SYLLABUS

MODULE	(NO. OF LECTURE HOURS)
Module – I Stress at a point on a plane, Stress transformation equation, Principal stresses, Mohr's circle of stresses, Strain transformation equation, principal strain, strain rosette.	**9**
Module – II Types of Beams, Types of loading and support, Relationship between Shear force, Bending Moment and intensity of loading, SFD, BMD, Point of Contraflexure, second moment of area, parallel axes theorem, Bending stress and shear stress in beam.	**9**
Module – III Deflection of Beams, Double integration method, Macaulay's method, Moment area method, Torsion of circular shafts.	**9**
Module – IV Buckling of columns. Strain energy method, Castigliano's theorem, application of energy method on different types of beams and thin circular ring.	**9**
Module – V Thin and thick cylinders: Radial and circumferential stresses, stresses produced due to shrink fit. Rotating Disc: Stresses in disc of uniform thickness and uniform strength.	**9**

TEXTBOOKS:
1. Strength of Materials by E J Hearn.
2. Strength of Materials by S. S. Rattan.
3. Mechanics of Material by Riley, Sturges, Morris

REFERENCE BOOKS:
1. Mechanics of Materials by S. Timoshenko and James M. Gere.
2. Strength of Materials by Ryder.
3. Advanced Mechanics of Material by Seely& Smith

GAPS IN THE SYLLABUS (TO MEET INDUSTRY/PROFESSION REQUIREMENTS)
Material nonlinearity, fatigue analysis, and real-world design standards and codes.

POS MET THROUGH GAPS IN THE SYLLABUS: PO 1-5.

TOPICS BEYOND SYLLABUS/ADVANCED TOPICS/DESIGN
Analysis and design of composite beams and advanced stress analysis.

POS MET THROUGH TOPICS BEYOND SYLLABUS/ADVANCED TOPICS/DESIGN: PO 1-5, PO 11-12.

COURSE OUTCOME (CO) ATTAINMENT ASSESSMENT TOOLS & EVALUATION PROCEDURE

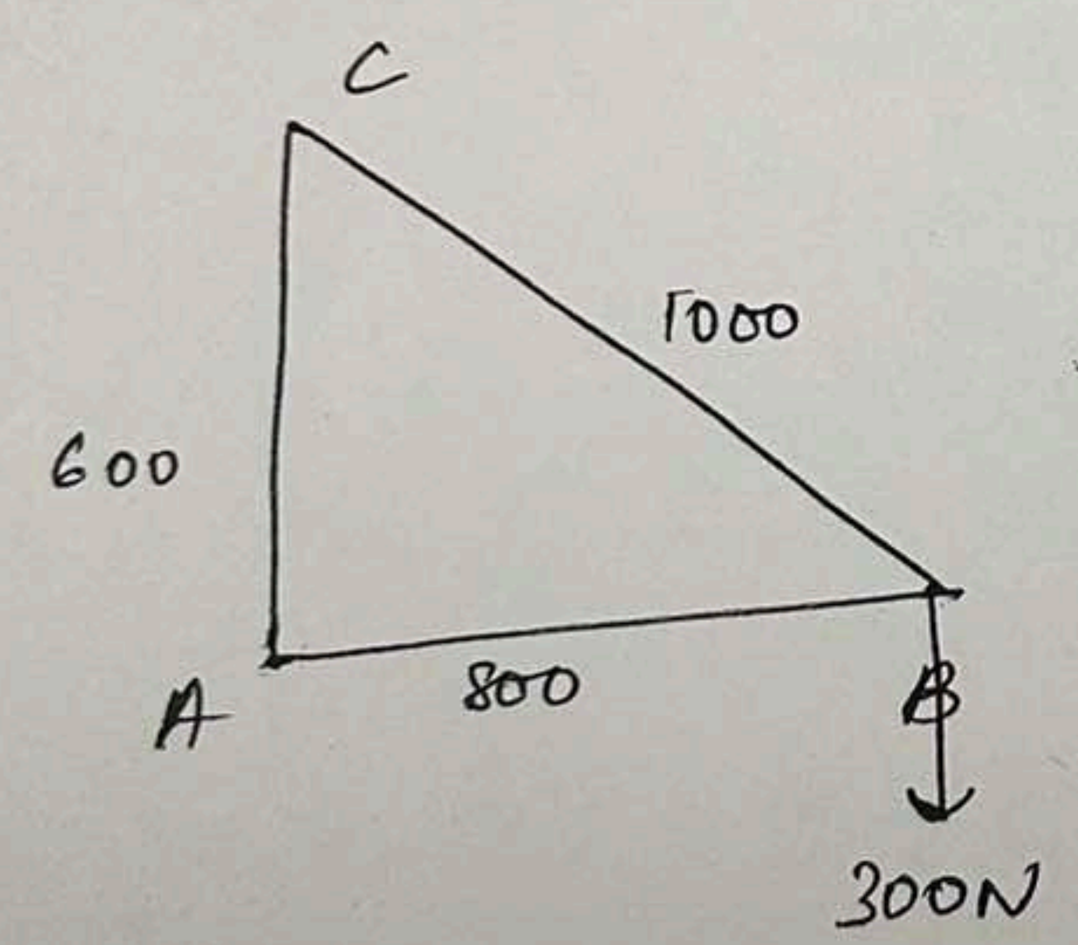
DIRECT ASSESSMENT

Assessment Tool	% Contribution during CO Assessment
Progressive Evaluation	50
End Semester Examination	50

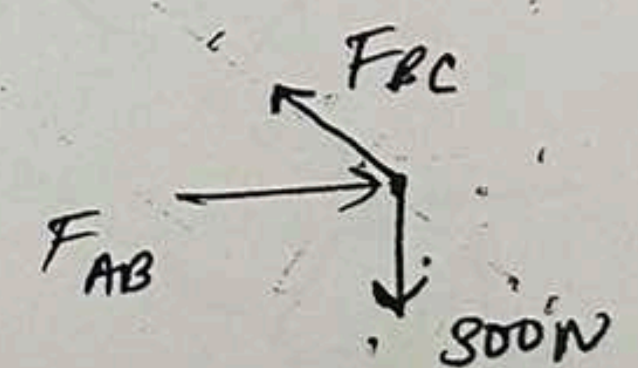
MODULE 1

- Mechanics of solids is a branch of applied mechanics that deals with behaviour of materials subjected to various types of loading.
- <u>Synthesis</u> - Determining or defining shape of a structure which can serve purpose.
- <u>Design</u> - Determining cross-section of members of structure.
- <u>Stress</u> - Qualitatively, internally distributed resisting force. We quantify it as intensity of force on a cross-section.

Ex:



At B,



$\Sigma F_y = 0$

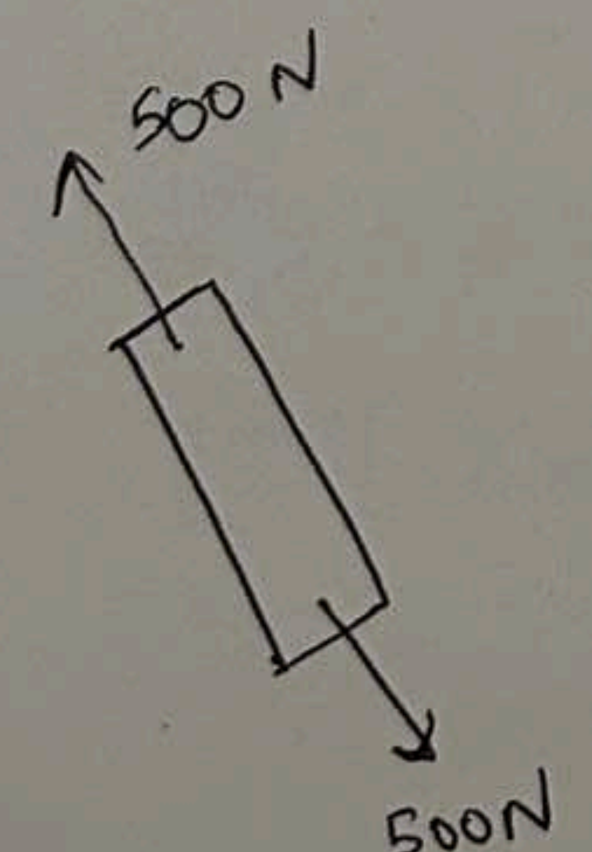
$F_{BC} \sin\theta = 300$

$F_{BC} = 500N$ (T)

$\Sigma F_x = 0$

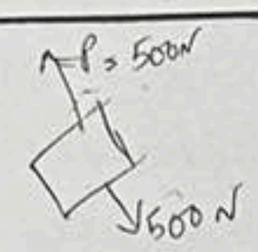
$F_{AB} = F_{BC} \cos\theta$

$F_{AB} = 400N$ (C)

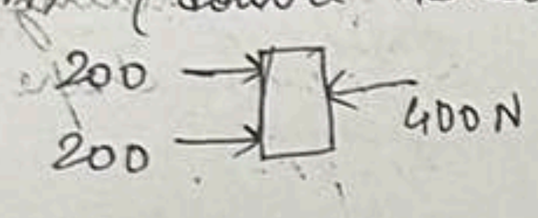


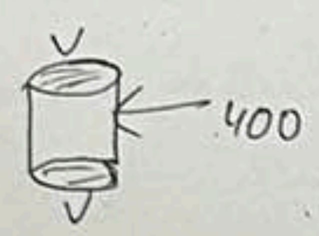
7/8 Pin at C, $\Rightarrow$ Case of single sheer

$\tau = \frac{P}{A} = \frac{500}{\pi \times \frac{25^2}{4}} = 1.02 \ N/mm^2$



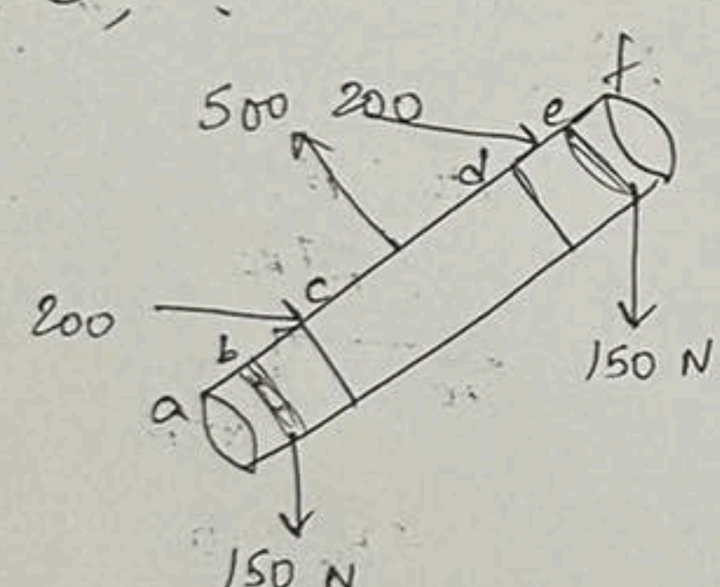
Pin at A (double sheer)



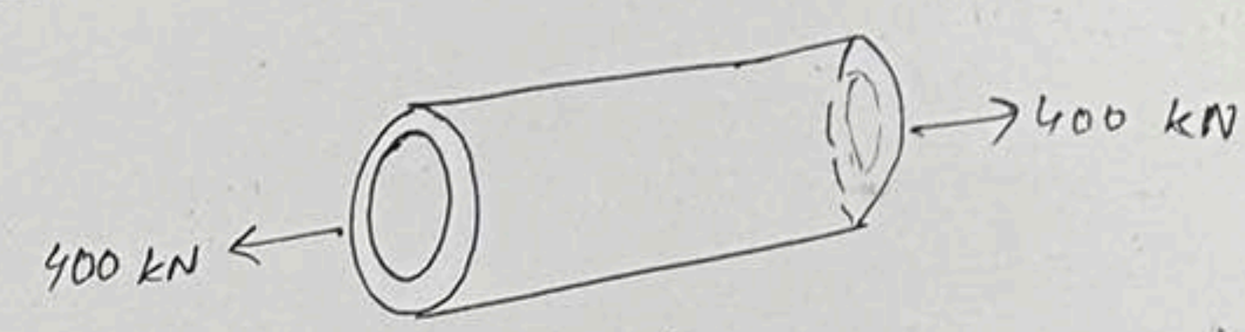


$\tau = \frac{V}{A} = \frac{200}{\pi \times \frac{25^2}{4}} = 0.407 \ N/mm^2$

Pin at B,



104)

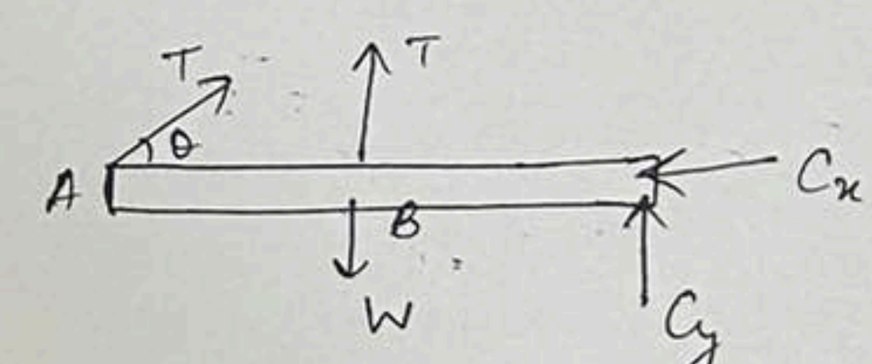


$\sigma = \frac{400 \ kN}{\frac{\pi(d_o^2 - 100^2)}{4}} \leq 120 \ MN/m^2$

$\Rightarrow \frac{400 \times 10^3}{\pi\left(\frac{d_o^2 - 100^2}{4}\right)} \leq 120 \ N/mm^2$

$\therefore d_o = 119 \ mm$

106)



$\Sigma F_x = 0$

$T\cos\theta = C_x$ — (i)

$\Sigma F_y = 0$

$\Rightarrow T\sin\theta + T = C_y + W$ — (ii)

$\Sigma M_C = 0$

$\Rightarrow 10T\sin\theta + 5T = 5W$ — (iii)

