERY METHODS

CD1	Lecture by use of boards/LCD projectors/OHP projectors	√
CD2	Assignments/Seminars	√
CD3	Laboratory experiments/teaching aids	
CD4	Industrial/guest lectures	
CD5	Industrial visits/in-plant training	
CD6	Self- learning such as use of NPTEL materials and internets	√
CD7	Simulation	

MAPPING BETWEEN COURSE OUTCOMES AND POs and PSOs

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	3	2	1	1	2	1	1	1	1	2	1	1	2	2	1
CO2	2	3	3	2	3	2	1	1	2	3	2	1	3	3	3
CO3	2	3	2	3	2	3	2	1	1	2	2	1	3	2	3
CO4	2	3	3	3	2	1	1	1	2	3	3	1	2	3	3
CO5	3	3	3	3	3	2	3	2	2	3	3	2	3	3	3

Grading: No correlation – 0, Low correlation - 1, Moderate correlation – 2, High Correlation - 3

MAPPING BETWEEN COURSE OUTCOMES AND COURSE DELIVERY METHOD

Course Outcomes	Course Delivery Method
CO1	CD1, CD2, CD 6
CO2	CD1, CD2, CD 6
CO3	CD1, CD2, CD 6
CO4	CD1, CD2, CD 6
CO5	CD1, CD2, CD 6

ROBOTICS

J J Greg

No of Joints = No. of DOF = no. of links − 1
= no. of actuators

$$\begin{bmatrix} \theta_1 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \phi_x \\ \phi_y \\ \phi_z \end{bmatrix}$$

Joint Variables

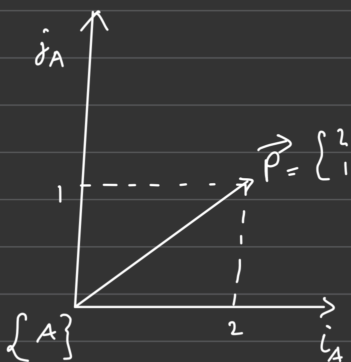
Backward Kinematics

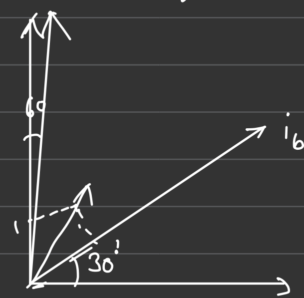
$$\begin{Bmatrix} x \\ y \\ z \\ \phi_x \\ \phi_y \\ \phi_z \end{Bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \vdots \end{Bmatrix}$$

Forward Kinematics
or direct

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1+b_1 \\ a_2+b_2 \\ a_3+b_3 \end{bmatrix}$$

* Cant add vectors when in diff co-ordinate sys.





$\cos 30 \times 1 + 2 = \frac{1.73}{2} + 2$

$1 \cos 60 + 1 = 1.5$

$= (2.86,\ 1.5)$

2.86

$\vec{O} = \hat{i}_B + \hat{j}_B \qquad \vec{P} = 2\hat{i}_A + \hat{j}_A$

$P = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \vec{O} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} {}^A O_x \\ {}^A O_y \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 30 - \sin 30 \\ \sin 30 + \cos 30 \end{bmatrix}$$

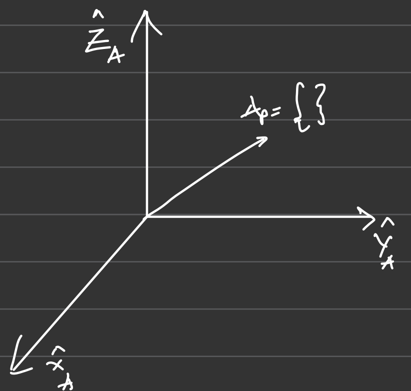
$$\begin{Bmatrix} {}^A P_x \\ {}^A P_y \end{Bmatrix} = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} {}^A P_x \\ {}^A P_y \end{Bmatrix} + \begin{Bmatrix} {}^A O_x \\ {}^A O_y \end{Bmatrix} = \begin{bmatrix} 2.366 \\ 2.360 \end{bmatrix}$$

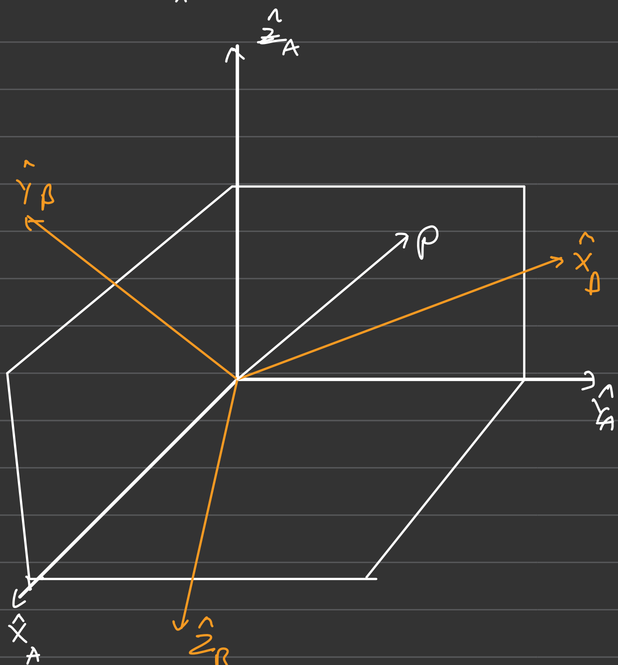
Spatial description

In the study of robotics we are constantly concerned with the location of obj in 3D space. These objects are the links of the manipulator, the parts & tools with which it deals in other objects. This concern can be addressed by the kine of robot arm.

Robo arm kinem is analytical study wrt to a fixed frame as a function of time without regard to the force or moments that cause the motion.



The leading superscript indicates the frame in which the vector is referenced



Let unit vectors in $\{A\}$ are $\hat{X}_A$ $\hat{Y}_A$ $\hat{Z}_A$

" " $\{B\}$ $\hat{X}_B$ $\hat{Y}_B$ $\hat{Z}_B$

$$\vec{P} = {}^{A}P_x \hat{X}_A + {}^{A}P_y \hat{Y}_A + {}^{A}P_z \hat{Z}_A$$

$$\vec{P} = {}^{B}P_x \hat{X}_B + {}^{B}P_y \hat{Y}_B + {}^{B}P_z \hat{Z}_B$$

If ${}^{B}P_x$ ${}^{B}P_y$ ${}^{B}P_z$ are known then ${}^{A}P_x$ ${}^{A}P_y$ ${}^{A}P_z$ can be

$$\vec{P} \cdot \hat{X}_A = {}^{B}P_x \hat{X}_B \cdot \hat{X}_A + {}^{B}P_y \hat{Y}_B \cdot \hat{X}_A + {}^{B}P_z \hat{Z}_B \cdot \hat{X}_A$$

$$\vec{P} \cdot \hat{Y}_A = {}^{B}P_x \hat{X}_B \cdot \hat{Y}_A + {}^{B}P_y \hat{Y}_B \cdot \hat{Y}_A + {}^{B}P_z \hat{Z}_B \cdot \hat{Y}_A$$

$$\vec{P} \cdot \hat{Z}_A = {}^{B}P_x \hat{X}_B \cdot \hat{Z}_A + {}^{B}P_y \hat{Y}_B \cdot \hat{Z}_A + {}^{B}P_z \cdot \hat{Z}_B \cdot \hat{Z}_A$$

$$\begin{Bmatrix} {}^{A}P_x \\ {}^{A}P_y \\ {}^{A}P_z \end{Bmatrix} \begin{bmatrix} \hat{X}_B \cdot \hat{X}_A & \hat{Y}_B \cdot \hat{X}_A & \hat{Z}_B \cdot \hat{X}_A \\ \hat{X}_B \cdot \hat{Y}_A & \hat{Y}_B \cdot \hat{Y}_A & \hat{Z}_B \cdot \hat{Y}_A \\ \hat{X}_B \cdot \hat{Z}_A & \hat{Y}_B \cdot \hat{Z}_A & \hat{Z}_B \cdot \hat{Z}_A \end{bmatrix} \begin{bmatrix} {}^{B}P_x \\ {}^{B}P_y \\ {}^{B}P_z \end{bmatrix}$$

$$= \begin{bmatrix} {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{bmatrix} \begin{bmatrix} {}^{B}P_x \\ {}^{B}P_y \\ {}^{B}P_z \end{bmatrix}$$

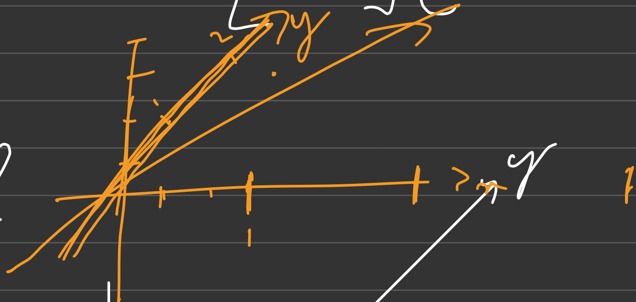
$A \to B$ describes B rel to A.

$$\vec{a} = \hat{i} + 2\hat{j} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

x-comp ↑ by 3 times

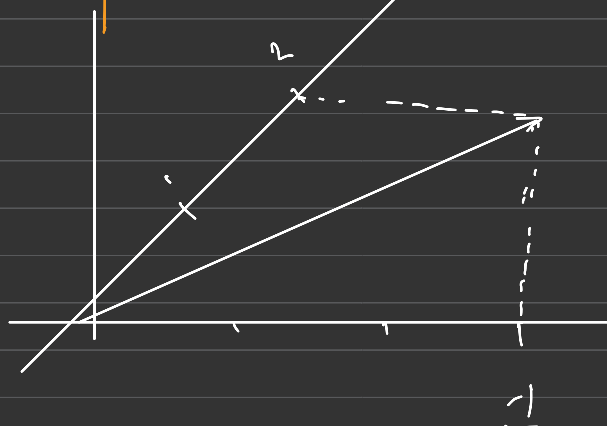
y-comp ↑ by 2 times

$$\vec{b} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$



$$\vec{b} = \begin{array}{c c} & \begin{array}{cc} \downarrow x & \downarrow y \end{array} \\ \begin{array}{c} x \\ y \end{array} & \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \end{array} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

$$= \begin{Bmatrix} 5 \\ 4 \end{Bmatrix} \approx$$



Wolfram Mathematica 9.0 - [Rotation about an arbitrary axis.nb *]

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Rotation about an arbitrary axis.nb *

```
R1[x_] := {{1, 0, 0}, {0, Cos[x], -Sin[x]}, {0, Sin[x], Cos[x]}}
R2[x_] := {{Cos[x], 0, Sin[x]}, {0, 1, 0}, {-Sin[x], 0, Cos[x]}}
R3[x_] := {{Cos[x], -Sin[x], 0}, {Sin[x], Cos[x], 0}, {0, 0, 1}}

R3[x] // MatrixForm
```

$$\begin{pmatrix} \mathrm{Cos}[x] & -\mathrm{Sin}[x] & 0 \\ \mathrm{Sin}[x] & \mathrm{Cos}[x] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
R = Simplify[((R3[α].R2[-β].R1[ϕ].R2[β].R3[-α]) /. {Cos[α] Cos[β] → kx, Sin[α] Cos[β] → ky}) /.
   {Sin[β] → kz, Cos[β] → Sqrt[1 - kz^2], Sin[α] → ky / Sqrt[1 - kz^2], Cos[α] → kx / Sqrt[1 - kz^2]}]
```

$$\Big\{\Big\{\frac{kx^2\,(-1+kz^2)-(ky^2+kx^2\,kz^2)\,\mathrm{Cos}[\phi]}{-1+kz^2},$$
$$\frac{kx\,ky\,(-1+kz^2)-kx\,ky\,(-1+kz^2)\,\mathrm{Cos}[\phi]+(kx^2+ky^2)\,kz\,\mathrm{Sin}[\phi]}{-1+kz^2},\ kx\,kz-kx\,kz\,\mathrm{Cos}[\phi]+ky\,\mathrm{Sin}[\phi]\Big\},$$
$$\Big\{\frac{kx\,ky\,(-1+kz^2)-kx\,ky\,(-1+kz^2)\,\mathrm{Cos}[\phi]-(kx^2+ky^2)\,kz\,\mathrm{Sin}[\phi]}{-1+kz^2},$$
$$\frac{ky^2\,(-1+kz^2)-(kx^2+ky^2\,kz^2)\,\mathrm{Cos}[\phi]}{-1+kz^2},\ ky\,kz-ky\,kz\,\mathrm{Cos}[\phi]-kx\,\mathrm{Sin}[\phi]\Big\},$$
$$\Big\{kx\,kz-kx\,kz\,\mathrm{Cos}[\phi]-ky\,\mathrm{Sin}[\phi],\ ky\,kz-ky\,kz\,\mathrm{Cos}[\phi]+kx\,\mathrm{Sin}[\phi],\ kz^2+\mathrm{Cos}[\phi]-kz^2\,\mathrm{Cos}[\phi]\Big\}\Big\}$$

```
In[6]:= R // MatrixForm
```

Out[6]//MatrixForm=

$$\begin{pmatrix} \frac{kx^2(-1+kz^2)-(ky^2+kx^2 kz^2)\mathrm{Cos}[\phi]}{-1+kz^2} & \frac{kx\,ky(-1+kz^2)-kx\,ky(-1+kz^2)\mathrm{Cos}[\phi]+(kx^2+ky^2)kz\,\mathrm{Sin}[\phi]}{-1+kz^2} & kx\,kz-kx\,kz\,\mathrm{Cos}[\phi] \\ \frac{kx\,ky(-1+kz^2)-kx\,ky(-1+kz^2)\mathrm{Cos}[\phi]-(kx^2+ky^2)kz\,\mathrm{Sin}[\phi]}{-1+kz^2} & \frac{ky^2(-1+kz^2)-(kx^2+ky^2 kz^2)\mathrm{Cos}[\phi]}{-1+kz^2} & ky\,kz-ky\,kz\,\mathrm{Cos}[\phi] \\ kx\,kz-kx\,kz\,\mathrm{Cos}[\phi]-ky\,\mathrm{Sin}[\phi] & ky\,kz-ky\,kz\,\mathrm{Cos}[\phi]+kx\,\mathrm{Sin}[\phi] & kz^2+\mathrm{Cos}[\phi]-kz \end{pmatrix}$$

```
Simplify[R[[1, 1]] /. ky^2 → (1 - kx^2 - kz^2)]
```

$kx^2 + \mathrm{Cos}[\phi] - kx^2\,\mathrm{Cos}[\phi]$

```
Simplify[R[[1, 2]] /. ky^2 → (1 - kx^2 - kz^2)]
```

$kx\,ky - kx\,ky\,\mathrm{Cos}[\phi] - kz\,\mathrm{Sin}[\phi]$