$\Rightarrow T = \frac{5W}{10\sin\theta + 5} = \frac{5 \times 6000}{10 \times \frac{3}{\sqrt{5^2+3^2}} + 5}$

$= 2957 \ lb$

$\sigma = \frac{T}{A} = \frac{2957}{\pi \times \frac{0.6^2}{4}} = 10.46 \ ksi$ (kilo pound/inch²)

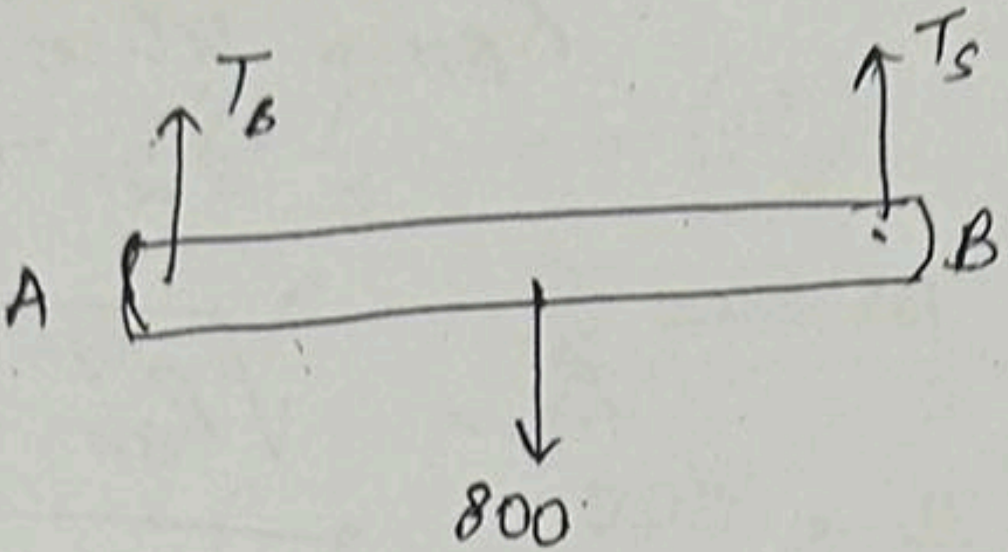
Step 1: Draw FBD

2: Find force on member where stress is req

3: Take sections where stress is needed

2: Write static eqlb eq

105)



1 kg = 9.8 N

1 Pa = 1 N/m^2

$$\frac{T_B}{A_B} \le 90 \text{ MPa}$$

$$\frac{400 \times 9.8}{A_B} \le 90 \times 10^6$$

$$A_B = 43.6 \text{ mm}^2$$

$$\frac{400 \times 9.8}{A_S} \le 120 \times 10^6$$

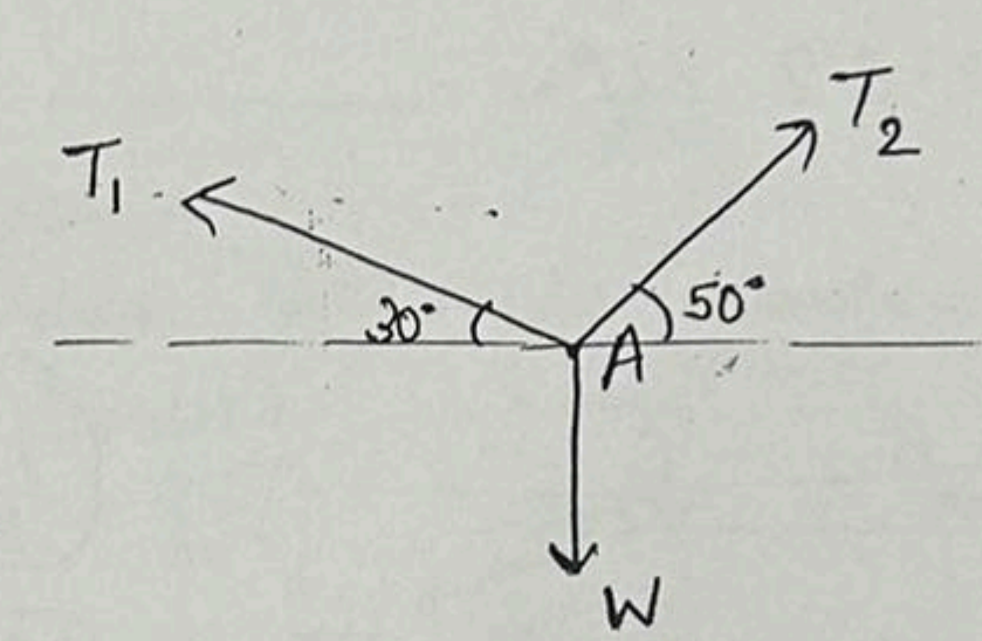
$$A_S = 32.7 \text{ mm}^2$$

HW - 109, 110, 114

109) Wire AB,

$\sigma_1 \le 30$ ksi

$$\frac{T_1}{0.4 \text{ in}^2} \le 30 \text{ ksi}$$



Wire AC, $\dfrac{T_2}{0.5 \text{ in}^2} \le 30$ ksi

$\Sigma F_x = 0 \Rightarrow T_1 \cos 30° = T_2 \cos 50°$ —— (i)

$\Sigma F_y = 0 \Rightarrow T_1 \sin 30° + T_2 \sin 50° = W$ —— (ii)

$T_2 = 0.879\,W$, $T_1 = 0.653\,W$

$$\frac{0.653\,W}{0.4 \text{ in}^2} \le 30 \text{ ksi}$$

$W \le 18.4$ kips (kilo pounds-force) $\frac{F}{A}$

$$\frac{0.879\,W}{0.5 \text{ in}^2} \le 30 \text{ ksi}$$

$W \le 17.1$ kips —— Ans

$1 kg = 9.8 N$

$1 Pa = 1 N/m^2$

$\frac{0 \times 9.8}{A_S} \le 120 \times 10^6$

$A_S = 32.7 \ mm^2$

T_2

50°

$50° \text{ —— (i)}$

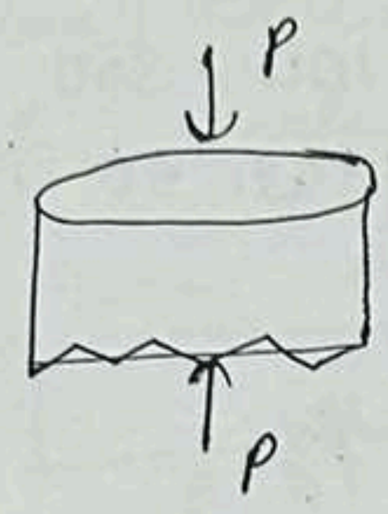
$= W \text{ —— (ii)}$

653 W

unds - force) $\frac{T}{A}$

110) $\sigma_W \le 1800$ psi $\qquad \sigma_c \le 650$ psi

Wood:



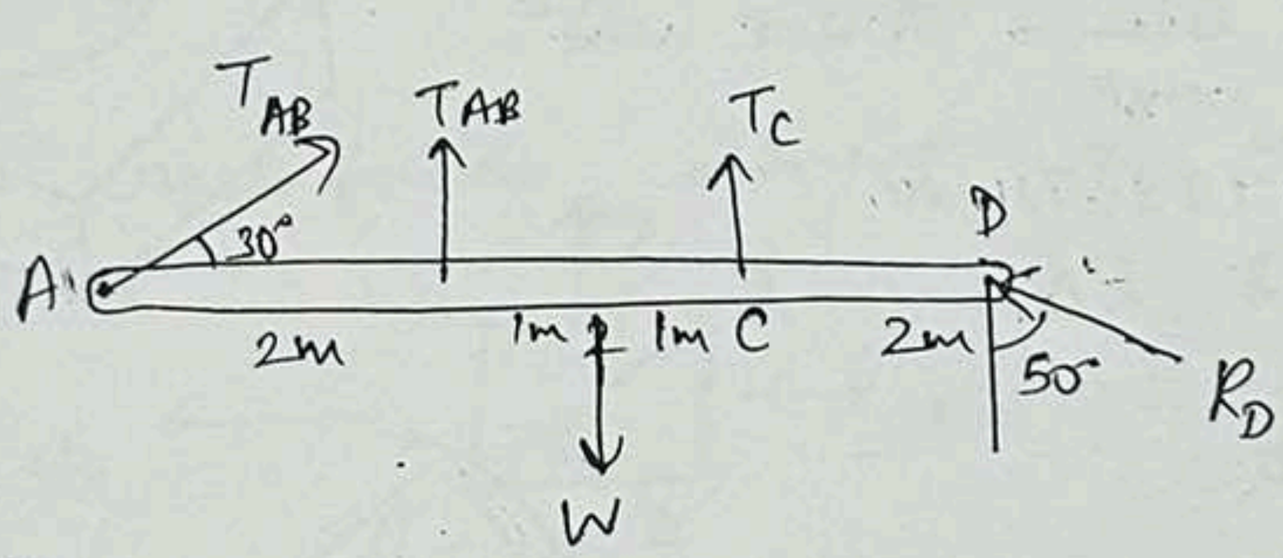
$$\frac{P}{\pi \times \frac{8^2}{4}} \le 1800 \text{ psi}$$ (pound/sq inch)

$P_W \le 90478$ lb —— Ans

Concrete: $\frac{P}{12 \times 12 \ in^2} \le 650$ psi

$P_C \le 93600$ lb

114)



$\Sigma F_x = 0$

$T_{AB} \cos 30° = R_D \sin 50° \Rightarrow R_D = 1.1305 \ T_{AB}$ —— (i)

$\Sigma F_y = 0$

$T_{AB}(1 + \sin 30°) + T_C + R_D \cos 50° = W$

$\Rightarrow (1 + \frac{1}{2}) T_{AB} + T_C + 1.1305 T_{AB} \cos 50° = W$

$\Rightarrow T_C = W - 2.2267 \ T_{AB}$ —— (ii)

$\Sigma M_D = 0$

$6 T_{AB} \sin 30° + 4 T_{AB} + 2 T_C = 3W$

$\Rightarrow 7 T_{AB} + 2(W - 2.2267 T_{AB}) = 3W$

$\Rightarrow T_{AB} = 0.3927 \ W$

Cable AB: $\sigma_{AB} = \frac{T_{AB}}{A_{AB}}$

$100 \ MPa = \frac{0.3927 W}{250 mm^2} \Rightarrow W = 63669.92$ N (ell

Cable at ... $T_2 = \sigma_c A_c$

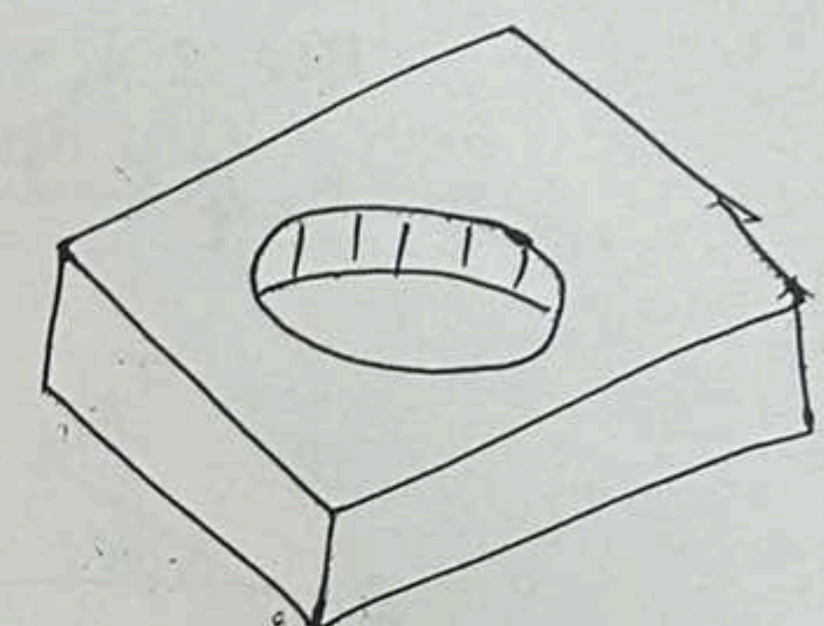
$\Rightarrow 0.1256\,W = 100 \times 300$

$\Rightarrow W = 238853.50$ N

$W = mg$

$\Rightarrow m = \dfrac{63669.92}{9.81} = 6490$ kg

115) $\tau = \dfrac{V}{\pi \times 20 \times 25}$

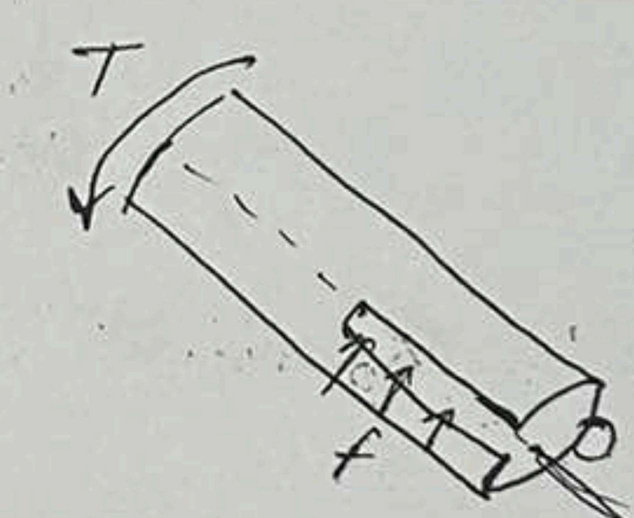


$\Rightarrow V = 350 \dfrac{N}{mm^2} \times 500\pi\ mm^2$

$= 549778.71$ N

$= 549.8$ kN

118) $\Sigma M_O = 0$



$T - \dfrac{fD}{2} = 0$

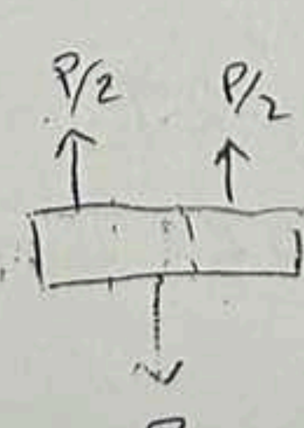
(i) $f = \dfrac{2 \times 2.2 \times 10^3 \times 10^3}{60}$