```
Simplify[R[[2, 1]] /. ky^2 → (1 - kx^2 - kz^2)]
```

$kx\,ky - kx\,ky\,\mathrm{Cos}[\phi] + kz\,\mathrm{Sin}[\phi]$

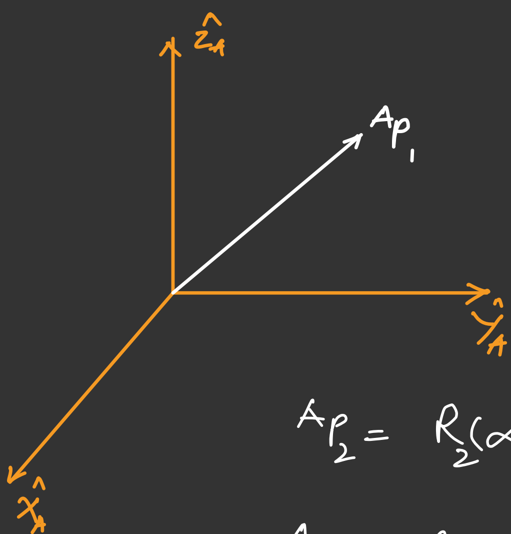
Composite Rotⁿ (* thoda kam samajh aaya)

i) Initialise the rot mat

$R = I$, i.e, {A} & {B} are coinciding

ii) If {B} is to be rotated by an amount ϕ about the k^{th} unit vector of the fixed frame {A}, then pre multiply R with R with $R_k(\phi)$

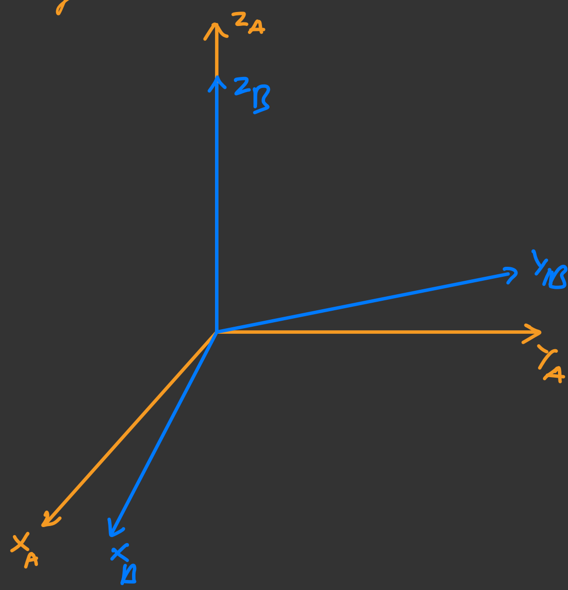
(iii) If B is to be rotated by an amount ϕ about its own k^{th} unit vector, then post multiply R with $R_k(\phi)$



$${}^{A}P_2 = R_z(\alpha)\ {}^{A}P_1$$

$${}^{A}P_3 = R_y(\beta)\ R_z(\alpha)\ {}^{A}P_1$$

(* now rotating the axis itself)



now consider a fram C rotated about Y_B by β

Position vector of P in (C)

$$\begin{Bmatrix} {}^{B}P_x \\ {}^{B}P_y \\ {}^{B}P_z \end{Bmatrix} = {}^{B}_{C}R \begin{Bmatrix} {}^{C}P_x \\ {}^{C}P_y \\ {}^{C}P_z \end{Bmatrix}$$

$$\begin{Bmatrix} {}^{A}P_x \\ {}^{A}P_y \\ {}^{A}P_z \end{Bmatrix} = {}^{A}_{B}R \begin{Bmatrix} {}^{B}P_x \\ {}^{B}P_y \\ {}^{B}P_z \end{Bmatrix}$$

$${}^{A}P = {}^{A}_{B}R \cdot {}^{B}_{C}R \; {}^{C}P$$

$${}^{A}_{C}R = {}^{A}_{B}R \cdot {}^{B}_{C}R$$

* <u>x-y-z fixed angles (Roll - Pitch - Yaw)</u>

One method of describing the orien^n of a fram B is :→

- Start with the frame coinciding with a known ref fr A. rotate B first about X_A by an angle "γ", then about Y_A by an angle β & finally about Z_A by angle "α"

$${}^{A}_{B}R_{xyz}(\gamma,\beta,\alpha) = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma)$$

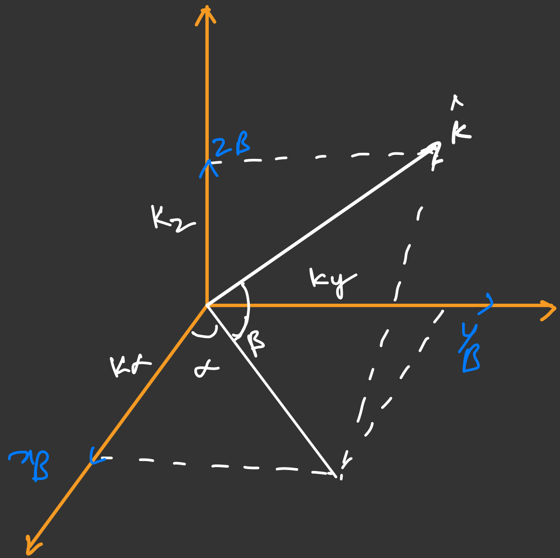
$$= \begin{bmatrix} \cos\alpha\cdot\cos\beta & \cos\alpha\cdot\sin\beta\cdot\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\cdot\sin\beta\cdot\cos\gamma + \sin\alpha\cdot\sin\gamma \\ \sin\alpha\cdot\cos\beta & \sin\alpha\cdot\sin\beta\cdot\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\cdot\sin\beta\cdot\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\cdot\sin\gamma & \cos\beta\cdot\sin\gamma \end{bmatrix}$$

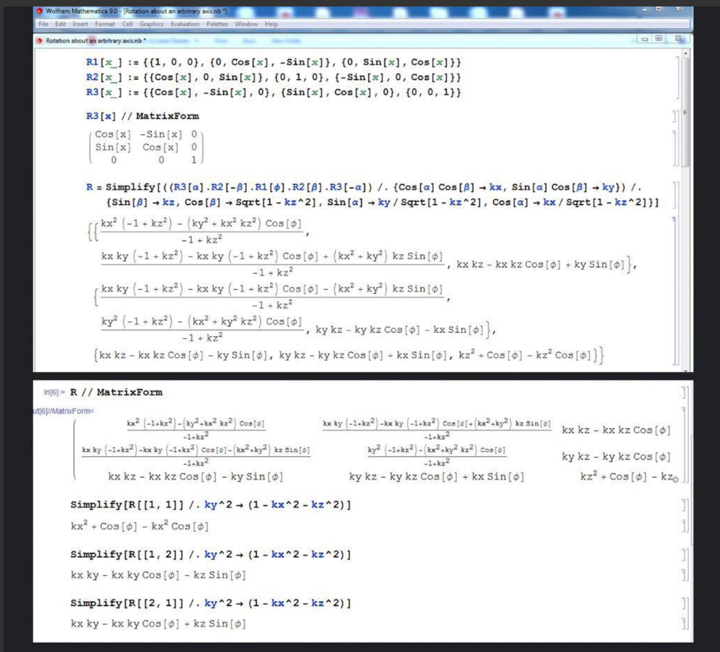
* Z-Y-X Euler angle rot^n

Another poss descr^n of from B as follows:-

- Start with a frame coincident with known frame A
- Rotate B first about Z_B by an angle α, then about Y_B by angle β & finally about X_B by angle γ

$${}^{A}_{B}R'(\alpha,\beta,\gamma) = R_z(\alpha)\cdot R_y(\beta)\cdot R_x(\gamma)$$





Wolfram Mathematica 9.0 - [Rotation about an arbitrary axis.nb *]

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Rotation about an arbitrary axis.nb *

```
R1[x_] := {{1, 0, 0}, {0, Cos[x], -Sin[x]}, {0, Sin[x], Cos[x]}}
R2[x_] := {{Cos[x], 0, Sin[x]}, {0, 1, 0}, {-Sin[x], 0, Cos[x]}}
R3[x_] := {{Cos[x], -Sin[x], 0}, {Sin[x], Cos[x], 0}, {0, 0, 1}}

R3[x] // MatrixForm
```

$$\begin{pmatrix} \mathrm{Cos}[x] & -\mathrm{Sin}[x] & 0 \\ \mathrm{Sin}[x] & \mathrm{Cos}[x] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
R = Simplify[((R3[α].R2[-β].R1[ϕ].R2[β].R3[-α]) /. {Cos[α] Cos[β] → kx, Sin[α] Cos[β] → ky}) /.
   {Sin[β] → kz, Cos[β] → Sqrt[1 - kz^2], Sin[α] → ky / Sqrt[1 - kz^2], Cos[α] → kx / Sqrt[1 - kz^2]}]
```

$$\Big\{\Big\{\frac{kx^2\,(-1+kz^2)-(ky^2+kx^2\,kz^2)\,\mathrm{Cos}[\phi]}{-1+kz^2},\ \frac{kx\,ky\,(-1+kz^2)-kx\,ky\,(-1+kz^2)\,\mathrm{Cos}[\phi]+(kx^2+ky^2)\,kz\,\mathrm{Sin}[\phi]}{-1+kz^2},\ kx\,kz-kx\,kz\,\mathrm{Cos}[\phi]+ky\,\mathrm{Sin}[\phi]\Big\},$$
$$\Big\{\frac{kx\,ky\,(-1+kz^2)-kx\,ky\,(-1+kz^2)\,\mathrm{Cos}[\phi]-(kx^2+ky^2)\,kz\,\mathrm{Sin}[\phi]}{-1+kz^2},\ \frac{ky^2\,(-1+kz^2)-(kx^2+ky^2\,kz^2)\,\mathrm{Cos}[\phi]}{-1+kz^2},\ ky\,kz-ky\,kz\,\mathrm{Cos}[\phi]-kx\,\mathrm{Sin}[\phi]\Big\},$$
$$\{kx\,kz-kx\,kz\,\mathrm{Cos}[\phi]-ky\,\mathrm{Sin}[\phi],\ ky\,kz-ky\,kz\,\mathrm{Cos}[\phi]+kx\,\mathrm{Sin}[\phi],\ kz^2+\mathrm{Cos}[\phi]-kz^2\,\mathrm{Cos}[\phi]\}\Big\}$$

In[6]:= `R // MatrixForm`

Out[6]//MatrixForm=

$$\begin{pmatrix} \frac{kx^2\,(-1+kz^2)-(ky^2+kx^2\,kz^2)\,\mathrm{Cos}[\phi]}{-1+kz^2} & \frac{kx\,ky\,(-1+kz^2)-kx\,ky\,(-1+kz^2)\,\mathrm{Cos}[\phi]+(kx^2+ky^2)\,kz\,\mathrm{Sin}[\phi]}{-1+kz^2} & kx\,kz-kx\,kz\,\mathrm{Cos}[\phi] \\ \frac{kx\,ky\,(-1+kz^2)-kx\,ky\,(-1+kz^2)\,\mathrm{Cos}[\phi]-(kx^2+ky^2)\,kz\,\mathrm{Sin}[\phi]}{-1+kz^2} & \frac{ky^2\,(-1+kz^2)-(kx^2+ky^2\,kz^2)\,\mathrm{Cos}[\phi]}{-1+kz^2} & ky\,kz-ky\,kz\,\mathrm{Cos}[\phi] \\ kx\,kz-kx\,kz\,\mathrm{Cos}[\phi]-ky\,\mathrm{Sin}[\phi] & ky\,kz-ky\,kz\,\mathrm{Cos}[\phi]+kx\,\mathrm{Sin}[\phi] & kz^2+\mathrm{Cos}[\phi]-kz \end{pmatrix}$$

```
Simplify[R[[1, 1]] /. ky^2 → (1 - kx^2 - kz^2)]
```

$kx^2+\mathrm{Cos}[\phi]-kx^2\,\mathrm{Cos}[\phi]$

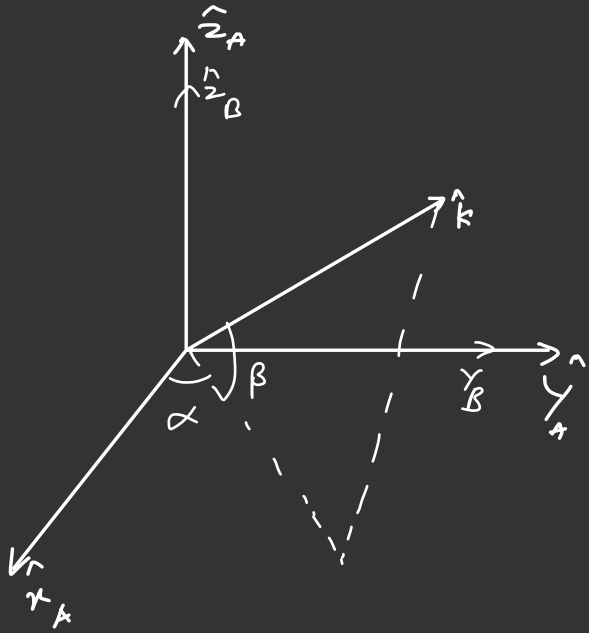
```
Simplify[R[[1, 2]] /. ky^2 → (1 - kx^2 - kz^2)]
```

$kx\,ky-kx\,ky\,\mathrm{Cos}[\phi]-kz\,\mathrm{Sin}[\phi]$

```
Simplify[R[[2, 1]] /. ky^2 → (1 - kx^2 - kz^2)]
```

$kx\,ky-kx\,ky\,\mathrm{Cos}[\phi]+kz\,\mathrm{Sin}[\phi]$

Rotation About An Arbitrary Axis



Let ${}^A\hat{k} = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$

* we begin with {B} coinciding with {A} ${}^A_B R = I$

<u>Step-1</u> Rotate $\hat{k}$ about $\hat{z}_A$ by an angle $(-\alpha)$ so that $\hat{k}$ falls in $\hat{X}_A \cdot \hat{Z}_A$ plane

Step 2 Rotate $\hat{k}$ about $\hat{Y}_A$ by an angle β so that $\hat{k}$ is colinear with $\hat{X}_A$

The new $^{A}_{B}R = R_y(\beta).\ R_z(\alpha)$

Step 3 Now rotate $\{B\}$ about $\hat{X}_A$ by an angle β

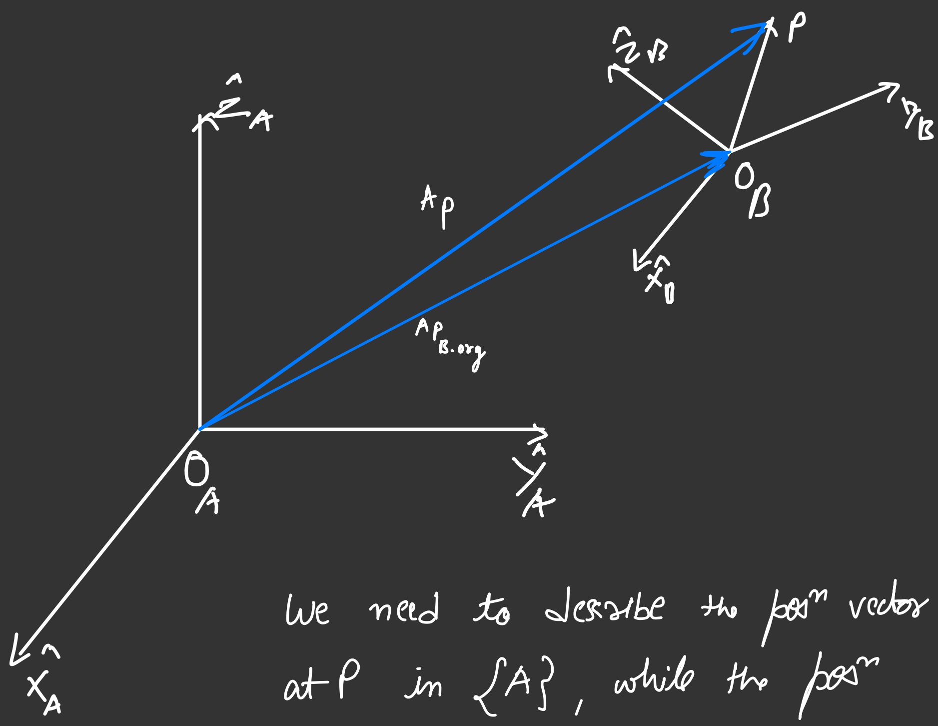
$$^{A}_{B}R = R_x(\phi)\, R_y(\beta)\, R_z(-\alpha)$$

finally

$$^{A}_{B}R = R_z(\alpha)\, R_y(-\beta)\, R_x(\phi)\, R_y(\beta)\, R_z(\alpha)$$

$$R_K(\phi) = \begin{bmatrix} k_x^2\, v\phi + c\phi & k_x k_y\, v\phi - k_z s\phi & k_x k_z v\phi + k_y s(\phi) \\ k_x k_y\, v\phi + k_z\, s(\phi) & k_x k_y\, v(\phi) + c\phi & k_y k_z v\phi - k_x\, s(\phi) \\ k_x k_z\, v(\phi) - k_y\, s\phi & k_y k_z\, v(\phi) + k_x s\phi & k_x k_z v\phi + c\phi \end{bmatrix}$$

Q. Under what condⁿ do 2 rotⁿ matrices represent finite rotⁿ commute?



We need to describe the posⁿ vector at P in {A}, while the posⁿ vector is {B} is given

$$^B P = \begin{Bmatrix} ^B P_x \\ ^B P_y \\ ^B P_z \end{Bmatrix}$$

$$^{A}P = {}^{A}P_{B.org} + {}^{A}_{B}R \, {}^{B}P$$

Q) A fram B is rotated relative to frame A about Z_A by 30° count clock translated 10 units along X and 5 units along Y. Find ^{A}P if ^{B}P is $\begin{Bmatrix} 3 \\ 7 \\ 0 \end{Bmatrix}$

$$^{A}P_{B.org} = \begin{Bmatrix} 10 \\ 5 \end{Bmatrix}$$

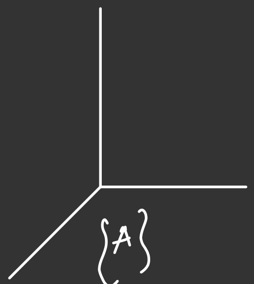
$$^{A}P_{Borg} = \sqrt{125}$$

$$^{A}P = {}^{A}P_{B.org} + {}^{A}_{B}R \, {}^{B}P$$

$$= \begin{Bmatrix} 10 \\ 5 \\ 0 \end{Bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix}$$

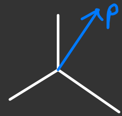
$$= \begin{Bmatrix} 10 \\ 5 \\ 0 \end{Bmatrix} + \begin{bmatrix} -0.901 \\ 7.562 \\ 0 \end{bmatrix} = \begin{bmatrix} 9.099 \\ 12.562 \\ 0 \end{bmatrix}$$

#



$\{A\}$

$\{B\}$



$\{C\}$

$^{B}p = {}^{B}p_{C\,org} + {}^{B}_{C}R\,{}^{C}p$

$^{A}p = {}^{A}p_{B.org} + {}^{A}_{B}R\,{}^{B}p$

→ Homogeneous Transformation

$$\begin{bmatrix} ^{A}p_x \\ ^{A}p_y \\ ^{A}p_z \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} {}^{A}_{B}R & {}^{A}p_{B.org} \\ \hline 0\ \ 0\ \ 0 & 1 \end{array}\right] \begin{Bmatrix} ^{B}p_x \\ ^{B}p_y \\ ^{B}p_z \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} ^{B}p \\ 1 \end{Bmatrix} = {}^{B}_{C}T \begin{Bmatrix} ^{C}p \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} ^{A}p \\ 1 \end{Bmatrix} = {}^{A}_{B}T\,{}^{B}_{C}T \begin{Bmatrix} ^{C}p \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} ^{A}p \\ 1 \end{Bmatrix} = {}^{A}_{B}T \begin{Bmatrix} ^{B}p \\ 1 \end{Bmatrix}$$

i) a "1" is added as the last element of the 4×1 vector

ii) a row "[0 0 0 1]"

2.15

$${}^{A}_{B}R = R_k(\theta)$$

$$R_k(\Delta\theta)\, R_k(\theta) = R_k(\theta+\Delta\theta)$$

$$R_k(\theta+\Delta\theta) - R_k(\theta) = \left(R_k(\Delta\theta) - I\right) R_k(\theta)$$

$$= \left(\begin{bmatrix} 1 & -k_z\Delta\theta & k_y\Delta\theta \\ k_z\Delta\theta & 1 & -k_x\Delta\theta \\ -k_y\Delta\theta & k_x\Delta\theta & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) R_k(\theta)$$

$$R_k(\theta+\Delta\theta) - R_k(\theta) = \Delta\theta \cdot k \cdot R_k(\theta)$$

$$\lim_{\Delta\theta\to 0} \frac{R_k(\theta+\Delta\theta) - R_k(\theta)}{\Delta\theta} = \Big|_{\Delta\theta\to 0} k \cdot R_k(\theta)$$

$$\frac{dR_k(\theta)}{d\theta} = k\, R_k(\theta)$$

$$\int_I^{R_k(\theta)} \frac{dR_k(\theta)}{R_k(\theta)} = \int_0^{\theta} k \cdot d\theta$$

$$\ln R_k(\theta) - \ln I = k\theta$$

$$\ln I = C$$

$$I = e^{C}$$

$$= I + C + \frac{C^2}{L^2} + \cdots$$

$$0 = C + \frac{C^2}{L^2} + \frac{C^3}{L^3}$$

$$C = 0$$

If λ_i are eigen values of A then eigen values of e^A are e^{λ_i}

$$Au = \lambda u$$

$$e^A u = \left(I + A + \frac{A^2}{\lfloor 2} + \text{---} \right) u$$

$$= \left(I + \lambda + \frac{\lambda^2}{\lfloor 2} + \frac{\lambda^3}{\lfloor 3} - - - \right) u$$

$$= e^{\lambda} u$$

$$A^2 u = AAu$$
$$= A\lambda u$$
$$= Au\lambda$$
$$= \lambda^2 u$$

Eigen values of $K\theta$

$$\det\left(\begin{bmatrix} 0 & -k_z\theta & k_y\theta \\ k_z\theta & 0 & -k_x\theta \\ -k_y\theta & k_z\theta & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) = 0$$

$$\begin{bmatrix} -\lambda & -k_z\theta & k_y\theta \\ k_z\theta & -\lambda & -k_x\theta \\ -k_y\theta & k_x\theta & -\lambda \end{bmatrix}$$

$$\Rightarrow -\lambda(\lambda^2 + K_x^2\theta^2) + K_z\theta(-K_z\theta - K_xK_y\theta^2) + K_y\theta(K_xK_z\theta^2 - \lambda K_y\theta) = 0$$

$$-\lambda^3 - (K_x^2 + K_y^2 + K_z^2)\lambda\theta^2 = 0$$

$$\lambda^3 + \lambda\theta^2 = 0$$

$$\lambda = 0, \ \pm i\theta$$

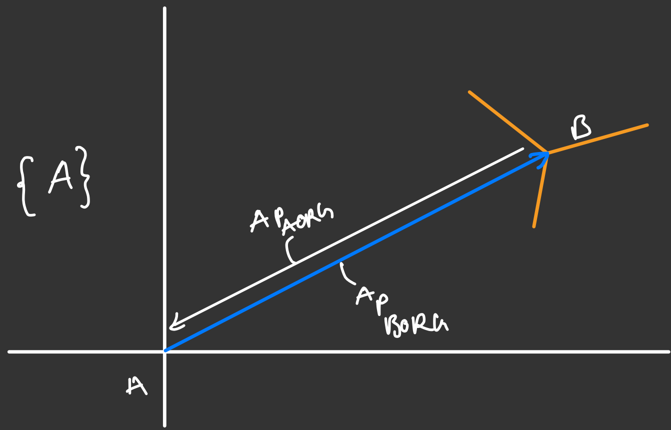
$$\therefore \quad R_K(\theta) = e^{K\theta}$$

Eigenvalues of $e^{K\theta}$ are $e^0, e^{\pm i\theta}$

or 1 & $e^{\pm i\theta}$

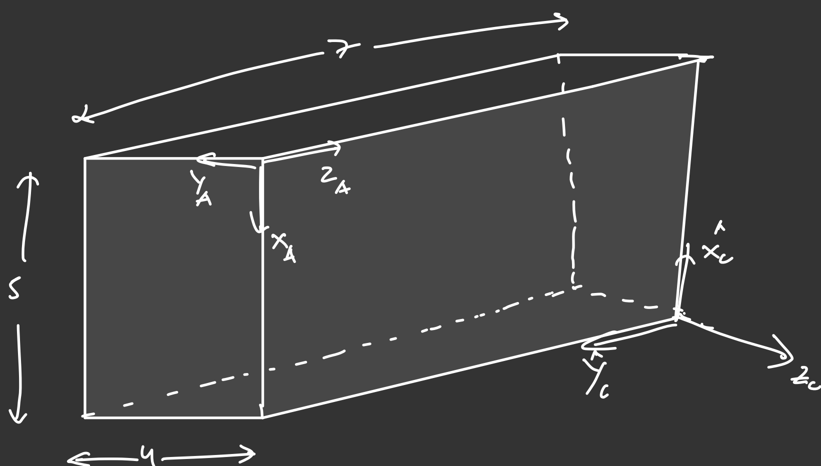
$${}^{A}_{B}T = \left[\begin{array}{c|c} {}^{A}_{B}R & {}^{A}P_{BORG} \\ \hline 0 \ 0 \ 0 & 1 \end{array}\right]$$

$${}^{B}_{A}T = \left[\begin{array}{c|c} {}^{B}_{A}R & {}^{B}P_{AORG} \\ \hline 0 \ 0 \ 0 & 1 \end{array}\right]$$



$$^A P_{AORG} = - {}^A P_{BORG}$$

$$^B P_{AORG} = - {}^B_A R \cdot {}^A P_{BORG}$$



$$R = R_y(180)\, R_z(90)$$

$$^C P_{BORG} = \begin{Bmatrix} 5 \\ 2 \\ 0 \end{Bmatrix}$$

$$
{}^{C}R_{A} = \left[\hat{C}_{x_A} \quad \hat{C}_{y_A} \quad \hat{C}_{z_A} \right]
$$

$$
= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}
$$

$$
\vec{v}_1 \xrightarrow{\text{mapped}} R v_1
$$

$$
\vec{v}_2 \xrightarrow{\text{mapped}} R v_2
$$

$$
\vec{v}_1 \cdot \vec{v}_2 = v_1^T v_2
$$

$$
(R v_1)^T R v_2 = v_1^T v_2
$$

$$
\cancel{v_1^T} R^T \cdot R \cancel{v_2} = \cancel{v_1^T} \cancel{v_2}
$$

$$
\cancel{v_1^T} \cancel{I v_2 = v_1^T v_2}
$$

$$
R^T R = I
$$

$$
\boxed{R^T = R^{-1}}
$$

Manipulator kinematics

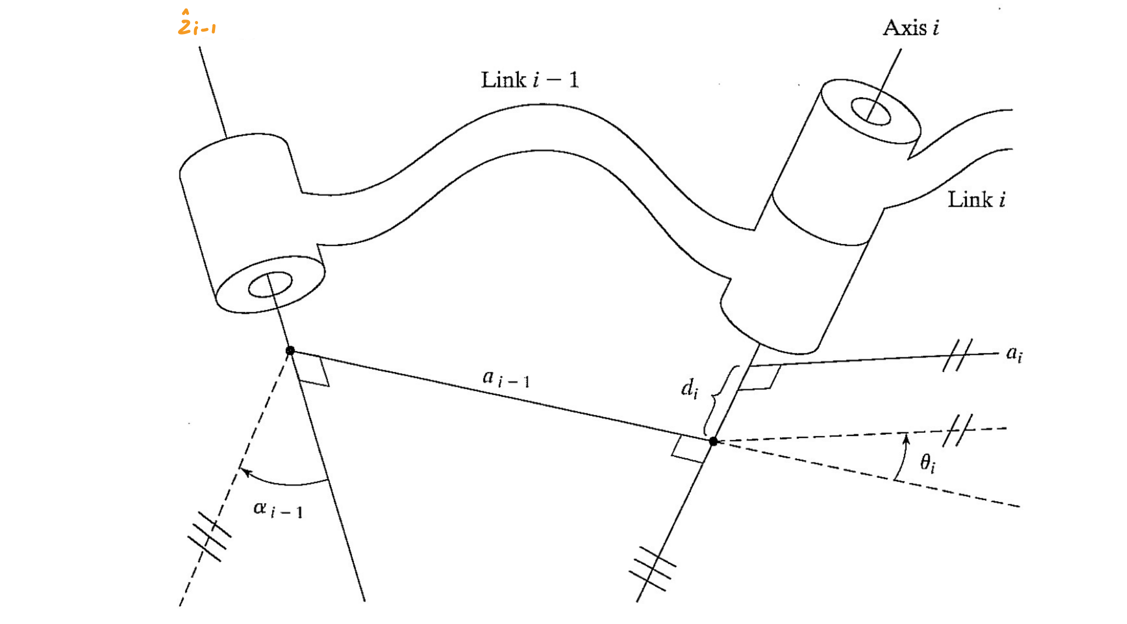
A robot manipulator
connected by usually lower pair (1DOF) say a revolute or prismatic joint
In order to control the end effector wrt the base, its necessary to find
reln b/w coordinate frames attached to end effector & the base

Obj

Compute the posn & orientation of the manipulator's end effector
relative to the base of the manipulator as a function of
joint variables connected to the base of the manipulator
as a fun of joint variables. For this purpose a systematic general
method is to be derived to define the relative posn & orientation of
2 consecutive links

(ii) Choose axis Z_i along the axis of joint I whose positive direction can be taken towards either direction

(iii) Locate origin O_{i-1} intersection of Z_{i-1} with the common normal to Z_{i-2} & Z_i along the common normal with the direction former to later

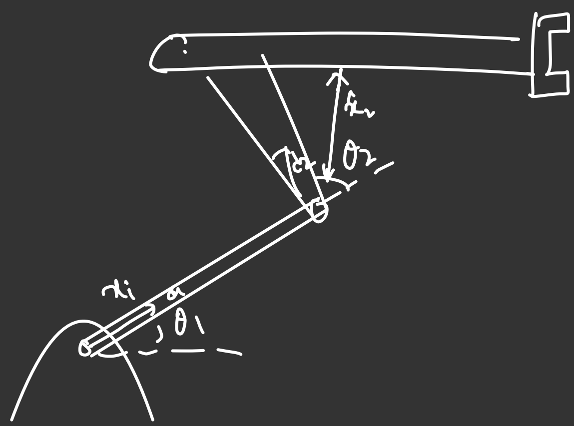


If $\hat{z}_i$ & $\hat{z}_{i-1}$ intersect then dir^n of x_i & x_{i-1} can be taken arbitrarily along $z_i \times z_{i-1}$ or $\hat{z}_{i-1} \times \hat{z}_i$.