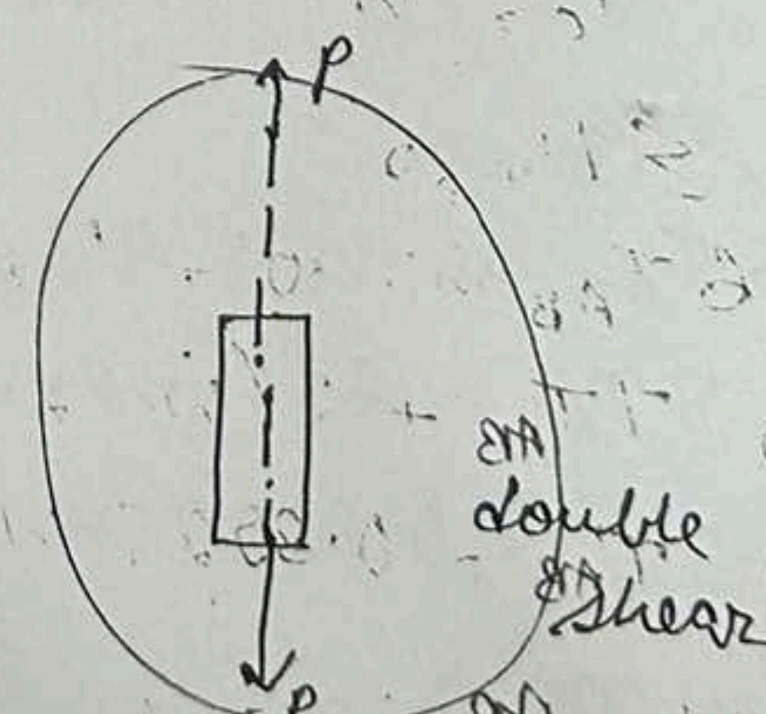
$= 73.3$ kN

60 MPa $= \dfrac{73.3 \times 10^3}{70 \times b}$

$b = 17.46\ mm^2$

117) 300 MPa $= \dfrac{400 \times 10^3}{2 \times \frac{\pi}{4} d^2}$





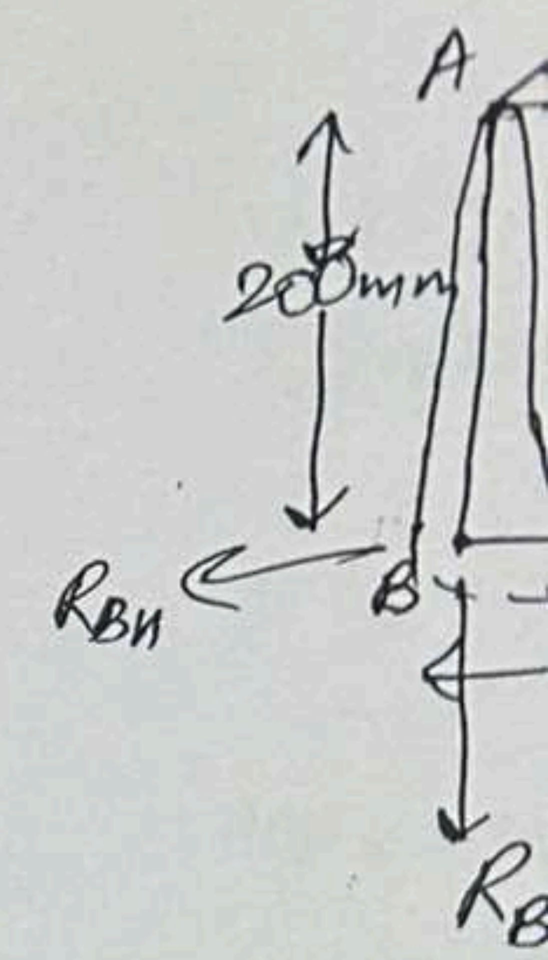
$\Rightarrow d^2 = \dfrac{8}{3} \times \dfrac{10^3}{\pi}$

$d = 29.13$ mm —— Ans

119) $\Sigma M_C = 0$

$\Rightarrow 0.25\,R_{BV} = 0.25(40 \sin 35°) + 0.2(40 \cos 35°)$

$R_{BV} = 49.156$ kN

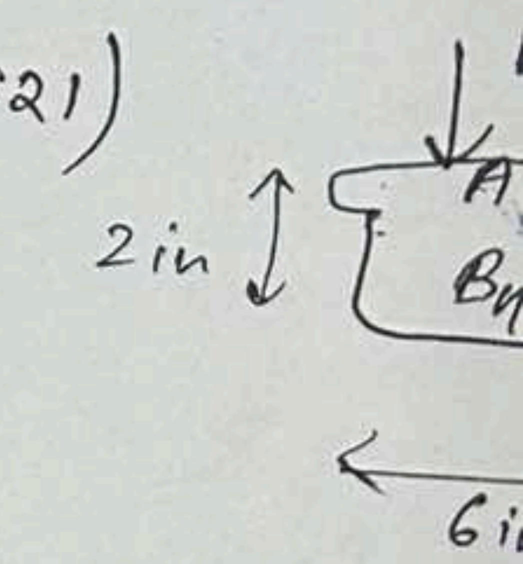


$V_B =$

59.076

$\Rightarrow \tau_B =$

121)



$\Sigma M_B = 0$

$\Rightarrow 6P = 2$

$\Sigma F_x = 0$

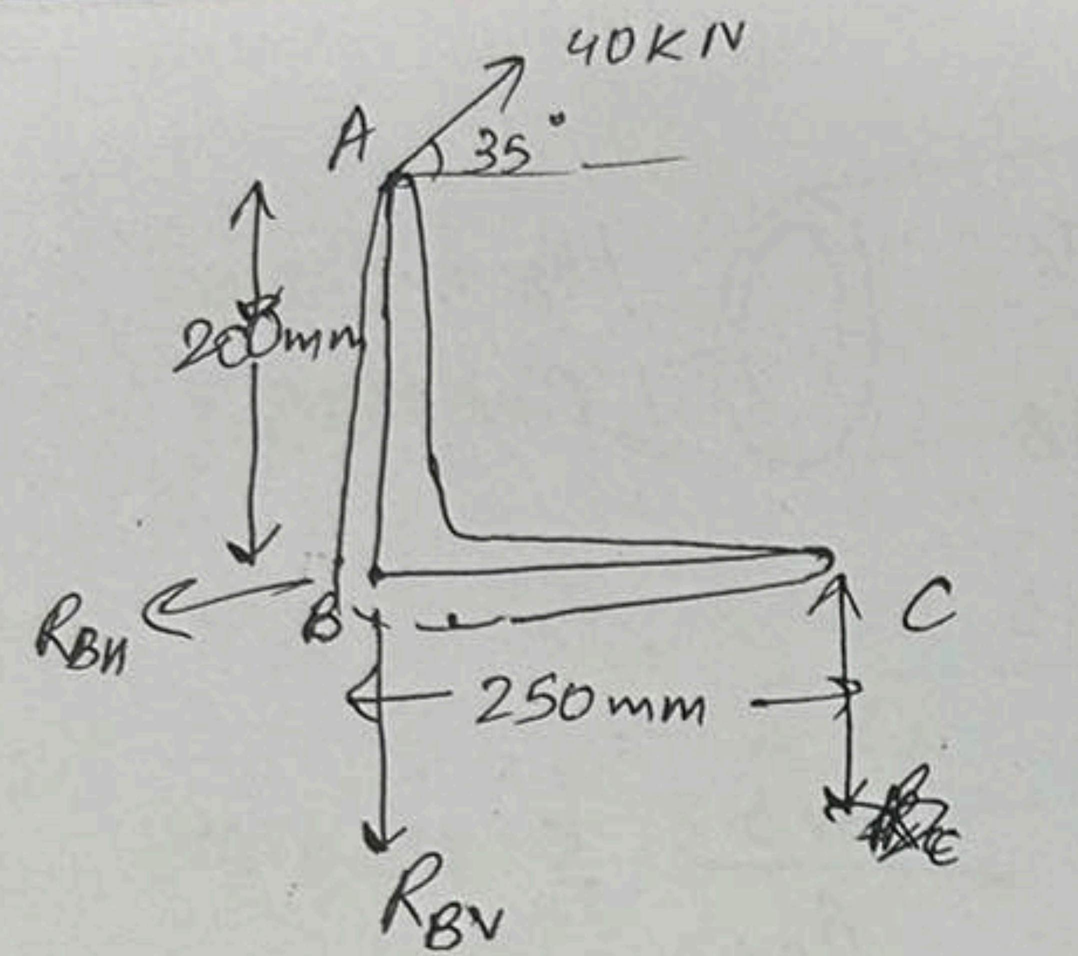
$\Sigma F_y = 0$

$\Rightarrow B_V = T s$

$\Rightarrow B_V = 3P$

$\Rightarrow B_V = 4P$

$R_B^2 =$



$\Sigma F_x = 0$

$R_{BH} = 40 \cos 35°$

$= 32.766$ kN

$R_B = \sqrt{R_{BH}^2 + R_{BV}^2}$

$= \sqrt{32.766^2 + 49.156^2}$

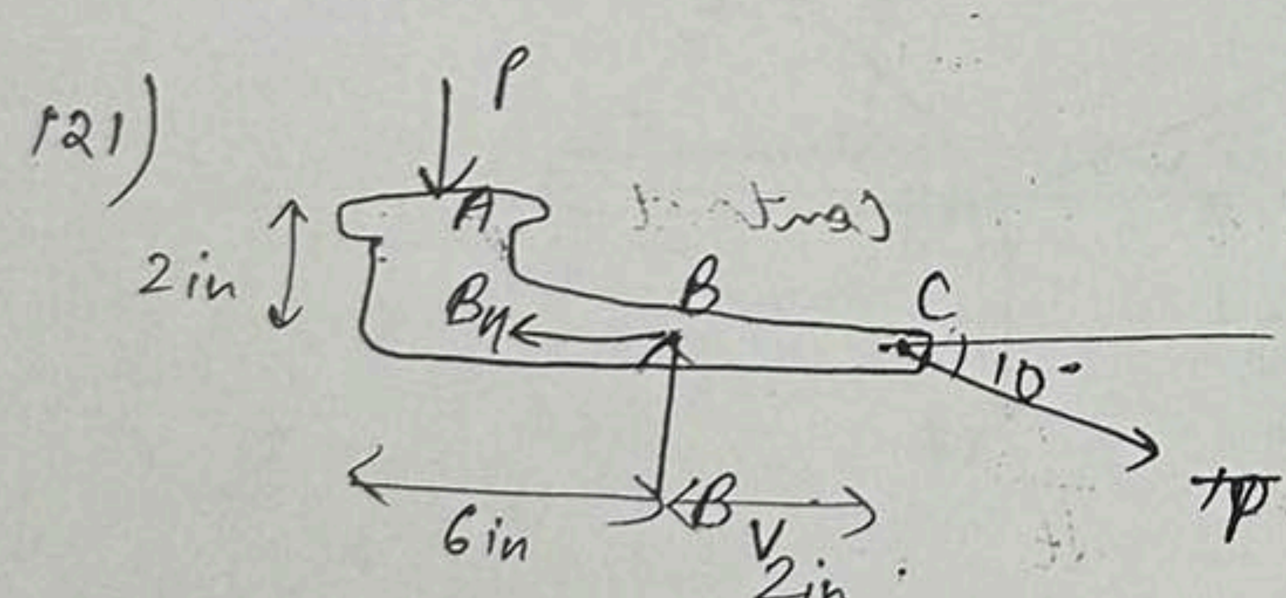
$= 59.076$ kN

$V_B = \tau_B A$ (double shear)

$59.076 \times 10^3 = \tau_B \left(\frac{2\pi}{4} \times 20^2\right)$

$\Rightarrow \tau_B = 94.02$ MPa ______ Ans

121)



$\frac{10}{10}$ 7/8/25

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$\Sigma M_B = 0$

$\Rightarrow 6P = 2T \sin 10°$ ______ (i)

$\Sigma F_x = 0 \Rightarrow B_H = T \cos 10°$

$B_H = \frac{3P \cos 10°}{\sin 10°}$ (from (i))

$B_H = 3P \cot 10°$ ______ (ii)

$\Sigma F_y = 0$

$\Rightarrow B_V = T \sin 10° + P$

$\Rightarrow B_V = 3P + P$

$\Rightarrow B_V = 4P$ ______ (iii)

$R_B^2 = B_H^2 + B_V^2$

$\Rightarrow R_B^2 = (3P\cos 10^\circ)^2 + 16P^2$

$\Rightarrow R_B^2 = 305.47 P^2$

$\Rightarrow R_B = 17.48\,P$ —— (iii)

From (i) $\Rightarrow P = \dfrac{T}{3}\sin 10^\circ$ (Control rod)

$$P = \frac{5000}{3} \times \frac{\pi}{4} \times 0.5^2 \sin 10^\circ$$

$$P = 56.83 \text{ lb}$$

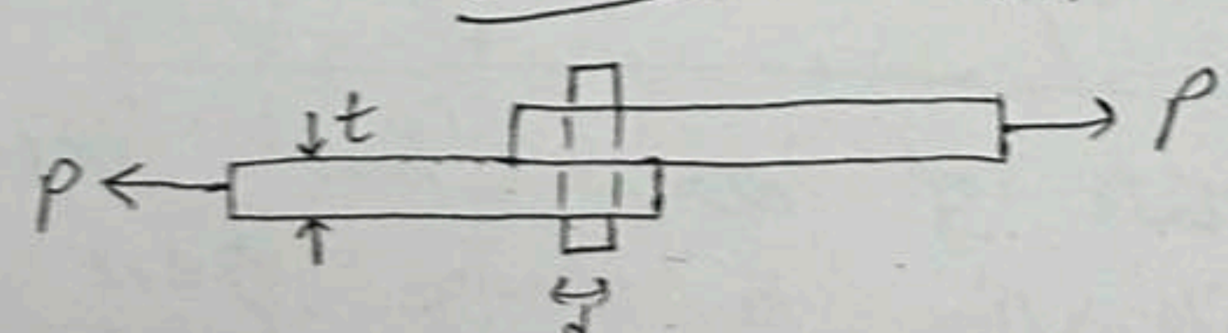
For the pin,

$$P = \frac{4000 \times \frac{\pi}{4} \times (0.25)^2}{17.48} \quad \text{(from eq (iii))}$$