The parameters α_{i-1} is the rotation of z_i measured from z_{i-1} along x_{i-1}. It is also known as link twist

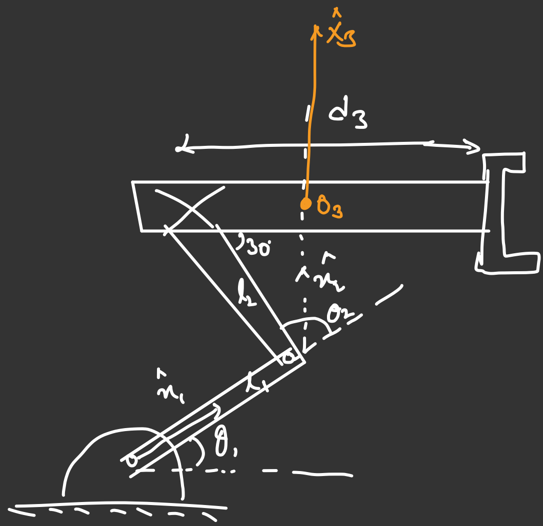
link offset → d_i is the distance from x_{i-1} to x_i measured alo z_i

The joint angle θ_i is angle b/w x_i & x_{i-1} measured about z_i



For frame I is attached to the fixed base i.e link #0.
Only the dirn of of axis z_i is specified then θ_1 & x_0
can be chosen arbitrarily, when there is no actuation $\alpha_1 = 0$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	l_1	0	θ_2
3	90	$l_2 \sin\theta_2$	d_3	30°



For 1^{st} Joint
$a_0 = 0$; $\alpha_0 = 0$
$d_1 = 0$ for revolute joint
or $\theta_1 = 0$ for prismatic joint.

for Joint # n (last joint)

If # n is revolute, the dir^n of $\hat{X}_n$ is chosen so that it alligns with $\hat{X}_{n-1}$ when $\theta_n = 0$ & the origin of # n is chosen so that $d_n = 0$

If # n is prismatic, the dir^n of X_n is chosen so that $\theta_n = 0$ & origin of and origin of frame n is chosen at the intersection of $\hat{X}_{n-1}$ & the joint axis of n i.e Z_n

$${}^{i-1}_{i}T = \text{Screw}_x(a_{i-1}; \alpha_{i-1})\ \text{Screw}_z(d_i; \theta_i)$$

$$\left[\begin{array}{ccc|c} & R_x(\alpha\text{-}i) & & a_{i-1} \\ & & & 0 \\ & & & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} & R_z(\alpha) & & 0 \\ & & & 0 \\ & & & d_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} c\theta_1 & -s\theta_i & 0 & a_{i-1} \\ s\theta_1 \, c\alpha_{i-1} & c\theta_1 \cdot c\alpha_{i-1} & -s\alpha_{i-1} & -d_i \, s\alpha_{i-1} \\ s\theta_i \, s\alpha_{i-1} & c\theta_i \, s\alpha_{i-1} & c\alpha_{i-1} & d_i \, c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_3T = {}^0_1T \; {}^1_2T \; {}^2_3T$$

$${}^0_1T = \text{Screw}_x(a_0, \alpha_0) \times \text{Screw}_z(d_1, \theta_1)$$

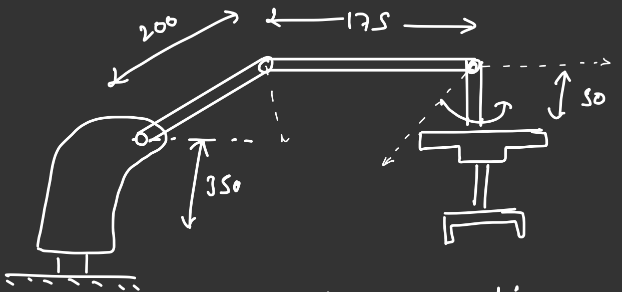
$a_0 = 0$

$\alpha_0 = 0$

$d_1 = 0$

$\theta_1 = 0$

$$\begin{bmatrix} c\theta_1 & s\theta_1 & 0 & a_0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^0_1T$$



Alpha II robotic arm

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90	a_1	0	θ_2
3	0	200	0	θ_3
4	0	175	0	θ_4
5	90	0	0	θ_5

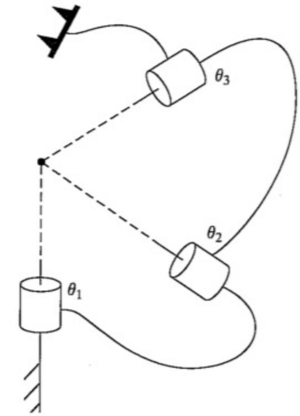


FIGURE 3.35: Schematic of a 3R manipulator (Exercise 3.15).

9?

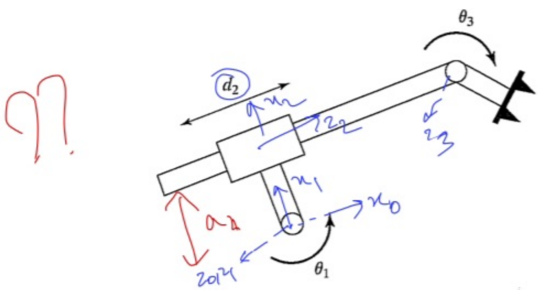
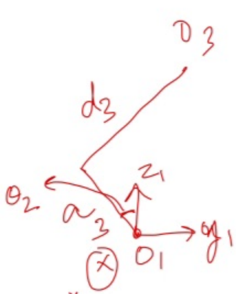
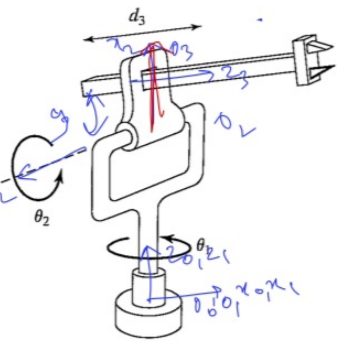


FIGURE 3.36: RPR planar robot (Exercise 3.16).

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90	a_2	d_2	0
3	-90	0	0	θ_3





	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	0	θ_2
3	90°	a_3	d_3	0

${}^0_3T =$

FIGURE 3.37: Three-link RRP manipulator (Exercise 3.17).

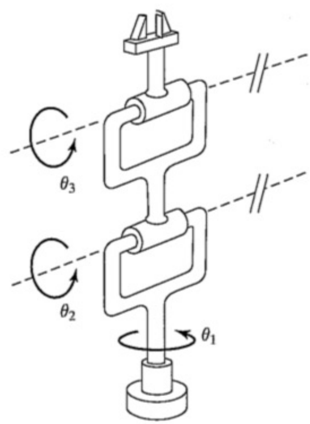


FIGURE 3.38: Three-link RRR manipulator (Exercise 3.18).

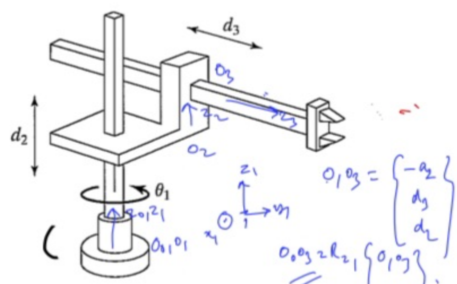


FIGURE 3.39: Three-link RPP manipulator (Exercise 3.19).

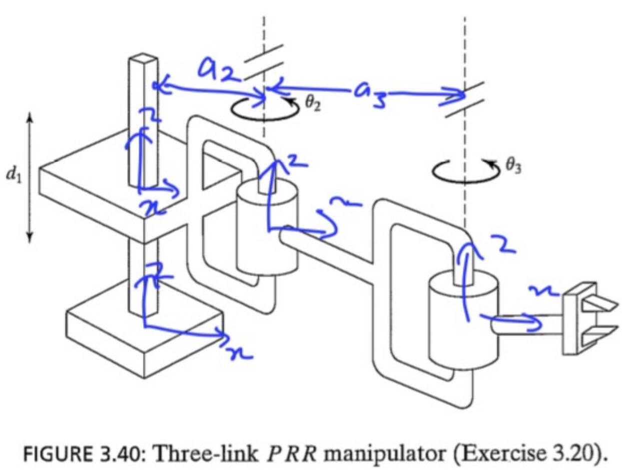


FIGURE 3.40: Three-link PRR manipulator (Exercise 3.20).

2.15

$${}^A_BR = R_K(\theta) = e^{K\alpha}$$

$$R_K(\Delta\theta)\, R_K(\theta) = R_K(\theta+\Delta\theta)$$

$$R_K(\theta+\Delta\theta) - R_K(\theta) = \left(R_K(\Delta\theta) - I\right) R_K(\theta)$$

$$= \left(\begin{bmatrix} 1 & -k_z\Delta\theta & k_y\Delta\theta \\ k_z\Delta\theta & 1 & -k_x\Delta\theta \\ -k_y\Delta\theta & k_x\Delta\theta & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) R_K(\theta)$$

$$R_K(\theta+\Delta\theta) - R_K(\theta) = \Delta\theta \cdot K \cdot R_K(\theta)$$

$$\begin{matrix} 0 & -k_z & k_y \\ & 0 & -k_x \\ & & 0 \end{matrix}$$

$$\lim_{\Delta\theta\to 0} \frac{R_K(\theta+\Delta\theta) - R_K(\theta)}{\Delta\theta} = \lim_{\Delta\theta\to 0} K \cdot R_K(\theta)$$

$$\frac{dR_K(\theta)}{d\theta} = K\, R_K(\theta)$$

$$\int_I^{R_K(\theta)} \frac{dR_K(\theta)}{R_K(\theta)} = \int_0^\theta K \cdot d\theta$$

$$\ln R_K(\theta) - \cancel{\ln I}^{\,0} = K\theta$$

$$\ln I = C$$

$$\ln R_K(\theta) = K\theta$$

rotation matrix that is equivalent to the Z–Y–Z Euler-angle set (α, β, γ). (The result is given by (2.72).)

2.20 [20] Imagine rotating a vector Q about a vector $\hat{K}$ by an amount θ to form a new vector, Q'—that is,

$$Q' = R_K(\theta)Q.$$

Use (2.80) to derive **Rodriques's formula,**

$$Q' = Q\cos\theta + \sin\theta(\hat{K} \times Q) + (1 - \cos\theta)(\hat{K} \cdot \hat{Q})\hat{K}.$$

read the approx again

2.21 [15] For rotations sufficiently small that the approximations $\sin\theta = \theta$, $\cos\theta = 1$, and $\theta^2 = 0$ hold, derive the rotation-matrix equivalent to a rotation of θ about a general axis, $\hat{K}$. Start with (2.80) for your derivation.

2.22 [20] Using the result from Exercise 2.21, show that two infinitesimal rotations commute (i.e., the order in which the rotations are performed is not important).

2.23 [25] Give an algorithm to construct the definition of a frame ${}^U_AT$ from three points ${}^UP_1$, ${}^UP_2$, and ${}^UP_3$, where the following is known about these points:

$m = {}^UP_3 - {}^UP_1$

$^UP_{A\,org} = {}^UP_1$

1. ${}^UP_1$ is at the origin of $\{A\}$;
2. ${}^UP_2$ lies somewhere on the positive $\hat{X}$ axis of $\{A\}$;
3. ${}^UP_3$ lies near the positive $\hat{Y}$ axis in the XY plane of $\{A\}$.

(Rot) $\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \end{bmatrix}$

check Six's method

$\hat{z} = \frac{m \times n}{|m \times n|}$

2.24 [45] Prove Cayley's formula for proper orthonormal matrices.

2.25 [30] Show that the eigenvalues of a rotation matrix are 1, $e^{\alpha i}$, and $e^{-\alpha i}$, where $i = \sqrt{-1}$.

2.26 [33] Prove that any Euler-angle set is sufficient to express all possible rotation matrices.

2.27 [15] Referring to Fig. 2.25, give the value of ${}^A_BT$.

$\hat{n} = \frac{\vec{n}}{|n|}$

2.28 [15] Referring to Fig. 2.25, give the value of ${}^A_CT$.

2.29 [15] Referring to Fig. 2.25, give the value of ${}^B_CT$.

$\hat{n} = \frac{{}^UP_2 - {}^UP_1}{|{}^UP_2 - {}^UP_1|}$

2.30 [15] Referring to Fig. 2.25, give the value of ${}^C_AT$.

2.31 [15] Referring to Fig. 2.26, give the value of ${}^A_BT$.

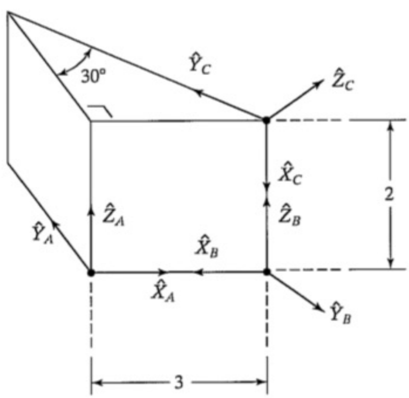


FIGURE 2.25: Frames at the corners of a wedge.

$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad |R - \lambda I| = 0$$

$$R - \lambda I = \begin{bmatrix} \cos\alpha - \lambda & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha - \lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$$(1-\lambda)\left((\cos\alpha - \lambda)^2 + \sin^2\alpha\right) = 0$$

$$(1-\lambda)\left(\underline{\cos^2\alpha} + \lambda^2 - 2\cdot\cos\alpha\cdot\lambda + \underline{\sin^2\alpha}\right) = 0$$

$$(1-\lambda)\left(1 + \lambda^2 - 2\cos\alpha\,\lambda\right) = 0$$

$$(1-\lambda)\left(\lambda^2 - 2\cos\alpha + 1\right) = 0$$

$$\lambda = \frac{\cancel{2}\cos\alpha \pm \sqrt{\cancel{4}\cos^2\alpha - \cancel{4}}}{\cancel{2}}$$

$$= \boxed{\cos\alpha \pm i\sin\alpha} \qquad 1$$

$$= e^{\alpha i},\ e^{-\alpha i}$$

$$\begin{bmatrix} \cos\alpha - \lambda & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha - \lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$$(\cos\alpha - \lambda)\left[(\cos\alpha - \lambda)(1-\lambda)\right] + \left[\sin\alpha\left(\sin\alpha(1-\lambda)\right)\right] = 0$$

$$(\cos\alpha - \lambda)\left[\cos\alpha - \lambda\cdot\cos\alpha - \lambda + \lambda^2\right] + \sin^2\alpha - \lambda\sin^2\alpha = 0$$

$$\underline{\cos^2\alpha} - \underline{\lambda\cos^2\alpha} - \lambda\cos\alpha + \underline{\lambda^2\cos\alpha} - \lambda\cdot\cos\alpha + \underline{\lambda^2\cos\alpha} + \lambda^2 \underline{-\lambda^3}$$

$$\underline{+\sin^2\alpha} - \underline{\lambda\sin^2\alpha} = 0$$

$$1 - \underline{\lambda} - \underline{\lambda^3} + \underline{2\lambda^2}\cos\alpha - 2\underline{\lambda\cdot\cos\alpha} = 0$$

$$\lambda^3 - 2\lambda^2\cos\alpha - \lambda(1+2\cos\alpha) + 1 = 0$$

$$\lambda^2(\lambda - 2\cos\alpha) - \lambda(1+2\cos\alpha) + 1 = 0$$

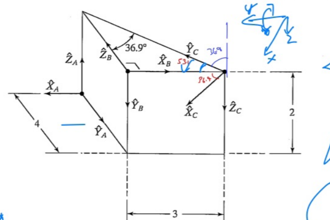


FIGURE 2.26: Frames at the corners of a wedge.

2.32 [15] Referring to Fig. 2.26, give the value of ${}^{A}_{C}T$.

2.33 [15] Referring to Fig. 2.26, give the value of ${}^{B}_{C}T$.

2.34 [15] Referring to Fig. 2.26, give the value of ${}^{C}_{A}T$.

2.35 [20] Prove that the determinant of any rotation matrix is always equal to 1.

2.36 [36] A rigid body moving in a plane (i.e., in 2-space) has three degrees of freedom. A rigid body moving in 3-space has six degrees of freedom. Show that a body in N-space has $\frac{1}{2}(N^2 + N)$ degrees of freedom.

2.37 [15] Given

$$
{}^{A}_{B}T = \begin{bmatrix} 0.25 & 0.43 & 0.86 & 5.0 \\ 0.87 & -0.50 & 0.00 & -4.0 \\ 0.43 & 0.75 & -0.50 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

what is the (2,4) element of ${}^{B}_{A}T$?

2.38 [25] Imagine two unit vectors, v_1 and v_2, embedded in a rigid body. Note that, no matter how the body is rotated, the geometric angle between these two vectors is preserved (i.e., rigid-body rotation is an "angle-preserving" operation). Use this fact to give a concise (four- or five-line) proof that the inverse of a rotation matrix must equal its transpose and that a rotation matrix is orthonormal.

2.39 [37] Give an algorithm (perhaps in the form of a C program) that computes the unit quaternion corresponding to a given rotation matrix. Use (2.91) as starting point.

2.40 [33] Give an algorithm (perhaps in the form of a C program) that computes the Z–X–Z Euler angles corresponding to a given rotation matrix. See Appendix B.

2.41 [33] Give an algorithm (perhaps in the form of a C program) that computes the X–Y–X fixed angles corresponding to a given rotation matrix. See Appendix B.