$\Rightarrow P = 11.23$ lb —— Ans

- <u>Bearing stress</u> - arises due to contact pressure b/w 2 separate bodies

$$\boxed{\sigma_b = \frac{P}{dt}}$$



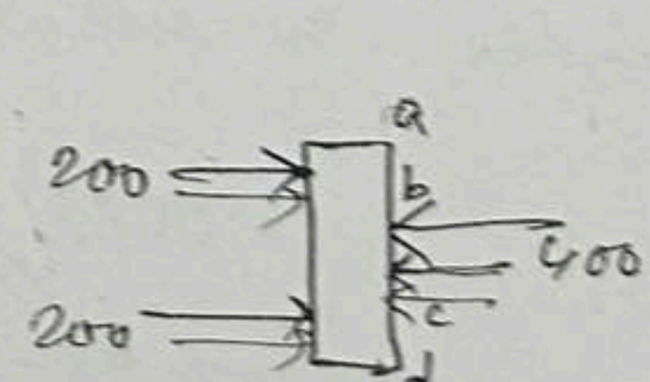
$A = dt$

At Point A,

Projected area b/w ab

$A_{ab} = 25 \times 25$

$\sigma_b = \dfrac{200}{25 \times 25}$ N/mm²



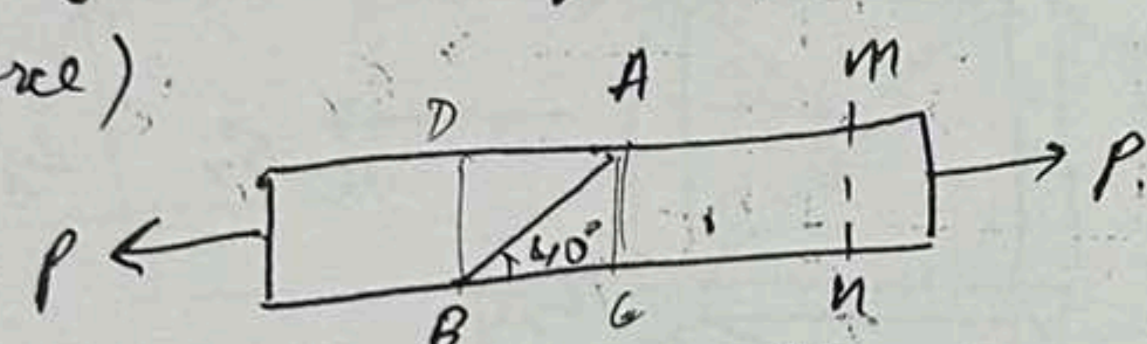
Projected area b/w bc $\Rightarrow A_{bc} = 25 \times 30$

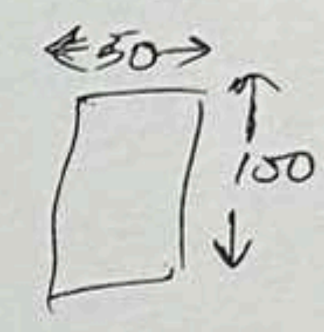
$\sigma_b = \dfrac{400}{25 \times 30}$ N/mm²

HW: Pg-1 $\Rightarrow$ Section 1-2

322 $\Rightarrow$ 9-5, 9-6 (read & understand)

~~Q25~~ Q) 2 wooden joists are joined together by glue as shown. Determine normal & shear stress at joint surface, if $P = 200$ kN (applied tensile force).

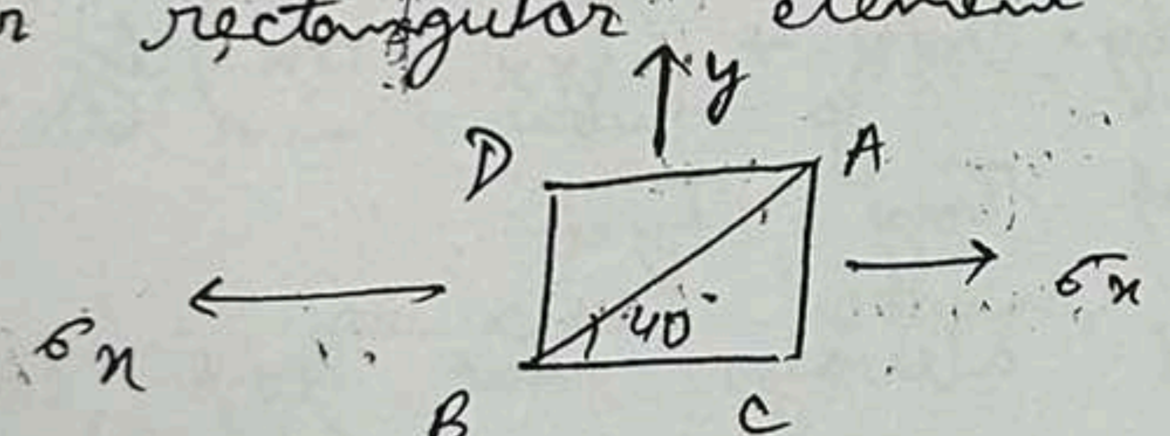




→ On section mn,

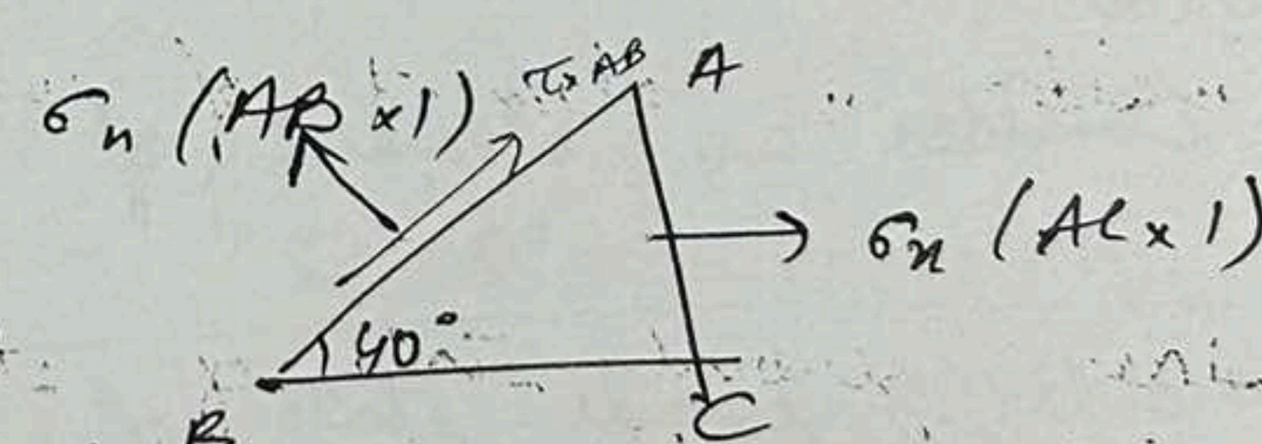
$$\sigma_x = \frac{P}{A} = \frac{200 \times 10^3}{50 \times 100} = 40 \text{ N/mm}^2$$

Consider rectangular element of unit thickness.

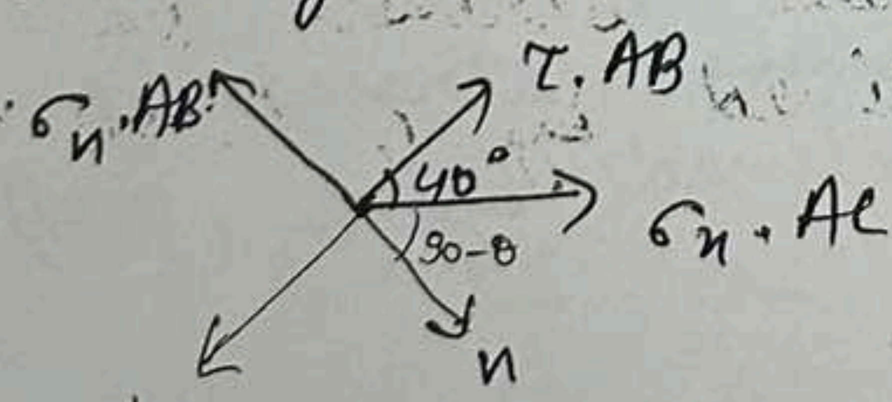


$\theta = 40°$

FBD of ABC part:



Forces diagram:



$\Sigma F_n = 0$

$\Rightarrow \sigma_n . AB = \sigma_x . AC \sin\theta$

$\sigma_n = \sigma_x \left(\frac{AC}{AB}\right) \sin\theta$

$= \sigma_x \sin^2\theta$

$= 40 \times \sin^2 40°$

$= 16.53$ N/mm²

$\Sigma F_t = 0$

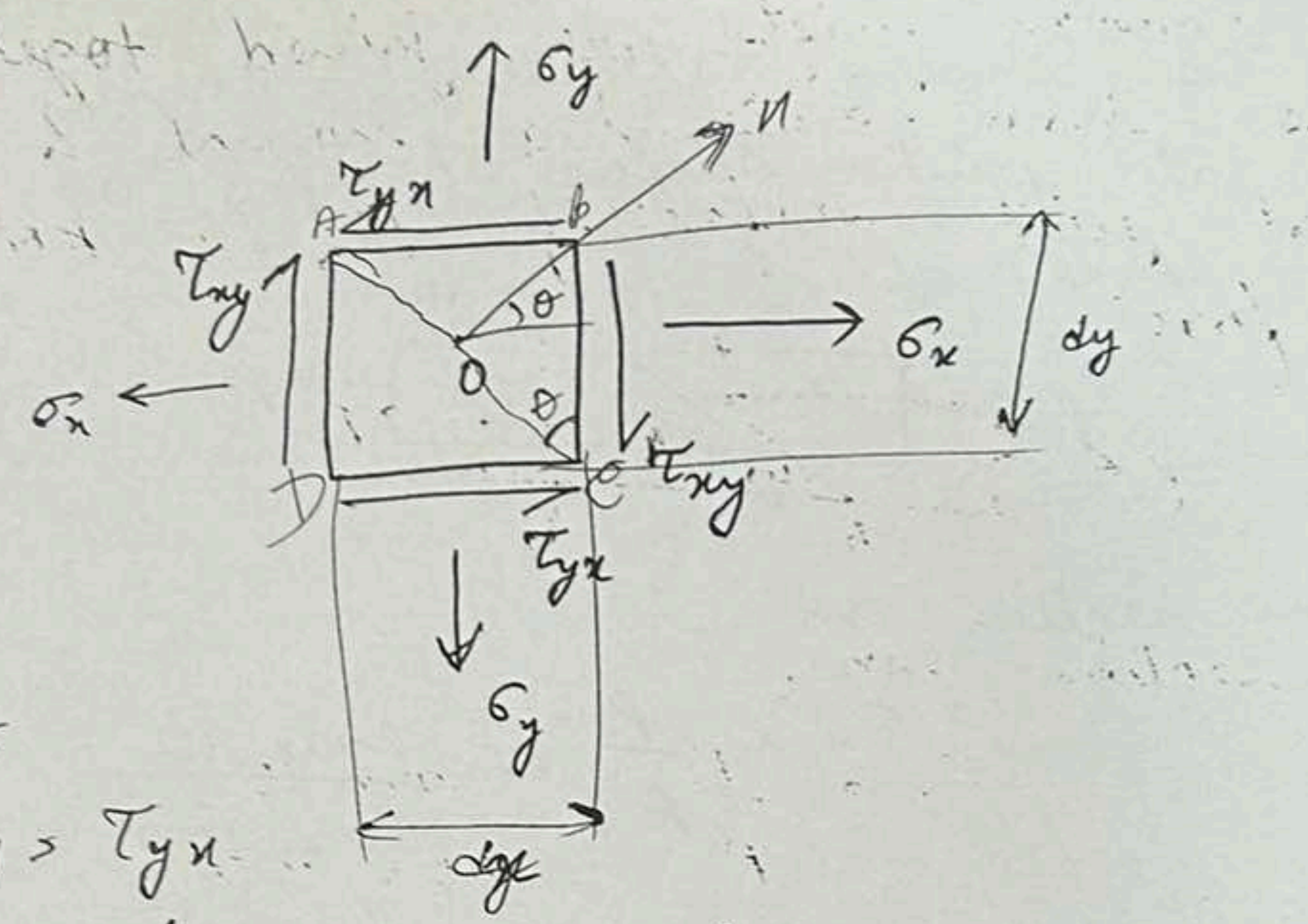
$\Rightarrow \tau . AB = -\sigma_x . AC \cos\theta$

$\Rightarrow \tau = -\frac{\sigma_x}{2} \sin 2\theta$

$= -20 \sin 80°$

$= -19.7$ N/mm²

~~Stress at a pt.~~



i) Prove that
$\tau_{xy} = \tau_{yx}$

$\Sigma M_O = 0 \Rightarrow -(\tau_{xy}(dy \times 1))dx + (\tau_{yx} \times dx)dy = 0$

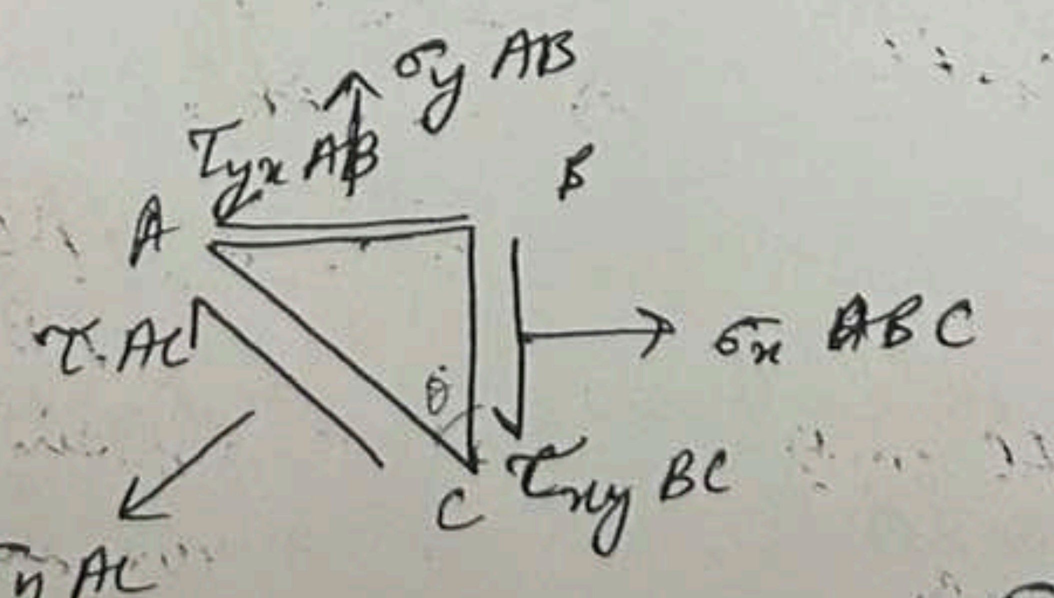
$\Rightarrow \tau_{xy} = \tau_{yx}$.

ii) Shear stress will always have opp complimentary stress

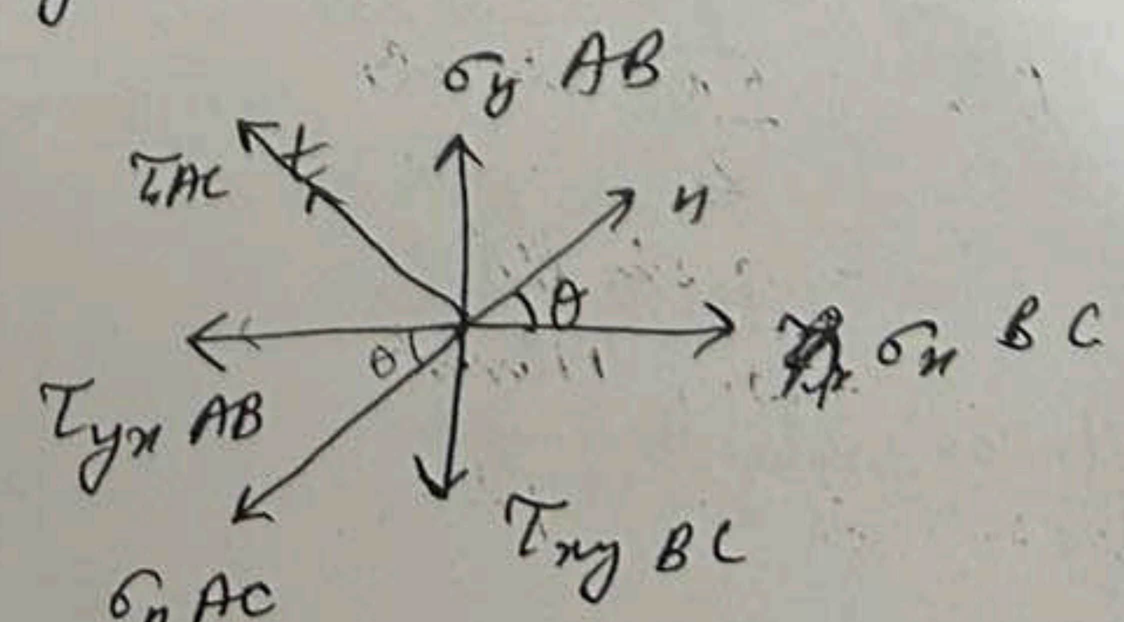
Consider an element of unit thickness (dimension $dx \times dy \times 1$)

Consider general state of stress:
Our interest lies to determine stresses on plane whose normal is inclined by θ (CCW) with x-axis

Case II



Force diagrams:



$\Sigma F_n =$

$\Rightarrow \sigma_n = \sigma_x$

$= \sigma_x ($

$\therefore \sigma(\theta) =$

ΣF_t

$\Rightarrow \tau . AC =$

$\Rightarrow \tau_n = (\sigma_x -$

$\tau(\theta) = \left(\frac{\sigma_x -}{2}\right.$

Remember st
under consid

Case II: $\sigma(90$
$\tau(90+\theta)$

$\sigma(90+\theta) =$

$=$

$$\Sigma F_n = 0 \Rightarrow \sigma_x (BC)\cos\theta + \sigma_y (AB)\sin\theta = \sigma_n AC + \tau_{yx} AB\cos\theta + \tau_{xy} BC\sin\theta$$

$$\Rightarrow \sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta - 2\tau_{xy}\sin\theta\cos\theta$$

$(\because \tau_{xy} = \tau_{yx})$

$$= \sigma_x\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) - \tau_{xy}\sin 2\theta$$

$$\boxed{\sigma(\theta) = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta}$$

$$\Sigma F_t = 0$$

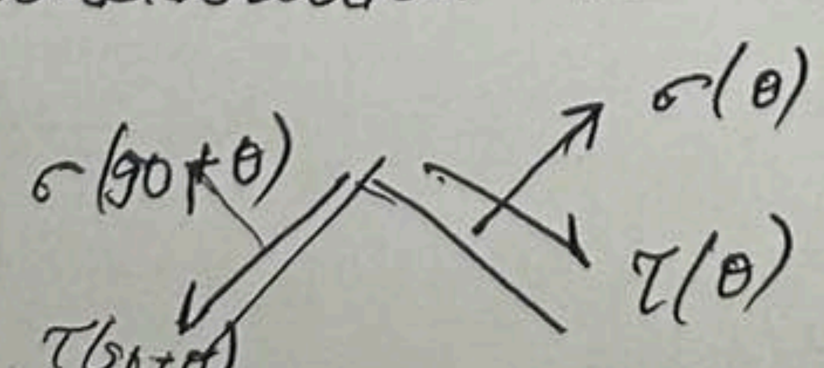
$$\Rightarrow \tau \cdot AC = \sigma_n (BC)\sin\theta - \sigma_y (AB)\cos\theta - \tau_{xy} AB\, S_\theta$$
$$\tau_{yx}(BC) C_\theta$$

$$\Rightarrow \tau_n = (\sigma_x - \sigma_y) S_\theta C_\theta + \tau_{xy}(C_\theta^2 - S_\theta^2) \quad (\because \tau_{xy} = \tau_{yx})$$

$$\boxed{\tau(\theta) = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta}$$

Remember, state of stress has been taken under consideration to derive these formulae

Case II:



$$\sigma(90+\theta) = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos\{2(90+\theta)\} - \tau_{xy}\sin\{2(90+\theta)\}$$

$$= \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

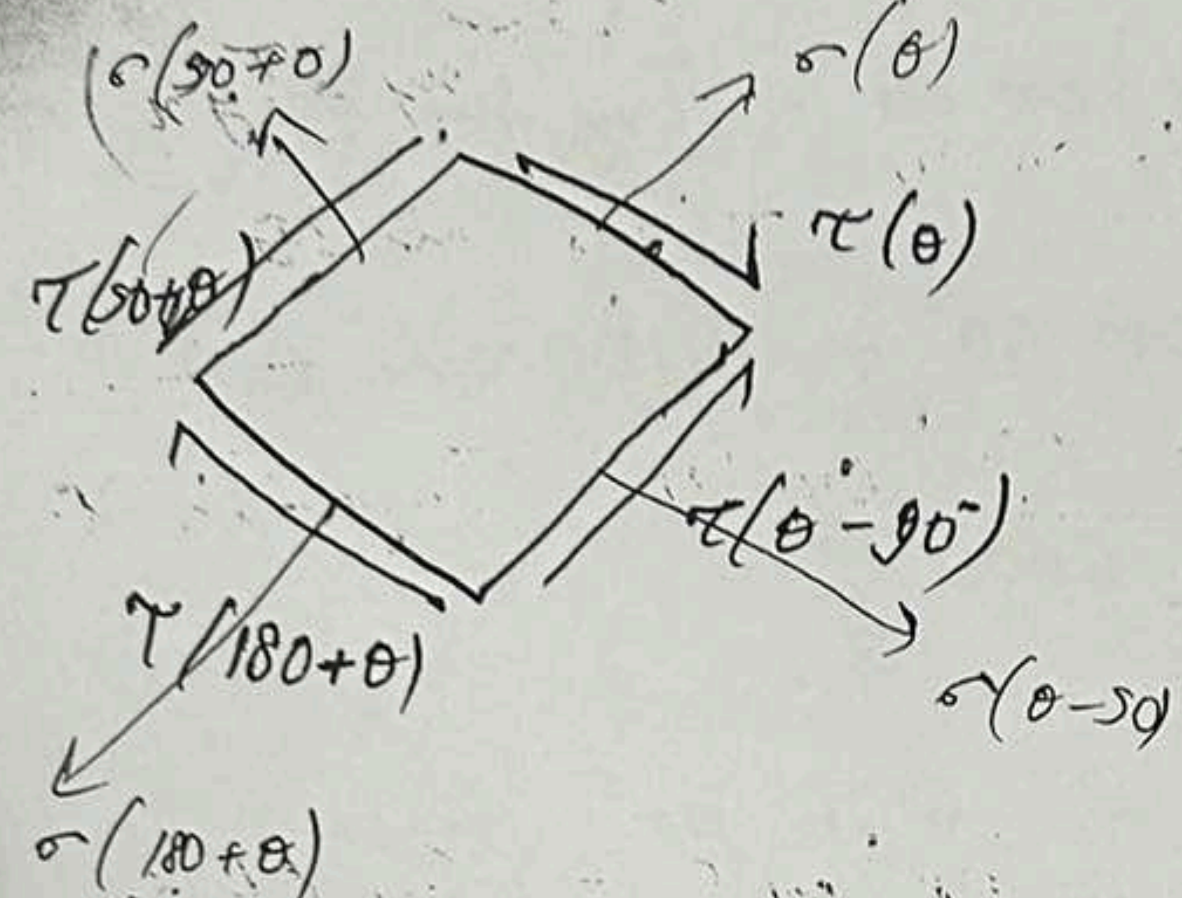
$$\tau(90+\theta) = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = -\tau(\theta)$$

so $\tau(90+\theta)$ ACW

Prove that

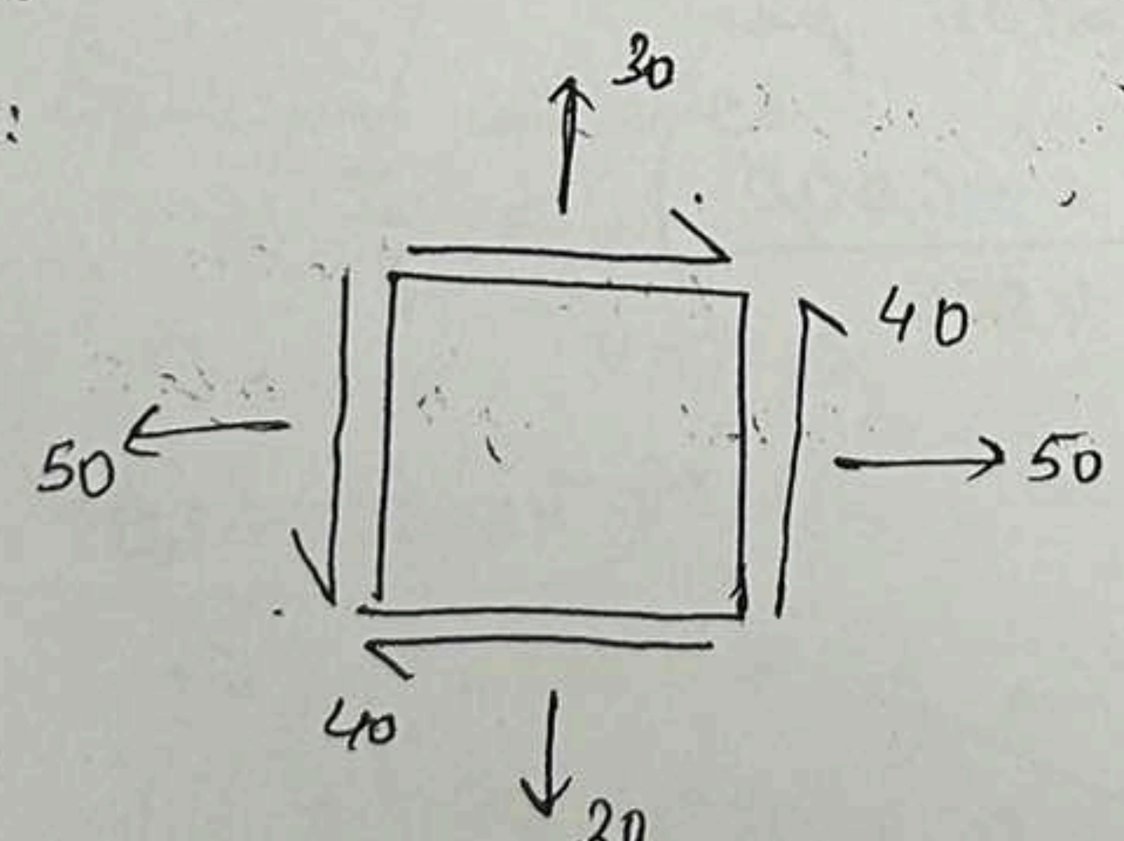
(i) $\sigma(180+\theta) = \sigma(\theta)$

(ii) $\sigma(\theta - 90^\circ) = \sigma(90+\theta)$



Conclusion: We transformed state of stress on an element, which is rotated θ (CCW) relative to original / known state of stress.