PROGRAMMING EXERCISE (PART 2)

1. If your function library does not include an Atan2 function subroutine, write one.
2. To make a friendly user interface, we wish to describe orientations in the planar world by a single angle, θ, instead of by a 2×2 rotation matrix. The user will always

Inverse kin

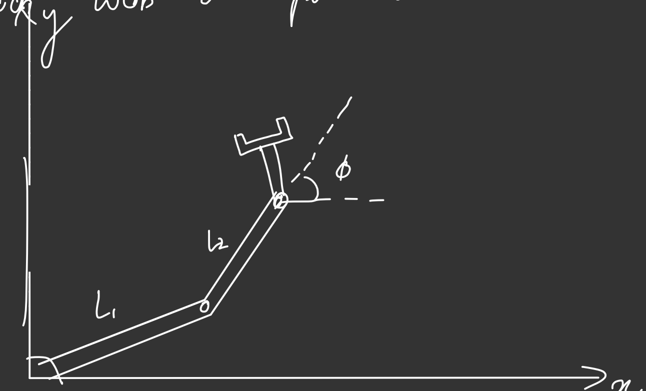
Given the desired posⁿ & orientation of tool end effector relative to the base, how do we conclude the set of joint variable which will achieve the result.

Workspace

Specified posⁿ & orientⁿ of the tool should be in the workspace of the manipulator. It is that vol of space in which the end effector can reach.

Dextrous Workspace

It is that vol of space that the robot's end effector can reach with all poss orientation



$$x = L_1 C\theta_1 + l_2 C_{12}$$

$$y = L_1 S\theta_1 + L_2 S_{12}$$

$$\phi = \theta_1 + \theta_2 + \theta_3$$

$$A\tan_2(y, x) = \tan^{-1}(y/x) \quad \forall x > 0$$

$$= 180 + \tan^{-1}(y/x)$$

$$x^2 + y^2 = L_1^2 + L_2^2 + 2L_1L_2C\theta_2$$

$$\cos\theta_2 = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}$$

Condition

$$\left| \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2} \right| \leq 1$$

$$\sin\theta_2 = \pm\sqrt{1 - C^2\theta_2}$$

$$\theta_2 = A\tan_2(S\theta_2, C\theta_2)$$

$$x = L_1C\theta_1 + L_2(C\theta_1C\theta_2 - S\theta_1S\theta_2)$$

$$y = L_1S\theta_1 + L_2(S\theta_1C\theta_2 + S\theta_2C\theta_1)$$

$$x = k_1 \cos\theta_1 + k_2 \sin\theta_2$$

$$y = k_1 \sin\theta_1 + k_2 \cos\theta_2$$

where $k_1 = L_1 + L_2 \cos\theta_2$

$k_2 = L_2 \sin\theta_2$

$4k_1 + S k_2$ gives :-

$k_1 = \gamma \cos\nu$

$k_2 = \gamma \sin\nu$

$\gamma = \sqrt{k_1^2 + k_2^2}$

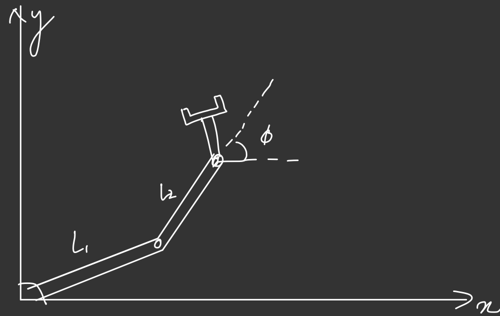
$$x/\gamma = \cos\gamma \cos\theta_1 - \sin\nu \sin\theta_1 = \cos(\gamma + \theta_1)$$

$$y/\gamma = \cos\nu \sin\theta_1 + \sin\gamma \sin\theta_1 = \sin(\nu + \theta_1)$$

$$\gamma + \theta_1 = A\tan 2(y/\gamma, x/\gamma)$$

$$\theta_1 = A\tan 2(y/\gamma, x/2) - A\tan 2(k_2, k_1)$$

$$\theta_3 = \phi - \theta_1 - \theta_2$$



$$A\tan 2(k_2, k_1) = \tan^{-1}(k_2/k_1) \quad \forall\ k_1 > 0$$

$$= \pi + \tan^{-1}(k_2/k_1) \quad \forall\ k_1 < 0$$

Start

Speed of x; y; L_1; L_2; ϕ

$$\epsilon = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}$$

$|\epsilon| \leq 1$

NO

Print No Solution

yes

$\epsilon = \cos\theta_2$

$\epsilon_y = \pm\sqrt{1-\epsilon^2} = \sin\theta_2$

$\theta_2 = A\cdot\tan 2(\epsilon_y, \epsilon)$

$K_1 = L_1 + L_2\cos\theta_2$

$K_2 = L_2\sin\theta_2$

$$\boxed{r = \sqrt{k_1^2 + k_2^2} \; ; \quad \gamma = A\tan2(k_1 ; k_2)}$$

$$\downarrow$$

$$\boxed{\theta_1 = A\tan2(y, x) - \gamma}$$

$$\downarrow$$

$$\boxed{\theta_3 = \phi - \theta_2 - \theta_1}$$

$$\downarrow$$

$$\boxed{\text{End}}$$

The result of forward kinematics is

$$^{base}_{tool}T = \left[\begin{array}{c|c} R & P \\ \hline 0\;0\;0 & 1 \end{array}\right]$$

$$R = \begin{array}{c} \begin{array}{ccc} \vec{r_1} & \vec{r_2} & \vec{r_3} \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \end{array}$$

$$\vec{r_1} \cdot \vec{r_2} = 0$$

$$\vec{r_2} \cdot \vec{r_3} = 0$$

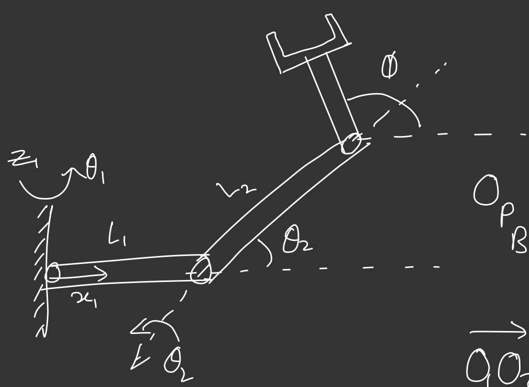
$$\vec{r_3} \cdot \vec{r_1} = 0$$

$$\sqrt{r_{11}^2 + r_{21}^2 + r_{31}^2} = 1$$

6 variables, 6 independent equations

$$\sqrt{r_{12}^2 + r_{22}^2 + r_{32}^2} = 1$$

$$\sqrt{r_{13}^2 + r_{23}^2 + r_{33}^2} = 1$$



${}^{O_P}_{B_{org}}$ is the last column of ${}^{base}_{tool}T$

$$\overrightarrow{O_1O_3} = {}^1P_{3\,org} = L_1 + L_2\cos\theta_2\,\hat{i}_1 + L_2\sin\theta_2\,\hat{k}_1$$

$$= \begin{Bmatrix} L_1 + L_2\cos\theta_2 \\ 0 \\ L_2\sin\theta_2 \end{Bmatrix}$$

$$\overrightarrow{O_0O_3} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (L_1 + L_2C\theta_2)\,C\theta_1 \\ (L_1 + L_2C\theta_2)\,S\theta_1 \\ L_2\,S\theta_2 \end{bmatrix}$$

$$= \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$x = (L_1 + L_2 C\theta_2) C\theta_1 \longrightarrow 1$$

$$y = (L_1 + L_2 C\theta_2)\, S\theta_1 \longrightarrow 2$$

$$z = L_2 S\theta_2 \longrightarrow 3$$

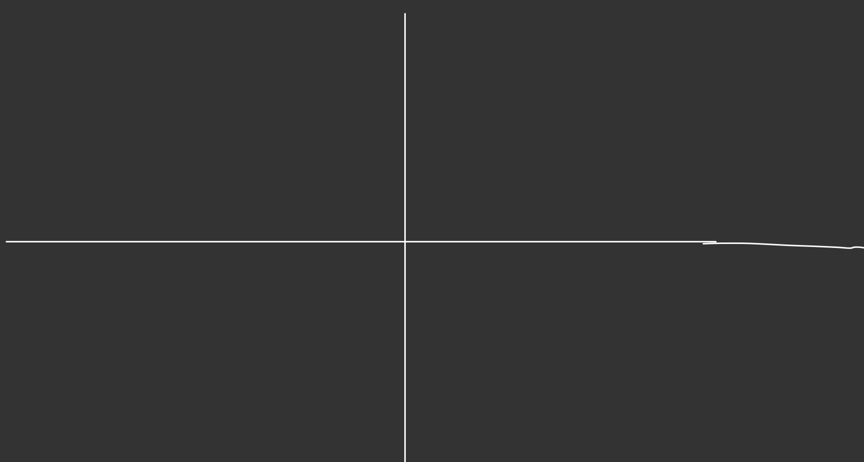
$$S\theta_2 = z / L_2$$

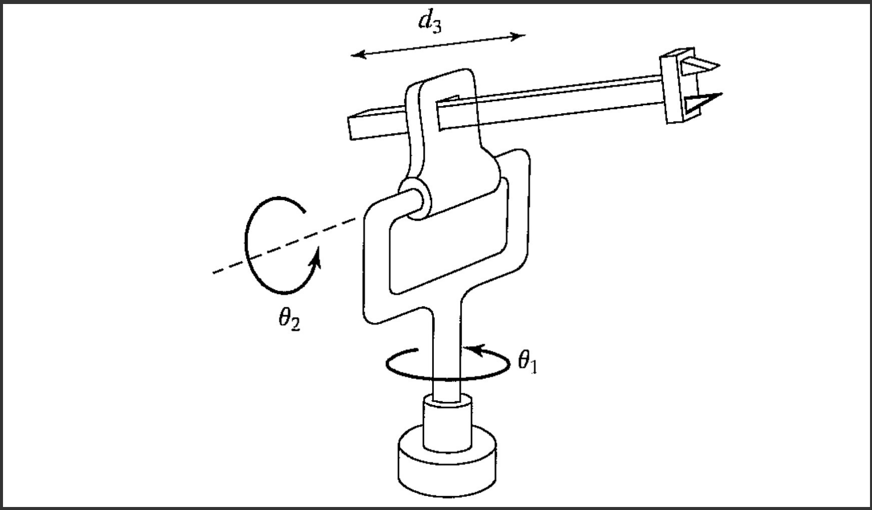
$$\cos\theta_2 = \pm \sqrt{1 - \sin^2\theta_2}$$

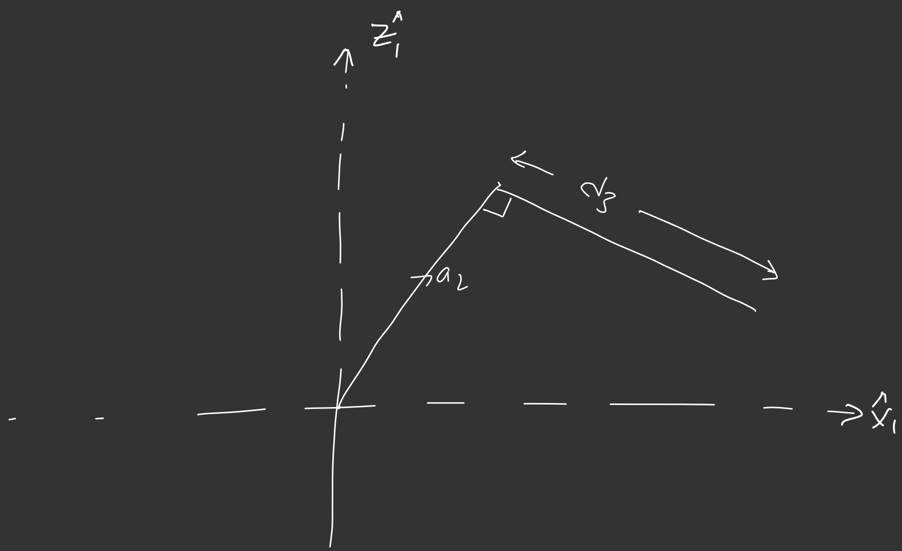
$$\theta_2 = A\tan 2 (S\theta_2, \cos\theta_2)$$

$$\theta_1 = A\tan 2 (y, x)$$

$$\theta_3 = \phi - \theta_2$$

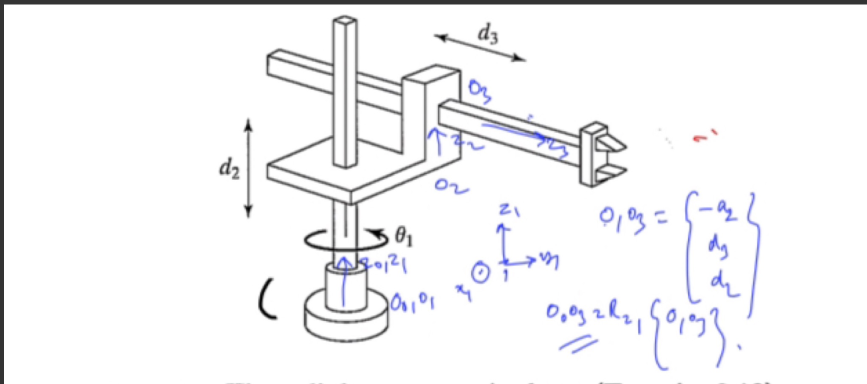






$$\overrightarrow{O_1O_3} = a_2 c\theta_2 \hat{i}_1 + a_2 s\theta_2 \hat{k}_1 + d_3 s\theta_2 \hat{i} - d_3 c\theta_2 \hat{k}_1$$

3.39



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	0	d_2	0
3	90	a_2	d_3	0

$$
{}^{0}T_{1} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}T_{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$^2T_3 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_2 = {}^0T_1 \, {}^1T_2$$

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = a_2 c\theta_1 + d_3 s\theta_1$$

$$y = a_2 s\theta_1 - d_3 c\theta_1$$

$$z = d_2$$

$z = d_2$

$d_2 = z$

$x^2 + y^2 = a_2^2 + d_3^2$

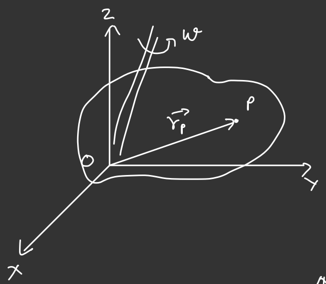
$d_3 = \pm \sqrt{x^2 + y^2 - a_2^2}$

$x = \sqrt{a_2^2 + d_3^2} \quad \cos(\theta_1 - \nu)$

$\tan \nu = \dfrac{d_3}{d_2}$

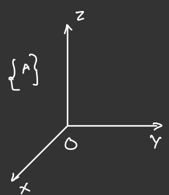
$y = \sqrt{a_2^2 + d_3^2} \quad \sin(\theta + \nu)$

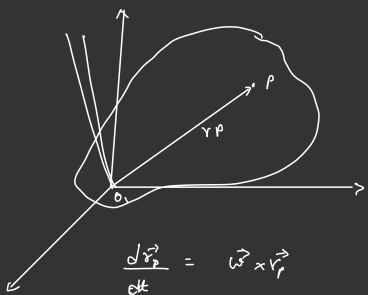
VELOCITY ANALYSIS



rigid body hinged at O, rotating with ang vel ω about an axis passing through O.

$$\frac{d\vec{r}_P}{dt} = \dot{\vec{r}}_P = \vec{v} = \vec{\omega} \times \vec{r}_P$$





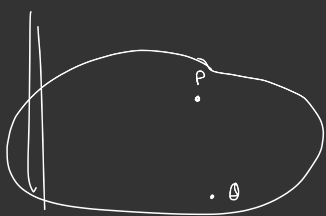
1

$$\frac{d\vec{r}_P}{dt} = \vec{\omega} \times \vec{r}_P$$

O_1 is not rotating rel to {A}

Let the frame {B} is fixed with the rigid body $\frac{d\vec{i}_B}{dt} = \vec{\omega} \times \vec{i}_B$

$$\frac{d\vec{j}_B}{dt} = \vec{\omega} \times \vec{j}_B$$



vel of pt P wrt Q

$$= \vec{\omega} \times (\vec{OP} - \vec{OQ})$$

$$= \vec{\omega} \times \vec{QP}$$

from (1) $\qquad$ from O_1 is also moving

$$\vec{OP} = \vec{OO_1} + \vec{O_1P}$$

$$\frac{d(\vec{OP})}{dt} = \frac{d(\vec{OO_1})}{dt} + \frac{d(\vec{O_1P})}{dt}$$

$$= V_{B\,org} + \frac{d}{dt}(x i_B + y j_B + z k_B)$$

$$\vec{V_P} = \vec{V}_{B.org} + \dot{x}\vec{i_B} + x\dot{\vec{i}}_B + \dot{y}\vec{j_B} + y\dot{\vec{j}}_B + \dot{z}\vec{k_B} + z\dot{\vec{k}}_B$$

$$= \vec{V}_{Borg} + \vec{\omega}(x\vec{i_B} + y\vec{j_B} + z\vec{k_B}) + \dot{x}\vec{i_B} + \dot{y}\vec{j_B} + \dot{z}\vec{k_B}$$

$$= \vec{V}_{B.org} + \vec{\omega} \times {}^A_B R\, {}^B P + {}^A_B R\, {}^B V_P$$

$${}^A P = {}^A P_{Borg} + {}^A_B R \cdot {}^B P$$

Diff wrt time

$$\frac{d({}^A P)}{dt} = {}^A V_{Borg} + \frac{d({}^A R_B)}{dt}\, {}^B P + {}^A_B R \cdot \frac{d({}^B P)}{dt}$$

$${}^A V_P = {}^A V_{Borg} + {}^A_B \dot{R} * {}^B P + {}^A_B R\, {}^B V_P$$

Let ${}^A_B R = R_k(\theta)$

let after $t + \Delta t$ $\qquad \theta \to \theta + \Delta\theta$

$$\frac{d({}^A_B R)}{dt} = \lim_{\Delta t \to 0} \frac{R_k(\theta+\Delta\theta) - R_k(\theta)}{\Delta t}$$

$$R_K(\theta+\Delta\theta) = R_K(\Delta\theta)\, R_K(\theta)$$

$$\frac{d({}^A_B R)}{dt} = \lim_{\Delta t \to 0} \frac{\left(R_K(\Delta\theta) - I\right)}{\Delta t} R_K(\theta)$$

$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \begin{bmatrix} 0 & -k_z \Delta\theta & k_y \Delta\theta \\ k_z \Delta\theta & 0 & -k_x \Delta\theta \\ -k_y \Delta\theta & k_x \Delta\theta & 0 \end{bmatrix} R_K(\theta)$$

$$= \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} R_K(\theta)$$

where $\lim_{\Delta t \to 0} \frac{k_x \Delta\theta}{dt} = k_x \dot{\theta} = \omega_x$

$$\frac{d({}^A_B R)}{dt} = \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \times R_K(\theta)$$

$$ {}^{A}v_{p} = {}^{A}V_{B\,org} + {}^{A}_{B}\dot{R}\ {}^{B}p + {}^{A}_{B}R \cdot {}^{B}v_{p} $$

$$ = {}^{A}V_{B\,org} + \vec{\omega} \times {}^{A}_{B}R \cdot {}^{B}p + {}^{A}_{B}R\ {}^{B}v_{p} $$

$$ {}^{A}v_{p} = {}^{A}\vec{v}_{B.org} + {}^{A}\omega_{B} \times (x\hat{i}_{B} + y\hat{j}_{B} + z\hat{k}_{B}) $$
$$ + \dot{x}\hat{i}_{B} + \dot{y}\hat{j}_{B} + \dot{z}\hat{k}_{B} $$

$$ {}^{A}\vec{v}_{p} = {}^{A}v_{B\,org} + {}^{A}\vec{\omega}_{B} \times {}^{A}_{B}R\ {}^{B}p + {}^{A}_{B}R\ {}^{B}v_{p} $$

$$ {}^{A}a_{p} = {}^{A}\left(\frac{d\vec{v_p}}{dt}\right) = {}^{A}\vec{a}_{B.org} + {}^{A}\omega_{B} \times (\hat{x}_{i_B} + \hat{y}_{j_B} + z\hat{k}_{B}) $$
$$ + {}^{A}\omega_{B}(\dot{x}\hat{i}_{B} + \dot{y}j_{B} + \hat{\dot{z}}k_{B}) $$
$$ + \ddot{x}\hat{i}_{K} + \ddot{y}\hat{j}_{B} + \ddot{z}\hat{k}_{B} $$

$$ {}^{A}\vec{a}_{p} = {}^{A}\vec{a}_{B.org} + {}^{A}\dot{\omega}_{B} \times \left({}^{A}\omega_{B} \times {}^{A}_{B}R\ {}^{B}p_{v}\right) $$
$$ + {}^{A}\omega_{B} \times {}^{A}_{B}R\ {}^{B}\vec{v}_{p} $$

$$= {}^A a_{B_{org}} + {}^A\dot{\omega}_B \times {}^A_B R\,{}^B p + {}^A\omega_B \times {}^A_B\dot{R}\,{}^B p$$

$$+ {}^A\omega_B \times {}^A_B R\,{}^B v_p + {}^A_B\dot{R}\,{}^B v_p$$

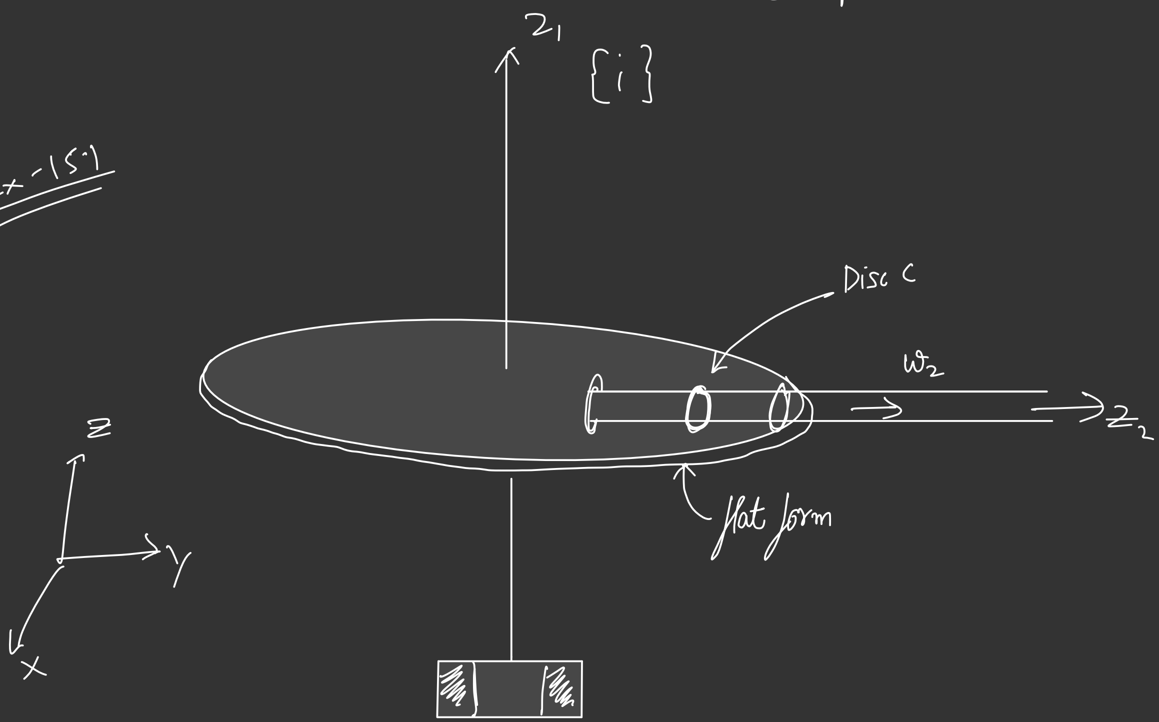
$$+ {}^A_B R\,{}^B a_p$$

$$A_{a_p} = {}^A a_{B_{org}} + {}^A\dot{\omega}_B \times {}^A_B R\,{}^B p + \left({}^A\omega_B \cdot {}^A_B R\,{}^B p\right) {}^A\omega_B$$

$$- \left({}^A\omega_B \cdot {}^A\omega_B\right) {}^A_B R\,{}^B p +$$

$$2\,{}^A\omega_B \times {}^A_B R\,{}^B v_p + {}^A_B R\,{}^B a_p$$

Ex-15.1

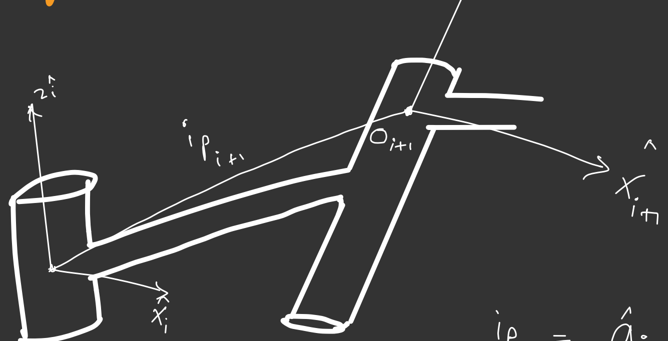


$$^A\vec{\omega} = {}^A\omega_1 + {}^A_2R\, {}^2\omega_2$$

$$= {}^A\omega_1 + {}^A_2R\, \omega_2\, {}^2\hat{z}_2$$

$$= {}^A\omega_1 + {}^A_2R\, \omega_2 \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

Velocity Propogation Link to Link



$${}^iP_{i+1} = \hat{a}_i\, {}^i\hat{x}_i + d_i\, {}^i\hat{z}_{i+1}$$

Vector joining O_i & O_{i+1}

$${}^i\omega_{i+1} = {}^i\omega_i + {}^iR_{i+1}\, \dot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1}$$

$${}^iP_{i+1} = a_i\, {}^i\hat{x}_{i+1} + {}^i_{i+1}R\, d_i\, {}^{i+1}\hat{z}_{i+1}$$

ang vel of $\{i+1\}$ relative
to the base expressed in $\{i\}$

$$\underbrace{{}^{i}\omega_{i+1}}_{} = {}^{i}\omega_i + \underbrace{{}^{i}R_{i+1}\,\dot{\theta}_{i+1}\,{}^{i+1}\hat{z}_{i+1}}_{}$$

${}^{i}\omega_i$: ang vel of $\{i\}$ rel to base expressed in $\{i\}$

${}^{i}R_{i+1}\,\dot{\theta}_{i+1}\,{}^{i+1}\hat{z}_{i+1}$: ang vel of $\{i+1\}$ rel to $\{i\}$ expressed in $\{i\}$

Pre multiplying both sides by ${}^{i+1}_{i}R$

$${}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{z}_{i+1}$$

$${}^{i}v_{i+1} = {}^{i}v_i + \underbrace{{}^{i}\omega_i \times {}^{i}P_{i+1}}_{}$$

${}^{i}v_{i+1}$: vel of O_{i+1} rel to the base expressed in $\{i\}$

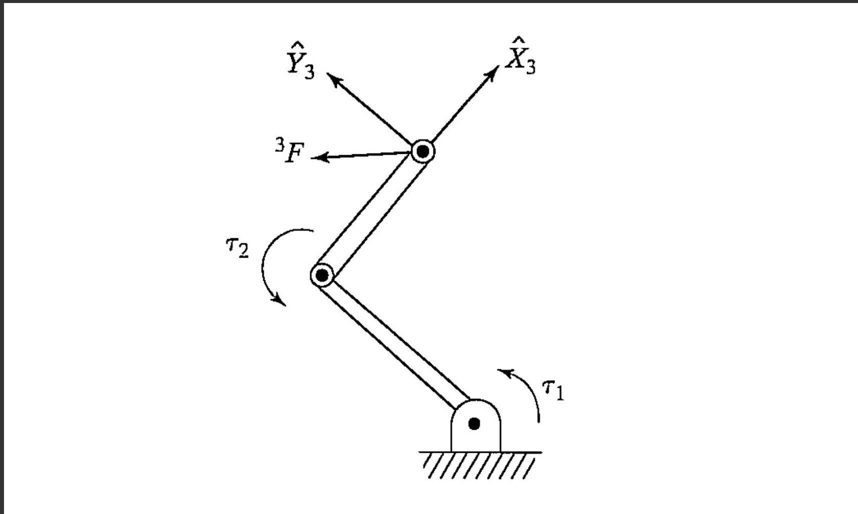
${}^{i}v_i$: vel of O_i rel to the base exp in $\{i\}$

${}^{i}\omega_i \times {}^{i}P_{i+1}$: vel of O_{i+1} exp in O_i

<u>for prismatic jt</u>

$${}^{i}v_{i+1} = {}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1} + \dot{d}_i\,{}^{i}\hat{z}_{i+1}$$

$$= {}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1} + {}^{i}_{i+1}R\,\dot{d}_i\,{}^{i+1}\hat{Z}_{i+1}$$



$$^0\omega_0 = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$^0\omega_1 = {}^0\omega_0 + {}^0_1R \ \dot{\theta}_1 \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$$^1\omega_1 = {}^0\omega_1 + {}^1R$$

$$^2v_3 = \begin{Bmatrix} -l_1 \sin\theta_2 \dot{\theta}_1 \\ l_1 c\theta_2 \dot{\theta}_1 + l_2(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{Bmatrix}$$

$$^3v_3 = {}^3_2R \ {}^2v_3 = \text{''} \qquad (\because {}^3R_2 = I)$$

$$
{}^{0}v_3 = {}^{0}R_3\,{}^{3}v_3 = \begin{bmatrix} l_1 S\theta_2 C_{12}\dot{\theta}_1 + l_1 S_{12} C\theta_2 \dot{\theta}_1 - l_2 S_{12}(\dot{\theta}_1+\dot{\theta}_2) \\ l_1 S_{12} S\theta_2 \dot{\theta}_1 + l_1 C_{12} C\theta_2 \dot{\theta}_1 + l_2 C_{12}(\dot{\theta}_1+\dot{\theta}_2) \\ 0 \end{bmatrix}
$$

$$
C_{12} = C(\theta_1+\theta_2)
$$

$$
{}^{0}v_3 = \begin{Bmatrix} -l_1 S\theta_1 \dot{\theta}_1 - l_2 S_{12}(\dot{\theta}_1+\dot{\theta}_2) \\ l_1 C\theta_1 \dot{\theta}_1 + l_2 C_{12}(\dot{\theta}_1+\dot{\theta}_2) \\ 0 \end{Bmatrix}
$$

$$
= \begin{bmatrix} -l_1 S\theta_1 - l_2 S_{12} & -l_2 S_{12} \\ l_1 C\theta_1 + l_2 C_{12} & l_2 C_{12} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix}
$$

$$
\{\dot{X}\} = J\{\dot{\theta}\}
$$

Derivative of joint variable vector

derivative of config vector

$$\{\dot{\theta}\} = J^{-1}\{\dot{X}\}$$

New Method

$$^{O}p_{3} = x\hat{i} + y\hat{j}$$

$$x = l_1 C\theta_1 + l_2 C_{12}$$

$$y = l_1 S\theta_1 + l_2 S_{12}$$

DBS

$$\dot{x} = -l_1 S\theta_1 - l_2 S_{12}(\dot{\theta}_1 + \dot{\theta}_2)$$

$$\dot{y} = l_1 \cos\theta_1 + l_2 C_{12}(\dot{\theta}_1 + \dot{\theta}_2)$$

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} = \begin{bmatrix} -l_1 S\theta_1 - l_2 S_{12} & -l_2 S_{12} \\ l_1 C\theta_1 + l_2 C_{12} & l_2 C_{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