Ex:



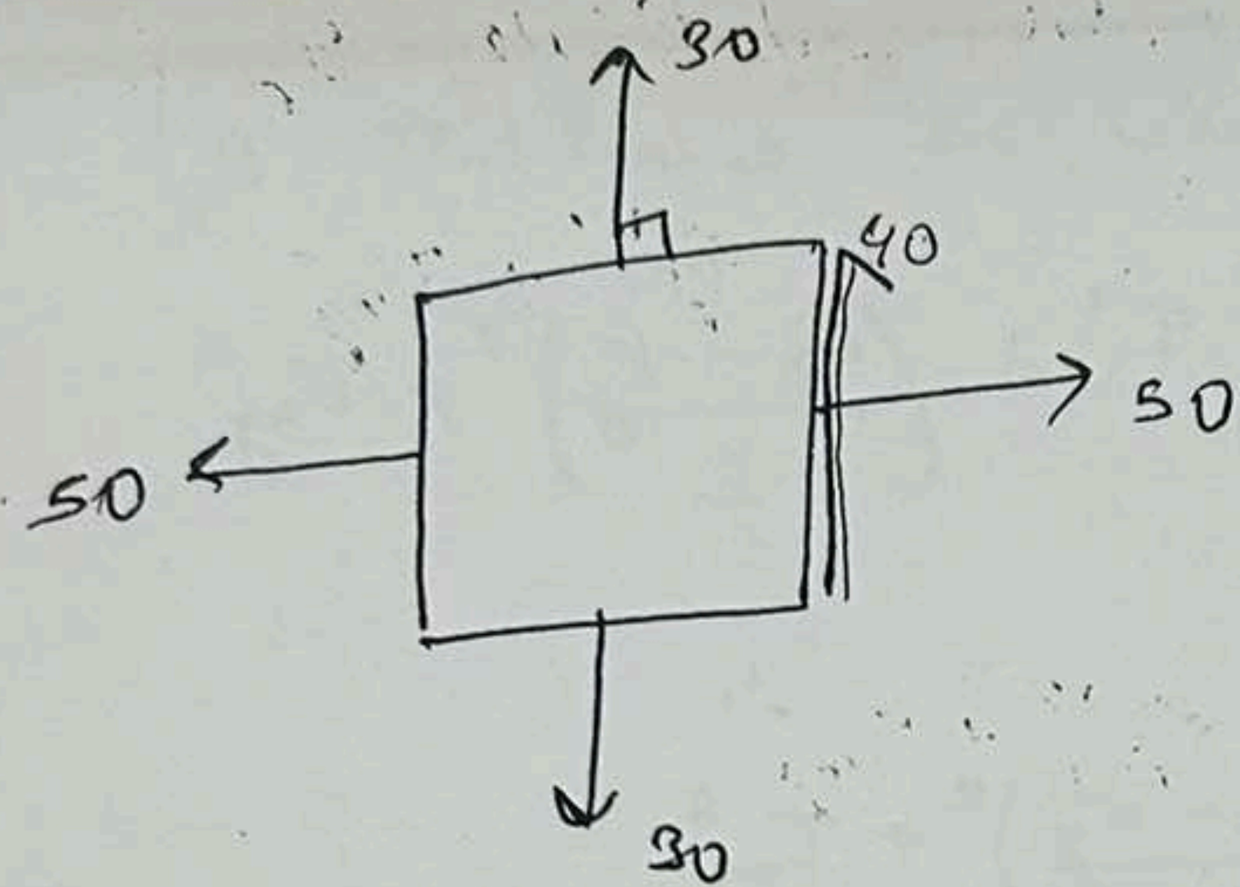
Draw state of stress when rotated by 90°

$$\Longrightarrow \sigma(90^\circ) = \frac{50+30}{2} + \frac{50-30}{2}\cos 180^\circ + (-40)\sin 180^\circ$$

$$= 30$$

$\sigma_x = 50 \qquad \sigma_y = 30 \qquad \tau_{xy} = -40 \qquad \theta = 90^\circ$

$\tau_{yx} = -40$



In prev ex, we observed state of stress is identical at $\theta = 0°, 90°$ while it varies with θ.
Thus, σ_n (normal stress) is max or min at some value ~~b/w $\theta = 0$ & $\theta = 90°$~~ $0° \le \theta \le 90°$.

- The max & min value of σ_n are principal stresses.

To determine principal stress, we define

$$\frac{d\sigma}{d\theta} = 0$$

$$\Rightarrow -2\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta - 2\tau_{xy} \cos 2\theta = 0$$

$$\Rightarrow \tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x - \sigma_y} \quad \text{———} \ (\theta)$$

$$\therefore \theta_\sigma = \frac{1}{2} \tan^{-1}\left(\frac{-2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

θ is orientation of plane relative to x-plane
θ is CCW inclination of normal of plane

- Principal planes: Planes on which ~~max~~ principal stresses appear.

Substituting (θ) in (τ) eq:

$$\tau(\theta_\sigma) = 0$$

$\therefore$ On principal planes, shear stress = 0

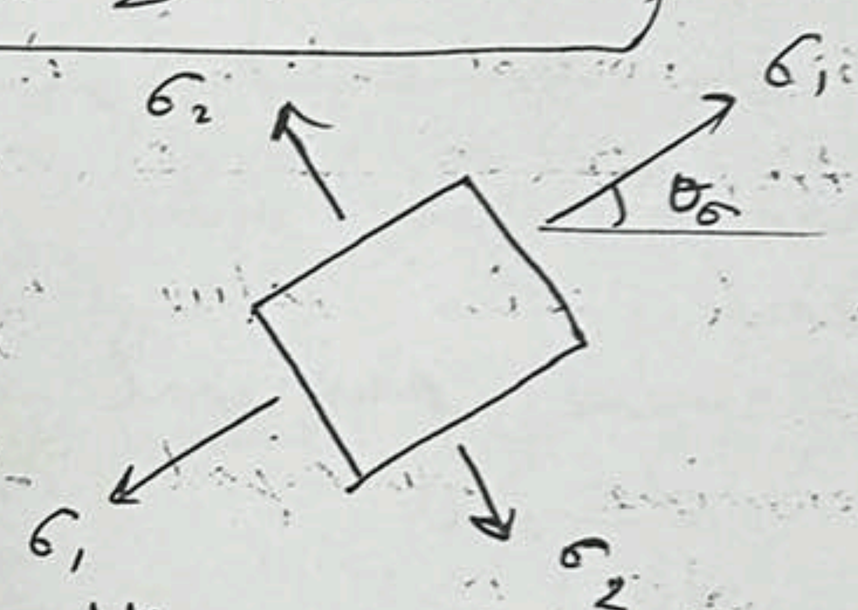
To determine principal stress, we substitute θ_σ in (6) eq

$$\left(\sigma - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

& at $\theta = \theta_\sigma \Rightarrow \tau = 0$

So, $$\left(\sigma - \frac{\sigma_x+\sigma_y}{2}\right)^2 \pm \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\boxed{\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right\} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$



Similarly, to find plane with max shear stress

$$\Rightarrow \frac{d\tau}{d\theta} = 0$$

$$\Rightarrow 2\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

$$\Rightarrow \tan 2\theta = \frac{\sigma_x-\sigma_y}{2\tau_{xy}}$$

$$\Rightarrow \theta_s = \frac{1}{2}\tan^{-1}\left(\frac{\sigma_x-\sigma_y}{2\tau_{xy}}\right)$$

926) $\sigma_x = 4000$ psi $\quad \sigma_y = -8000$ psi

$\tau_{xy} = -6000 = \tau_{yx} \quad \tau_{yx} = -6000$ psi

$$\sigma_1 = \frac{-4000}{2} + \sqrt{\left(\frac{12000}{2}\right)^2 + (6000)^2}$$

$$= -2000 + 6000\sqrt{2} \approx 6484 \text{ psi}$$

$\sigma_2 = -2000 - 6000\sqrt{2} \approx -10484$ psi

$\theta = 30^\circ$

$$\sigma(\theta) = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= -\frac{4000}{2} + \frac{12000}{2}\cos 60^\circ + 6000 \sin 60^\circ$$

$$= -2000 + \frac{6000}{2} + 6000\frac{\sqrt{3}}{2}$$

$$= 1000 + 3000\sqrt{3} \approx 6196 \text{ psi}$$

$$\tau(\theta) = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= 6000 \sin 60^\circ \cancel{+} - 6000 \cos 60^\circ$$

$$= 3000(\sqrt{3} - 1) \approx 2196 \text{ psi}$$

$$\theta_\sigma = \frac{1}{2}\tan^{-1}\left(\frac{-2 \times -6000}{-4000}\right) = \frac{\pi}{8}$$

05/05 A

HW: 927-929

BTECH/10998/24

927) $\sigma_y = 60$ MPa $\qquad \tau_{yx} = -30$ MPa

$\tau_{xy} = -30$ MPa $\qquad \sigma_x = 0$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{-2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{-2 \times -30}{0 - 60}\right)$$

$$= -22.5^\circ$$

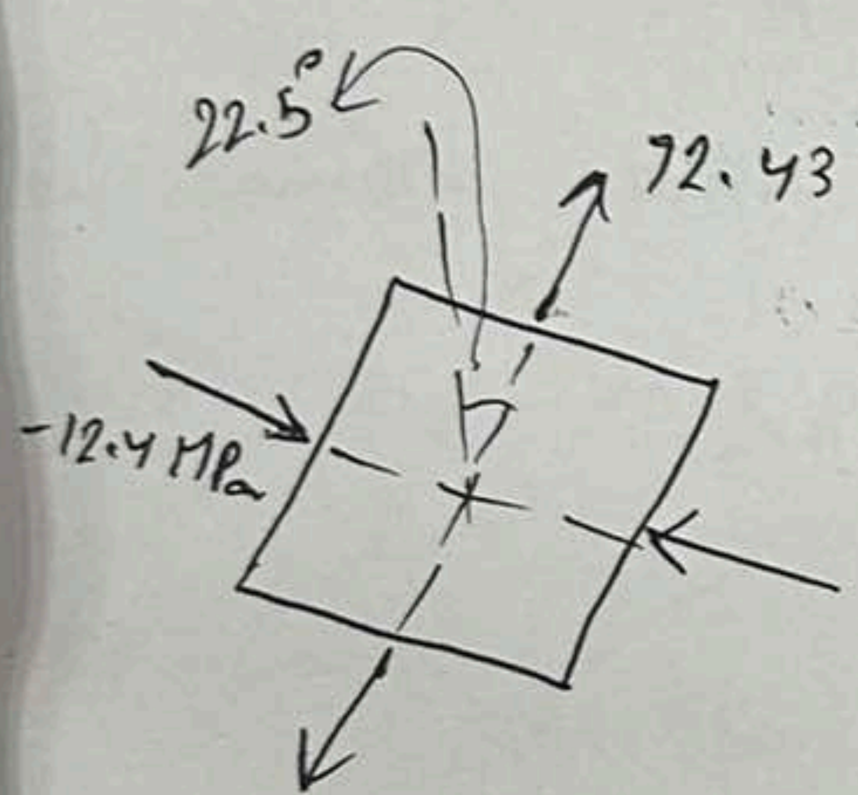
$$\left.\begin{matrix}\sigma_1 \\ \sigma_2\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 30 \pm \sqrt{900 + 900}$$

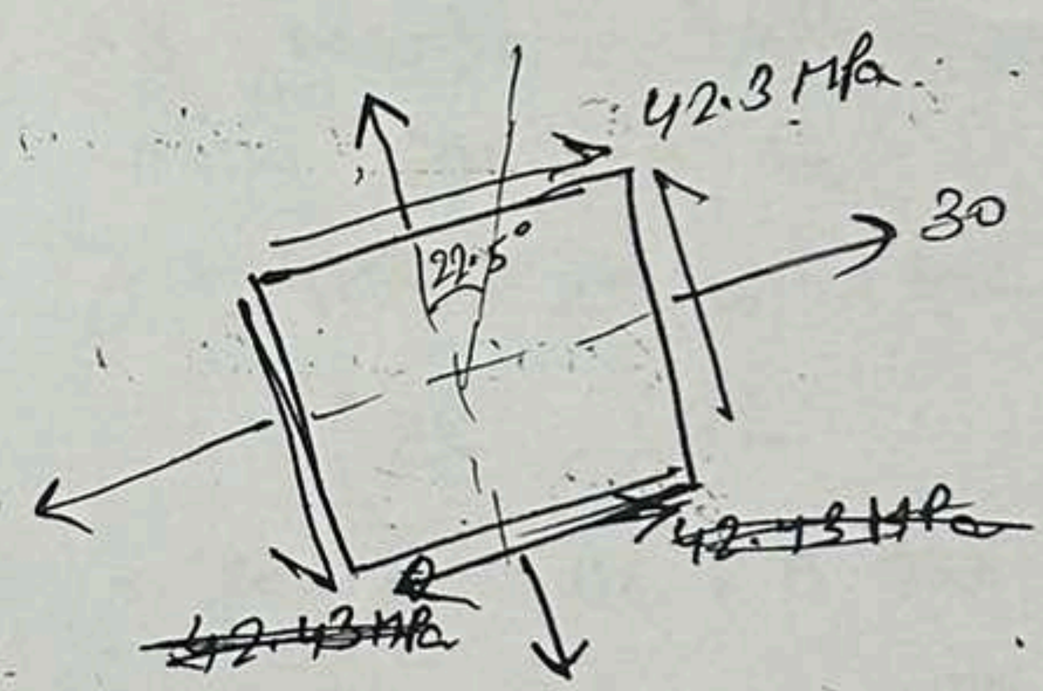
$$= 30\,(1 \pm \sqrt{2})$$

$\sigma_1 = 72.43$ MPa $\qquad\qquad \sigma_2 = -12.43$ MPa

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 42.43 \text{ MPa}$$



Principal stresses



Max shearing stress
(diagram - doubt?)