The configuration vector of the end effector can be written as :-

$$\{X\} = \begin{Bmatrix} x \\ y \\ z \\ \phi_x \\ \phi_y \\ \phi_z \end{Bmatrix}$$

$$x = f_1(\theta_1 ; \theta_2 ; \theta_3 \text{ ---- } \theta_6)$$

$$y = f_2(\theta_1 ; \theta_2 ; \text{ ---- } \theta_6)$$

$$z = f_3(- \quad . \quad . \quad)$$

$$\phi_x = f_4(\quad)$$

$$\phi_y = f_5(\quad)$$

$$\phi_z = f_6(\quad)$$

$$\boxed{\delta X = \begin{Bmatrix} \delta x \\ \delta y \\ \delta z \\ \delta\phi_x \\ \delta\phi_y \\ \delta\phi_z \end{Bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial\theta_1} & \frac{\partial f_1}{\partial\theta_2} & \frac{\partial f_1}{\partial\theta_3} & -- & \frac{\partial f_1}{\partial\theta_6} \\ \frac{\partial f_2}{\partial\theta_1} & - & - & - & - \\ \vdots & & & & \\ \frac{\partial f_6}{\partial\theta_1} & - & - & - & - \end{bmatrix} \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \\ \delta\theta_3 \\ \vdots \\ \delta\theta_6 \end{bmatrix}}$$

$$\delta x = \frac{\partial f_1}{\partial\theta_1}\delta\theta_1 + \frac{\partial f_1}{\partial\theta_2}\delta\theta_2 \text{ -- -- } \frac{\partial f_1}{\partial\theta_6}(\delta\theta_6)$$

$$\delta y = \frac{\partial f_2}{\partial\theta_1}\delta\theta_1 + \frac{\partial f_2}{\partial\theta_2}(\delta\theta_2) + \text{ - -- } \partial$$

$$\lim_{\delta t\to 0}\frac{\delta X}{\delta t} = \lim_{\delta t \to 0}\frac{1}{\delta t}\begin{Bmatrix}\delta x\\ \delta y\\ \delta z\\ \delta\phi_x\\ \delta\phi_y\\ \delta\phi_z\end{Bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial\theta_1} & \frac{\partial f_1}{\partial\theta_2} & \frac{\partial f_1}{\partial \theta_3} & \cdots & \frac{\partial f_1}{\partial\theta_6}\\ \frac{\partial f_2}{\partial\theta_1} & - & - & - & -\\ \vdots & & & & \\ \frac{\partial f_6}{\partial\theta_1} & - & - & - & -\end{bmatrix}\begin{bmatrix}\delta\theta_1\\ \delta\theta_2\\ \delta\theta_3\\ \vdots\\ \delta\theta_6\end{bmatrix}$$

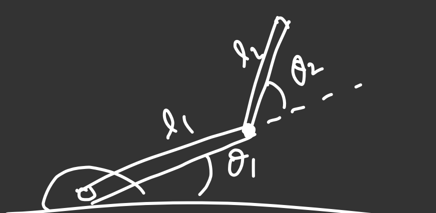
∂)

$$\begin{Bmatrix}\dot x\\ \dot y\\ \dot z\\ \dot\phi_x\\ \dot\phi_y\\ \dot\phi_z\end{Bmatrix} = J\begin{Bmatrix}\dot\theta_1\\ \dot\theta_2\\ \dot\theta_3\\ \vdots\\ \vdots\end{Bmatrix}$$

$$\dot\theta = J^{-1} X$$

$x = l_1 c\theta_1 + l_2 c_{12}$

$y = l_2 s\theta_1 + l_2 s_{12}$



$$J = \begin{bmatrix} -l_1 S\theta_1 - l_2 S_{12} & -l_2 S_{12} \\ l_1 C\theta_1 + l_2 C_{12} & l_2 C_{12} \end{bmatrix}$$

$$= l_1 l_2 S\theta_2$$

Singularity when $\theta_2 = 0^\circ$ or 180°

To find desired vel of end eff what is req. speed of joint variables

Jacobian

$$x = f_1(\theta_1, \theta_2 \text{ --- } \theta_6)$$
$$y = f_2(\theta_1 \text{ --- } \theta_6)$$
$$z = f$$
$$\phi_x = f_3(\quad)$$
$$\phi_y = f_4(\quad)$$
$$\phi_z = f_5(\quad)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi}_x \\ \dot{\phi}_y \\ \dot{\phi}_z \end{bmatrix} = J \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \vdots \\ \dot{\theta}_6 \end{Bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} & \cdots & \frac{\partial f_1}{\partial \theta_6} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} & \cdots & \frac{\partial f_2}{\partial \theta_6} \\ \frac{\partial f_6}{\partial \theta_1} & \cdots & \cdots & \frac{\partial f_6}{\partial \theta_6} \end{bmatrix}$$

Hence $\{\dot{X}\} = J\{\dot{\theta}\}$

$$\{\dot{\theta}\} = J^{-1}\{\dot{x}\}$$

For Singularity we seek θ_i for which $\det(J) = 0$

---

* If J is sq matrix then the operation such as J^{-1}, singular cond^n valuations are straight forward.

If J is not a sq matrix then for inverse of J we determine pseudoinverse of J

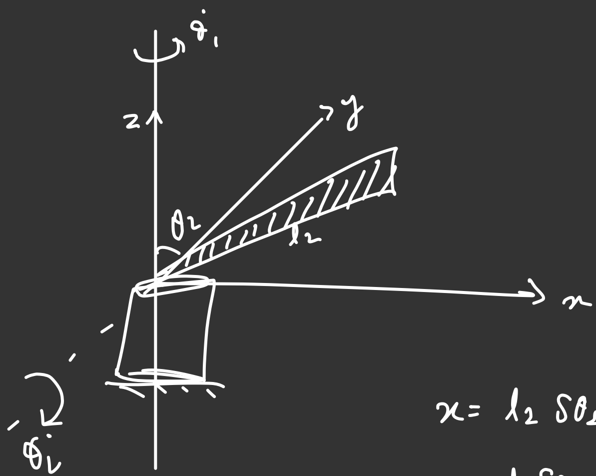
(Q) what is pseudo inverse

A → If A is a $m \times n$ matrix then A^+ the pseudoinverse of A is an $n \times m$ matrix determine

$$A^+ \begin{cases} A^T(AA^T)^{-1} & \text{for } m \le n \\ A^{-1} & m = n \\ (A^TA)^{-1}A^T & m \geq n \end{cases}$$

For Singularity we seek θ_i for which atleast one row turns to zero

Ex → P-8 ; fig 1.9



$x = l_2 S\theta_2 C\theta_1$

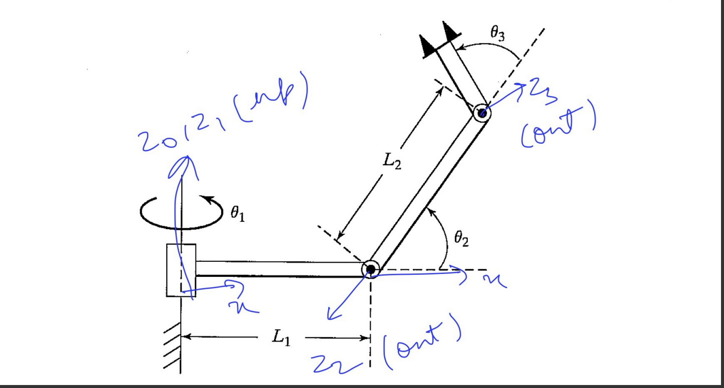
$y = l_2 S\theta_2 S\theta_1$

$z = l_2 C\theta_2$

$$J = \begin{bmatrix} -l_2 S\theta_2 S\theta_1 & l_2 C\theta_2 C\theta_1 \\ l_2 S\theta_2 C\theta_1 & l_2 C\theta_2 S\theta_1 \\ 0 & -l_2 S\theta_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ 0 \end{bmatrix}$$

$\theta_1 \in (0, 180)$
$\theta_2 \in (0, 90)$

$\theta_2 = 0$ is the Singularity condition for which one row become 0.



$x = l_1 + l_2 C\theta_2$

$y = l_2 S\theta_2$

$[R]$

$\phi = \theta_2 + \theta_3$

for $\theta_2 = 0$

the 3r row becs 0

Thus $\theta_2 = 0$ is the singularity condition.

x

y

ϕ

$$[R] = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore x = L_1 C\theta_1 + L_2 C\theta_1 C\theta_2$

$y = L_1 S\theta_1 + L_2 S\theta_1 C\theta_2$

$z = L_2 S\theta_2$

$\phi = \theta_2 + \theta_3$

$$\begin{bmatrix} -l_1 S\theta_1 - l_2 C\theta_2 S\theta_1 & -l_2 C\theta_1 S\theta_2 & 0 \\ l_1 C\theta_1 + l_2 C\theta_1 C\theta_2 \dot{q} & -l_2 S\theta_1 S\theta_2 & 0 \\ 0 & l_2 C\theta_2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Ex - 5.7

Formal Process

Ans The config vector of the end effector can be given as:-

$$X = \begin{bmatrix} x & y & z & \phi \end{bmatrix}^T$$

$$x = (l_1 + l_2 C\theta_2)\ \hat{i}_1$$

$$y = 0$$

$$z = l_2 S\theta_2\ \hat{k}$$

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = R_z(\theta) \begin{Bmatrix} l_1 + l_2 C\theta_2 \\ 0 \\ l_2 S\theta_2 \end{Bmatrix}$$

$$x = (l_1 + l_2 C\theta_2)\, C\theta_1 = f_1(\theta_1 ; \theta_2 ; \theta_3)$$

$$y = (l_1 + l_2 C\theta_2)\, S\theta_1 = f_2(\theta_1 ; \theta_2 ; \theta_3)$$

$$z = l_2\, z\theta_2 = f_3($$

$$\phi = \theta_2 + \theta_3 = f_4$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} & \frac{\partial f_1}{\partial \theta_3} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} & \frac{\partial f_2}{\partial \theta_3} \\ \frac{\partial f_3}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} & \frac{\partial f_3}{\partial \theta_3} \end{bmatrix}$$

$$J = \begin{bmatrix} -L_1 S\theta_1 - L_2 C\theta_2 S\theta_1 & -L_1 C\theta_1 S\theta_2 & 0 \\ L_1 C\theta_1 + L_2 C\theta_1 C\theta_2 & -L_2 S\theta_1 S\theta_2 & 0 \\ 0 & l_2 S\theta_2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Assuming range of θ_1

$$0 \leq \theta_1 \leq 2\pi$$

$$-\pi/2 \leq \theta_2 \leq \pi/2$$

$$0 \leq \theta_3 \leq \pi/2$$

we see for $\theta_2 = \pi/2$, the 3^{rd} row turns to 0

Hence singularity arises at $\theta_2 = \pi/2$

Virtual Work.

On the system of ~~par~~ N particle suppose that force components $F_1, F_2, \dots F_{3N}$ are applied at the corresponding coordinates in a positive sense (increasing direction of x_i).

The virtual work δW of these forces in a virtual displacement δx is given by

$$\delta W = \sum_{j=1}^{3N} F_j \, \delta x_j$$

& $\delta W = \vec{F_j}\,\delta\vec{x_j}$

An alternate form of the expression for the virtual work is

$$\delta W = \sum_{i=1}^{N} \vec{F_i} \cdot \vec{\delta r_i}$$

Where $\vec{F_i}$ is the force applied at the ith particle, and where r_i is the position vector of this particle.

Note: In the expression of virtual work, it is important to realize that the forces are assumed to remain const. throughout the virtual disp.

P

Virtual Work

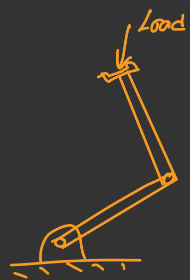
Virtual Displacement:

Suppose the configuration of a system of N particles is given by 3N cartesian coordinates $x_1, x_2 \ldots, x_{3N}$ which are measured relative to an inertial frame and may be subjected to constraints. At any given time, let us assume that the coordinates move ~~tt~~ through infinitesimal displacement $\delta x_1, \delta x_2 \ldots$ δx_{3N} which are ~~virch~~ virtual or imaginary in the sense that they are assumed to occur without the passage of time, and dos not necessarily conform (follow or abide) to the constraints. This small change δx in the configuration of the system is known as a virtual displacement.

* Virtual dispacement is not, in general, a possible real displacement.

Principle of Virtual Work

It States that, when a body is in equl^m, the vir^l work due to external force = the vir^l work due to internal resisting force



$$\delta U_i = \delta W_e$$

$$\sum f_i \, \delta x_i = F \delta y$$

Jacobian in force domain

Consider a manipulator is in Static equl^n. Applying principle of virtual work -

$$\vec{F} \cdot \delta \vec{x} = \vec{\tau} \cdot \delta \vec{\theta}$$

$$F^T \delta x = \tau^T \delta \theta$$

$$F^T J \delta \theta = \tau^T \delta \theta$$

$$F^T J = \tau^T$$

taking transpose both sides

$$\tau = J^T F$$

EXAMPLE 5.7

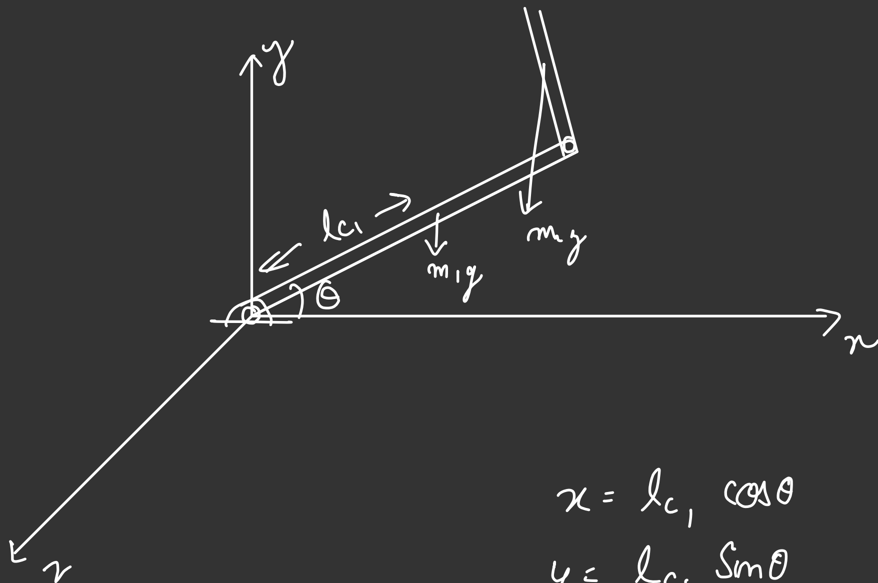
The two-link manipulator of Example 5.3 is applying a force vector ${}^3F$ with its end-effector. (Consider this force to be acting at the origin of {3}.) Find the required joint torques as a function of configuration and of the applied force. (See Fig. 5.12.)

$$ {}^0F = {}^0_3R \, {}^3F = \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} f_x \\ f_y \\ 0 \end{Bmatrix} $$

$$ X = \begin{Bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{Bmatrix} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} $$

$$ J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \\ 0 & 0 \end{bmatrix} $$

$$ \tau = J^T \, {}^0F = \begin{bmatrix} l_1 s\theta_2 & l_2 + l_1 c_2 \\ 0 & l_2 \end{bmatrix} \begin{Bmatrix} f_x \\ f_y \end{Bmatrix} $$



$$x = l_{c_1} \cos\theta$$

$$y = l_{c_1} \sin\theta$$

$$J' = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} \end{bmatrix}$$

$$J_1 = \begin{bmatrix} -l_{c_1} \sin\theta & 0 \\ l_{c_1} \cos\theta & 0 \end{bmatrix}$$

$$x = l_1 C\theta_1 + l_{c_2} c_{12} = P_1(\theta_1 ; \theta_2)$$

$$y = l_1 S\theta_1 + l_{c_2} S_{12} = P_2(\theta_1 ; \theta_2$$

$$J_2 = \begin{bmatrix} \dfrac{\partial P_1}{\partial \theta_1} & \dfrac{\partial P_1}{\partial \theta_2} \\ \dfrac{\partial P_2}{\partial \theta_1} & \dfrac{\partial P_2}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -l_1 S\theta_1 - l_{c_2} S_{12} & -l_{c_2} S_{12} \\ l_1 C\theta_1 + l_{c_2} C_{12} & l_{c_2} C_{12} \end{bmatrix}$$

$$t^T \delta\theta = F_1^T \delta x_1 + F_2^T \delta x_2$$

$$t = J^T F$$

$$= J_1^T F_1 + J_2^T F_2$$

$$= \begin{bmatrix} -l_{c_1} \sin\theta & l_{c_1} \cos\theta \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -m_1 g \end{bmatrix} + \begin{bmatrix} -l_1 S\theta_1 - l_{c_2} S_{12} & -l_{c_2} S_{12} \\ l_1 C\theta_1 + l_{c_2} C_{12} & l_{c_2} C_{12} \end{bmatrix} \begin{bmatrix} 0 \\ m_2 g \end{bmatrix}$$