928) $\sigma_1 = \sigma_x = 40$ MPa

$\sigma_2 = \sigma_y = -30$ MPa $\qquad\qquad \tau_{xy} = 0$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$$

$$= 40\cos^2 30^\circ - 30\sin^2 30^\circ = 22.5 \text{ MPa}$$

$$\tau_n = (\sigma_x - \sigma_y)\, s_\theta\, c_\theta$$

$$= 70 \sin 30^\circ \cos 30^\circ = 30.31 \text{ MPa}$$

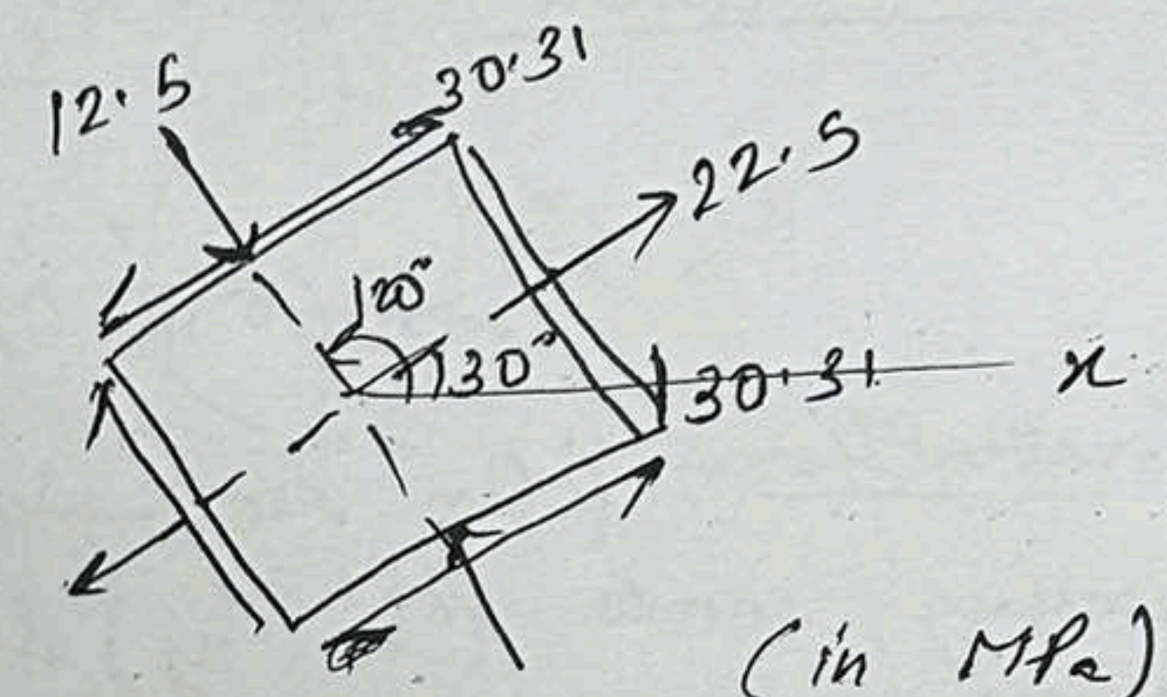
For $\theta = 120^\circ$

$$\sigma_n = 40\cos^2 120^\circ - 30\sin^2 120^\circ$$

$$= 10 - 22.5$$

$$= -12.5 \text{ MPa}$$

$\tau_n = -30.31$ MPa



(in MPa)

(9.25) $\tau_{xy} = -8000$ psi $\qquad \sigma_x = \sigma_y = 0$

$\tau_{yx} = -8000$ psi

i) At $\theta = 30°$,

$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta - \tau_{xy} \sin 2\theta$

$= 0 + 0 + 8000 \sin 60°$

$= 4000\sqrt{3} = 6930$ psi

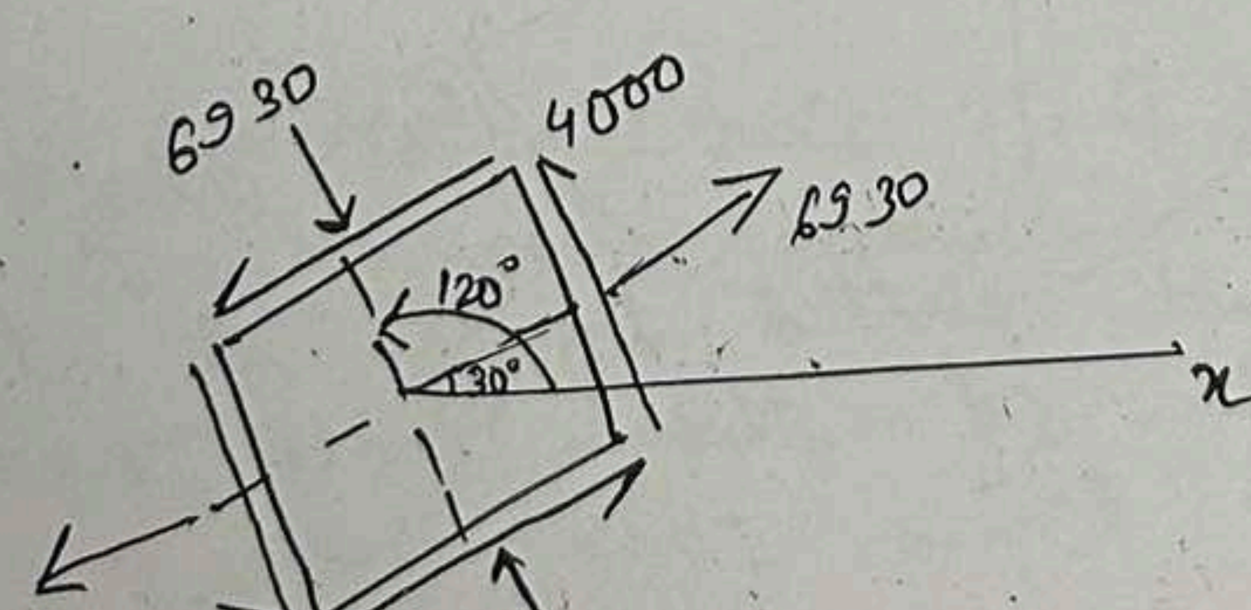
$\tau_n = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$

$= -8000 \cos 60° = -4000$ psi

ii) At $\theta = 120°$,

$\sigma_n = 8000 \sin 240° = -6930$ psi

$\tau_n = -8000 \cos 240° = 4000$ psi



(in MPa)

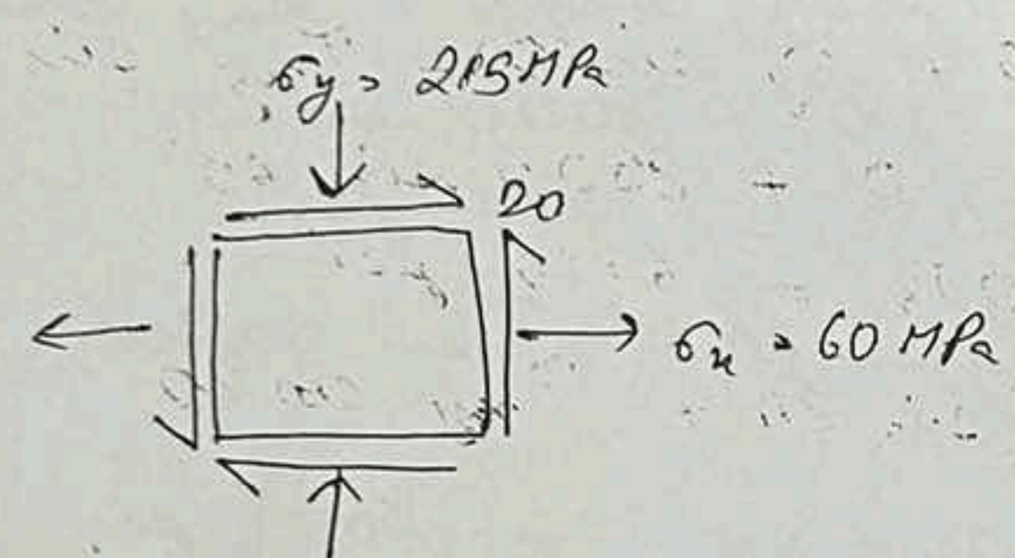
Mohr circle: $\left(\sigma - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$

$$\Rightarrow (\sigma - c_\sigma)^2 + \tau^2 = R_\sigma^2$$

Construction: i) On rectangular σ - τ axis, plot pts having coods (σ_x, τ_{xy}) & (σ_y, τ_{yx}). These pts represent normal & shear stresses acting on x & y-faces of an element.

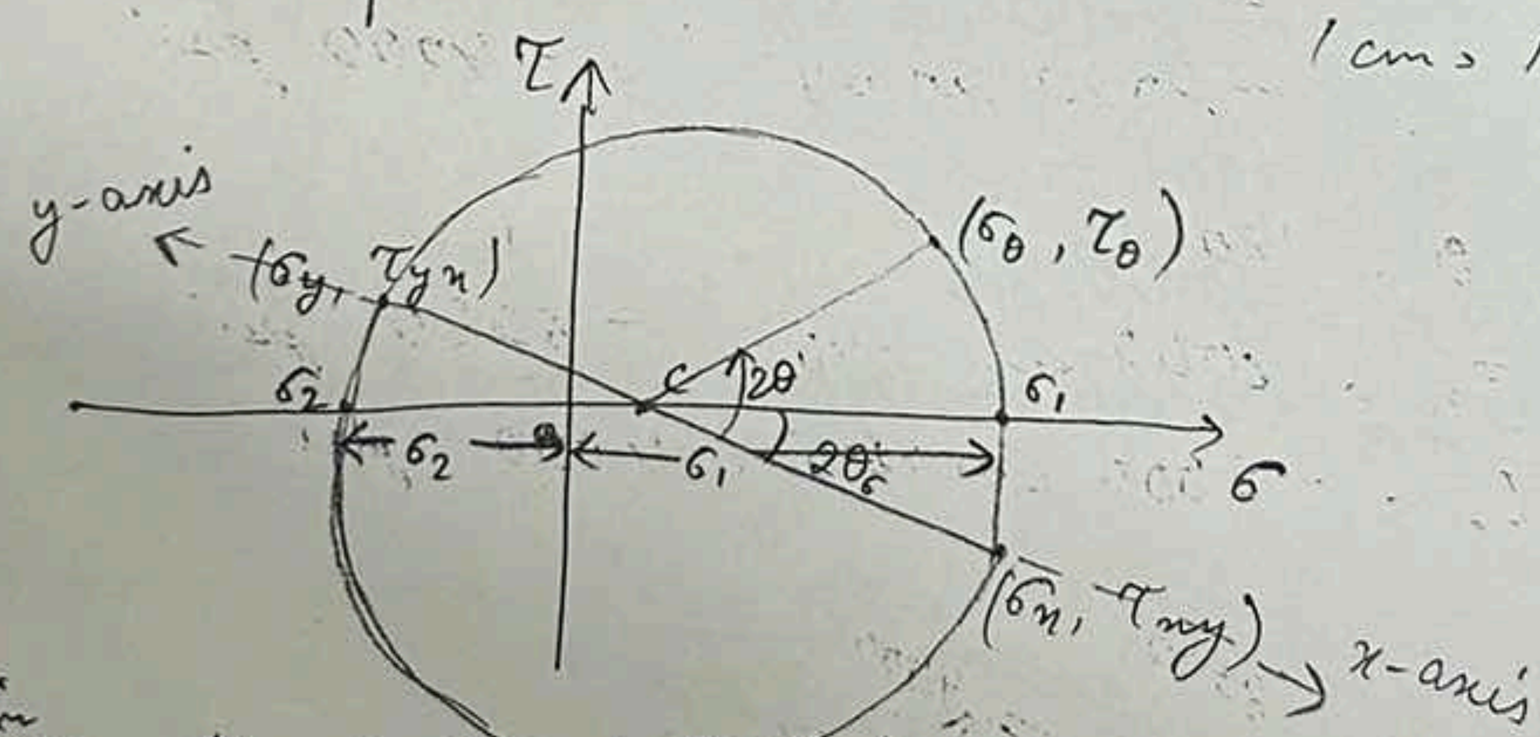
While plotting these pts, assume tension as +ve, compression as -ve & shearing stress +ve when its moment about centre is clockwise.

Ex:



$\theta = 30°$

1 cm = 10 MPa



ii) ~~Drawing~~ Join pts just plotted by a st line. Line joining them is dia. of Mohr circle whose centre falls on σ axis. Draw Mohr circle.

iii) Radius of circle to any pt on its circumference represents axis directed normal to plane whose stress components are given by coods of that pt.

iv) Angle b/w radii
twice angle b/w
represented by t

v) In order to dete
normal axis (n) is
x-axis then on
is laid off at a
x-axis. Coods of 'n
represent stresses

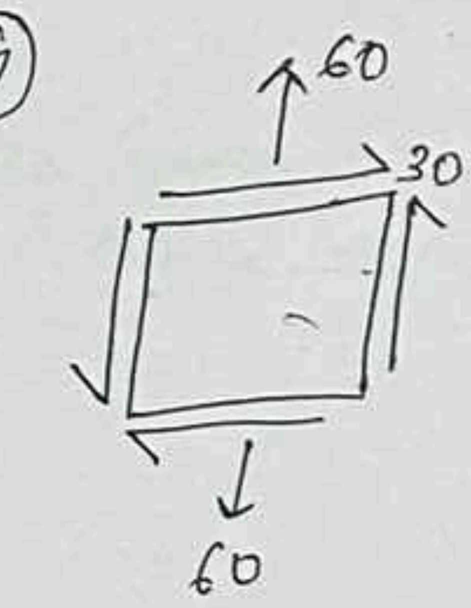
Ex: Find stress o
at $\theta = 30°$ with

$\rightarrow$ $\sigma_1 = 64$ MPa

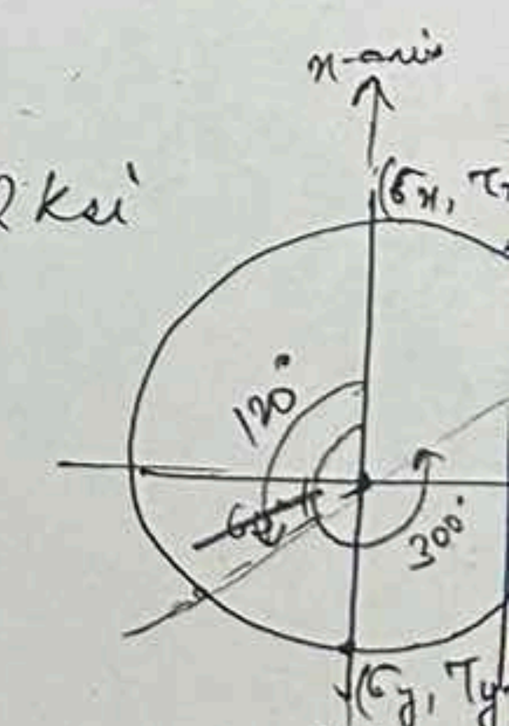
$2\theta_\sigma = 25°$

At $\theta = 30°$ $\Rightarrow$ $\sigma_\theta =$

(927)