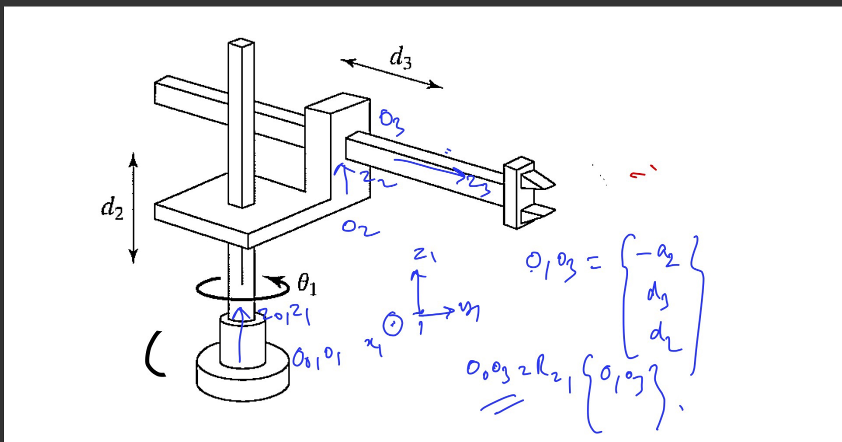
$$t^{T}\delta\theta = f_1^{T} J\,\delta\theta_1 + f_2^{T} J_2\,\delta\theta_2$$

$$t^{T} = f_1^{T} J + f_2^{T} J_2$$

$$= \begin{bmatrix} -l_{c_1} m_1 g\, c\theta \\ 0 \end{bmatrix} + \begin{bmatrix} -l_{c_2} m_2 g\, s_{12} \\ l_{c_2}\, c_{12} \end{bmatrix}$$

MANIPULATOR DYNAMICS

fig - 3.34



$x = f_1(\theta ; d_2, d_3)$

$y = f_2(\theta_2 ; d_2 ; d_3)$

$z = f_3(\theta_3 ; d_2 ; d_3)$

$x = a_2$

$y = d_3$

$z = d_2$

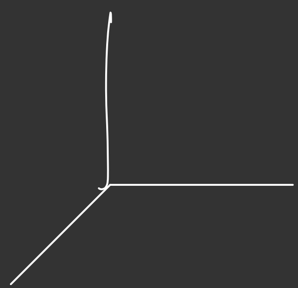
$$\begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3} \begin{bmatrix} a_2 \\ -d_3 \\ d_2 \end{bmatrix}_{3\times1}$$

$$J = \begin{bmatrix} a_2 C\theta_1 + d_3 S\theta_1 \\ a_2 S\theta_1 - d_3 C\theta_1 \\ d_2 \end{bmatrix}_{3\times1}$$

Manipulator Dynamics

Generalized coord: The min no of independent co-ord req to describe config^n sys uniquely called

Single Particle Sys

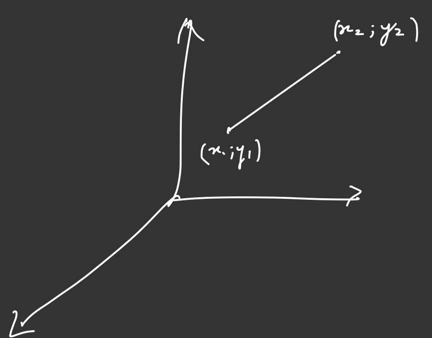


If the particle is subjected to a constraint

$$z^2 - xy = 0$$

no. of degree of freedom

Two particle Sys



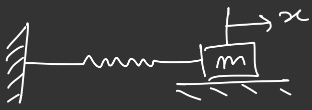
$z_1 = 0$

$z_2 = 0$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = 0$$

In serial robot

No of Joint = No of gen co-ord
= No of degree of freedom



$$m\ddot{x} + kx = 0$$

$$L = T - V$$

Lagrangian

KE

PE

<u>Eq^n of Motion</u>

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$T = KE = \frac{1}{2} m \dot{x}^2$$

$$V = PE = \frac{1}{2} k x^2$$

$$L = T - V = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} kx^2$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \; ; \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}$$

$$\frac{\partial L}{\partial x} = -kx$$

$$m\ddot{x} + kx = 0$$

<u>Euler Lagrange eq^n of Motion</u>

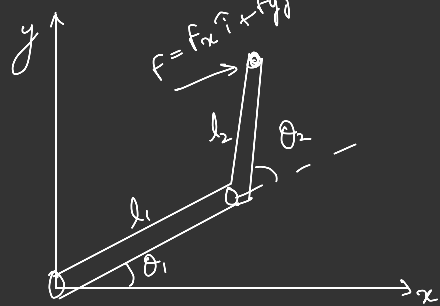
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$$

generalised force

q_i is the i^{th} generalized coord

Virtual Work

$$\delta W = Q_1 \, \delta q_1 + Q_2 \, \delta q_2 + - - - - -$$



$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\delta\vec{r} = \delta x \, \hat{i} + \delta y \, \hat{j}$$

$$\delta W = \vec{F} \cdot \delta\vec{r}$$

$$= F_x \delta x + F_y \delta y$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F_x$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = F_y$$

If θ_1 & θ_2 are generalized coordinates

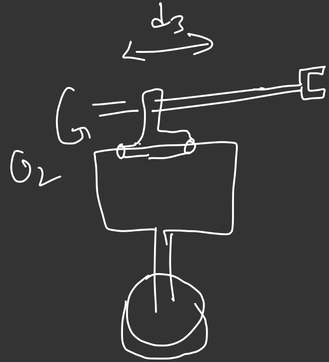
$$\vec{r} = (l_1 C\theta_1 + l_2 C_{12})\hat{i} + (l_1 S\theta_1 + l_2 S_{12})\hat{j}$$

$$\delta\vec{r} = \left(-l_1 S\theta_1 \cdot \delta\theta_1 - l_2 S_{12}(\delta\theta_1 + \delta\theta_2)\right)\hat{i}$$
$$+ \left(l_1 C\theta_1 \cdot \delta\theta_1 + l_2 C_{12}(\delta\theta_1 + \delta\theta_2)\right)\hat{j}$$

$$\delta W = \vec{F} \cdot \delta\vec{r}$$

$$= F_x\left(-l_1 S\theta_1 \delta\theta_1 - l_2 S_{12}(\delta\theta_1 + \delta\theta_2)\right)$$
$$+ F_y\left(l_1 C\theta_1 \cdot \delta\theta_1 + l_2 C_{12}(\delta\theta_1 + \delta\theta_2)\right)$$

Fig 3.37



$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = \tau_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) \frac{\partial L}{\partial \theta_2} = \tau_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial d_3} - \frac{\partial L}{\partial d_3}\right) = \tau_3$$

The jt variables are generalized co-ordinates & actuation at the joints are generalized forces

KE of rigid bodies

$$KE = T = \sum_{i=1}^{n} \frac{1}{2} m_i c_i^2 + \frac{1}{2} \sum_{i=1}^{n} I_c \omega_i^2$$

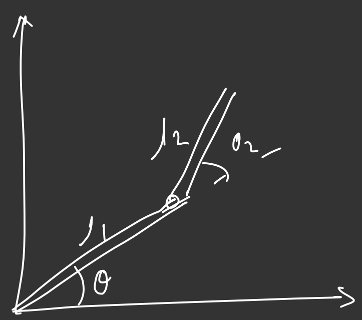
KE of rigid bodies in general motion

$$KE = T = \sum \frac{1}{2} m V_c^2 + \frac{1}{2} \sum I_c \omega^2$$

$$T = \frac{1}{2} m_c \, T \, v_c \cancel{+} \frac{1}{2} \omega^T I_c \, \omega$$

where $I_c = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$

$$\omega = \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix}$$



Assuming the masses of links are concentrated of distant end of link

$$\vec{r}_1 - l_1 \begin{Bmatrix} -S\theta_1 \\ C\theta_1 \\ 0 \end{Bmatrix} \dot{\theta}_1$$

$$\vec{r}_2 = \begin{Bmatrix} l\,S\theta_1 + l_2 C_{12} \\ l_1 S\theta_1 + l_2 S_{12} \\ 0 \end{Bmatrix}$$

$$= \frac{1}{2} m_1 \begin{Bmatrix} -s\theta_1 & c\theta_1 & 0 \end{Bmatrix} \begin{Bmatrix} s\theta_1 \\ c\theta_1 \\ 0 \end{Bmatrix} l_1^2 \dot{\theta}_1^2$$

$$\vec{V}_{c_2} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

$$\vec{r}_{c_2} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$x = (l_1 + l_2 C\theta_2) C\theta_1$$

$$y = (l_1 + l_2 C\theta_2) S\theta_1$$

$$z = l_2 S\theta_2$$

$$\dot{x} = -(l_1 + l_2 C\theta_2) S\theta_1 \cdot \dot{\theta}_1 - l_2 C\theta_2 S\theta_2 \dot{\theta}_2$$

$$\dot{y} = (l_1 + l_2 C\theta_2) S\theta_1 \cdot \dot{\theta}_1 - l_2 S\theta_1 \cdot S\theta_2 \cdot \dot{\theta}_2$$

now, $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = (l_1 + l_2 C\theta_2)^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_1^2$

$$T = \frac{1}{2} m_1 l_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2 \dot{\theta}_2^2 + \frac{1}{2} m_2 (l_1 + l_2 C\theta_2)^2 \dot{\theta}_1^2$$

$$V = m_2 g l_2 S\theta_2$$

$$L = T - V$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = m_1 l_1^2 \dot{\theta}_1^2 + m_2 (l_1 + l_2 C\theta_2)^2 \dot{\theta}_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = m_1 l_1^2 \ddot{\theta}_1 + m_2 (l_1 + l_2 C\theta_2)^2 \ddot{\theta}_1 - m_2 l_2 S\theta_2 \dot{\theta}_1 \dot{\theta}_2$$

$$\tau_1 = m_1 l_1^2 \ddot{\theta}_1 + m_2 (l_1 + l_2 c\theta_2)^2 \ddot{\theta}_1 - m_2 l_2 s\theta_2 \dot{\theta}_1 \dot{\theta}_2 - m_1 l_1^2 \dot{\theta}_1^2 - m_2 (l + l_2 c\theta_2)^2 \dot{\theta}_1$$

now

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2 \dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2 l_2^2 \ddot{\theta}_2$$

$$\frac{\partial L}{\partial \theta_2} = -m_2 (l + l_2 c\theta_2) l_2 s\theta_2 \dot{\theta}_1^2 - m_2 g l_2 c\theta_2$$

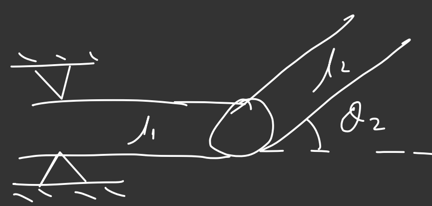
$$m_2 l_2^2 \ddot{\theta}_2 + m_2 (l_1 + l_2 c\theta_2) l_2 s\theta_2 \dot{\theta}_1^2$$

$$+ m_2 g l_2 c\theta_2 = \tau_2$$

6.16

$$KE_{m_1} = \frac{1}{2} m_1 \dot{d}_1^2$$

$$KE_2 = \frac{1}{2} I_1 \dot{\theta}_2^2 + \frac{1}{2} m_2 \dot{d}_1^2$$

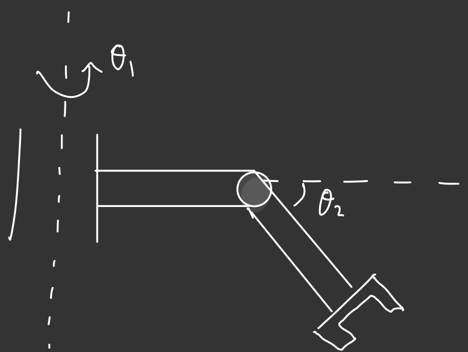


$$T = \frac{1}{2} m_1 \dot{d}_1^2 + \frac{1}{2} m_2 \dot{d}_1^2$$

$$+ \frac{1}{2} \begin{bmatrix} 0 & 0 & \dot{\theta}_2 \end{bmatrix} \begin{bmatrix} I_{xx_2} & 0 & 0 \\ 0 & I_{yy_2} & 0 \\ 0 & 0 & I_{zz_2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$$

TRAJECTORY GENERATION

Sequence of pts is path and the history of vel posⁿ & acceleration of joint variables referred as trajectory



<u>The process of robotic operⁿ</u>

Identify the task

↓

Trajectory planning ← Take decisions to choose traj to comply with time constraints smooth operⁿ

↓

Kinematics

Given end effector path $\begin{Bmatrix} X_t \\ Y_t \\ Z_t \end{Bmatrix}$

Trajectory of degree of freedom $\begin{Bmatrix} \theta_1(t) \\ \theta_2(t) \end{Bmatrix}$

$$\begin{Bmatrix} x_t(t) \\ y_t(t) \\ z_{(t)}(t) \end{Bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \\ J_{31} & J_{32} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}$$

$$\begin{Bmatrix} \theta_1(t) \\ \theta_2(t) \end{Bmatrix} = J^{-1} \begin{Bmatrix} x_t(t) \\ y_{(t)}(t) \\ z_{(t)}(t) \end{Bmatrix}$$

Pseudo inv

$J_{11} = -l_1 S\theta_1 - l_2 C\theta_2 S\theta_1$

$J_{12} = -l_2 C\theta_1 S\theta_2$

$J_{21} = l_1 C\theta_1 + l_2 C\theta_1 C\theta_2$

$J_{22} = -l_2 S\theta_1 S\theta_2$

$J_{31} = 0$

$J_{32} = l_2 C\theta_2$

Quiz - 2

- Dyna, Jacobian

$$\begin{Bmatrix} \dot\theta_1(t) \\ \dot\theta_2(t) \end{Bmatrix} = J^{-1} \begin{Bmatrix} x_E(t) \\ y(t) \\ z(t) \end{Bmatrix}$$

* The controller checks the singularity for all the θ while calculating the Jacobian and this increases the computational time & power consumption. To tackle the problem we will use jt space scheme

Joint Space Scheme

Each path pt is usually specified interms of desired pos^n & orientation of the tool frame rel to the station frame. Each of these via points is converted into a set of desired joint angles by application of the inverse kinematics. Then a smooth fuction is found for each of the joints

Cubic Poly

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{bmatrix} a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ a_0^2 + a_1^2 t + a_2^1 t + a_3^2 t^3 \\ a_0^3 + a_1^3 t + a_2^3 t^2 + a_3^3 t^3 \end{bmatrix}$$

$$\theta_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

I $\theta(t)\Big|_{t=0} = \theta_0$

III $\theta_t\Big|_{t=t_f} = \theta_f$

II $\dot{\theta}(t)\Big|_{t=0} = 0$

IV $\dot{\theta}_t\Big|_{t=t_f} = 0$

applying I

$\theta_0 = a_0$

Applying II

$0 = a_1$

$$\theta_t = \theta_0 + a_2 t_f^2 + a_3 t_f^3 \quad (1)$$

Applying IV

$$0 = 2a_2 t_f + 3a_3 t_f^2$$

$$a_3 = \frac{-2}{3}\frac{a_2}{t_f} \quad \text{— (11)}$$

Subtracting (2) − (1)