(931) 1cm = 2 ksi



iv) Angle b/w radii to selected pts on circle is twice angle b/w normal to actual plane represented by these pts.

v) In order to determine stresses on a face whose normal axis (n) is at angle 'θ' (say ccw) with x-axis then on Mohr circle, radius along 'n' is laid off at a CCW (same sense) at 2θ from x-axis. Coods of 'n' on circumference of circle represent stresses on face of plane

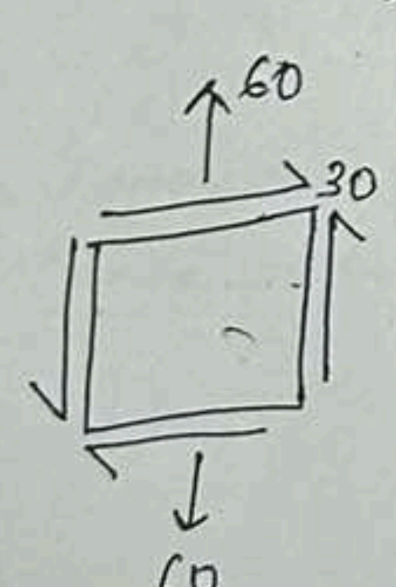
Ex: Find stress on plane at whose normal is at $\theta = 30°$ with x-axis

→ $\sigma_1 = 64$ MPa $\quad \sigma_2 = 29$ MPa

$2\theta_\sigma = 25°$

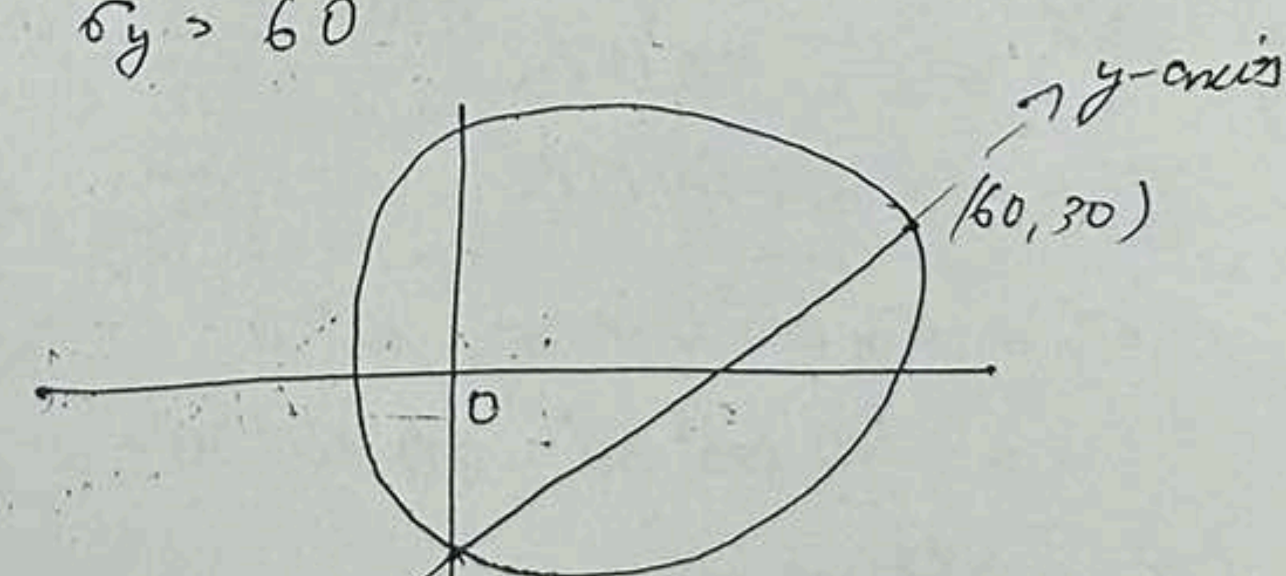
At $\theta = 30° \Rightarrow \sigma_\theta = 56$ MPa, $\tau_\theta = 27$ MPa

(927)

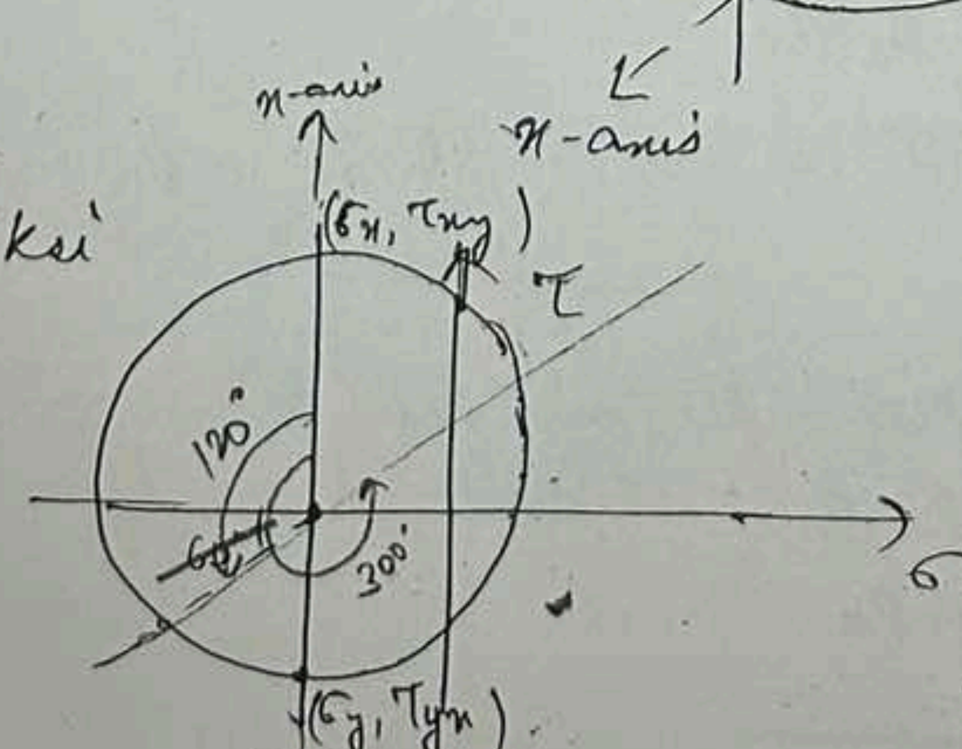


$\sigma_x = 0 \qquad \tau_{xy} = -30$

$\sigma_y = 60$



(931) 1cm = 2 ksi



$C_\sigma = \frac{\sigma_x + \sigma_y}{2}$

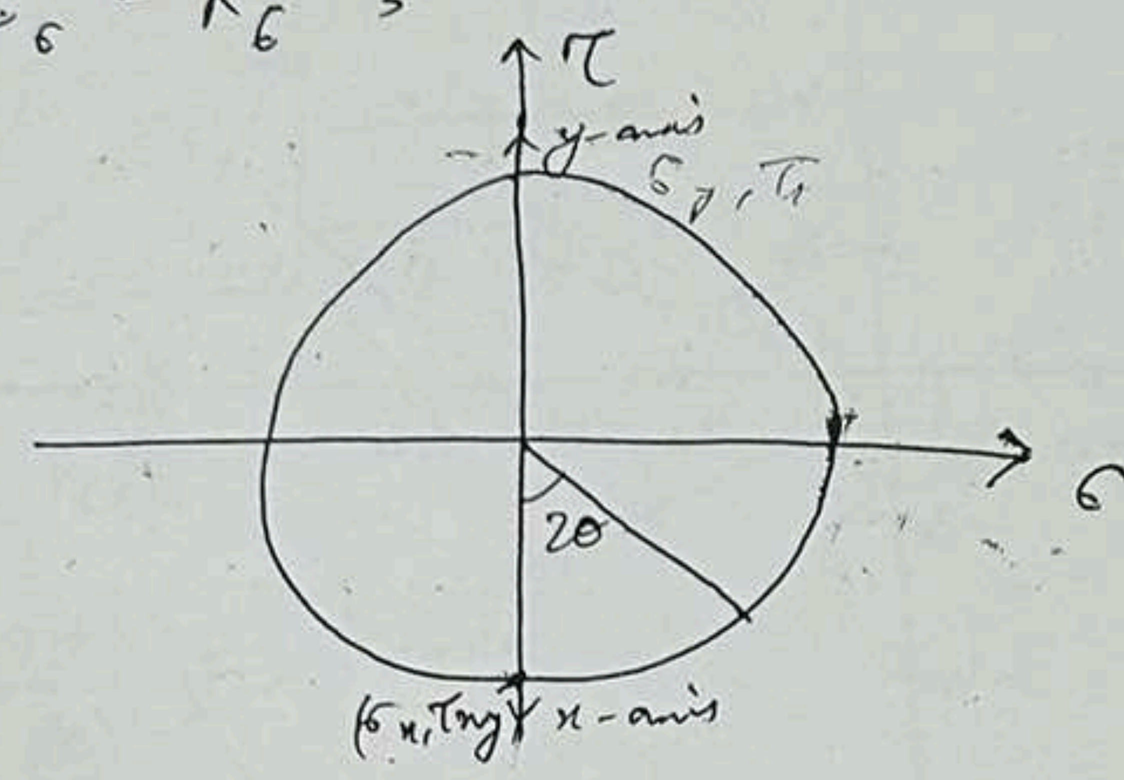
$= \frac{-8-8}{2} = -8$

$R_\sigma = 10$ ksi

$\sigma_1 = C_\sigma + R_\sigma = -8 + 10 = 2$ ksi

$\sigma_2 = C_\sigma - R_\sigma = -18$ ksi

929



$\sigma_x = -8$

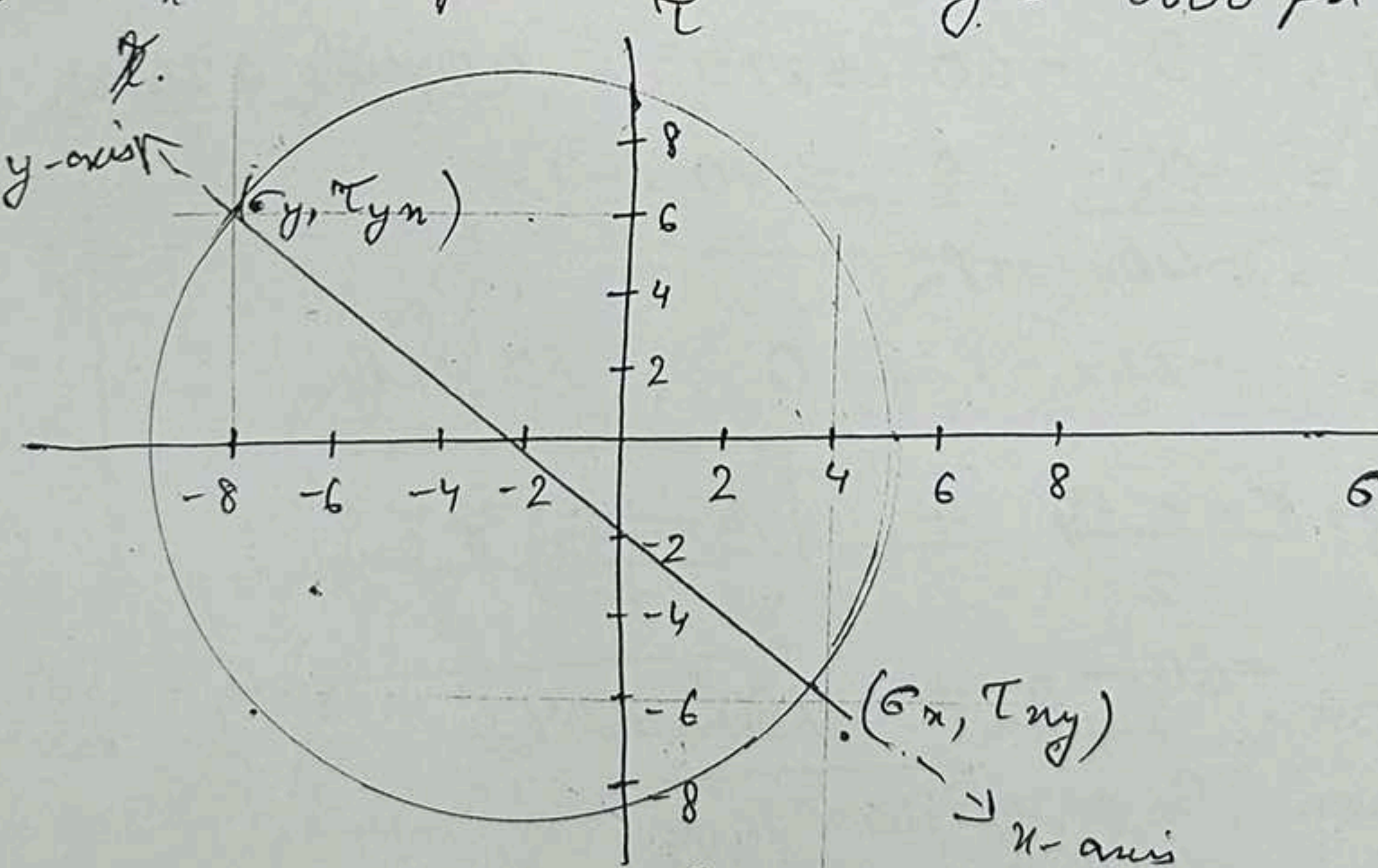
$\sigma_y = 8$

$\theta = 30^\circ$

$\sigma = R_\sigma \sin 2\theta = 4\sqrt{3}$ ksi

$\tau = R \cos 2\theta = 4$ ksi

932 $\sigma_x = 4000$ psi $\quad \sigma_y = -8000$ psi $= -8$ ksi



934 ~~$\sigma_x = 2400$ lb~~ ~~$\sigma_y = 1280$ lb~~

$\theta = \tan^{-1}\left(\frac{1.2}{1.6}\right) = 37^\circ$

X-axis → 1.2×0.2

Y-axis → 1.6×0.2

$\sigma_x = \frac{2400}{1.2 \times 0.2} = 10$ ksi

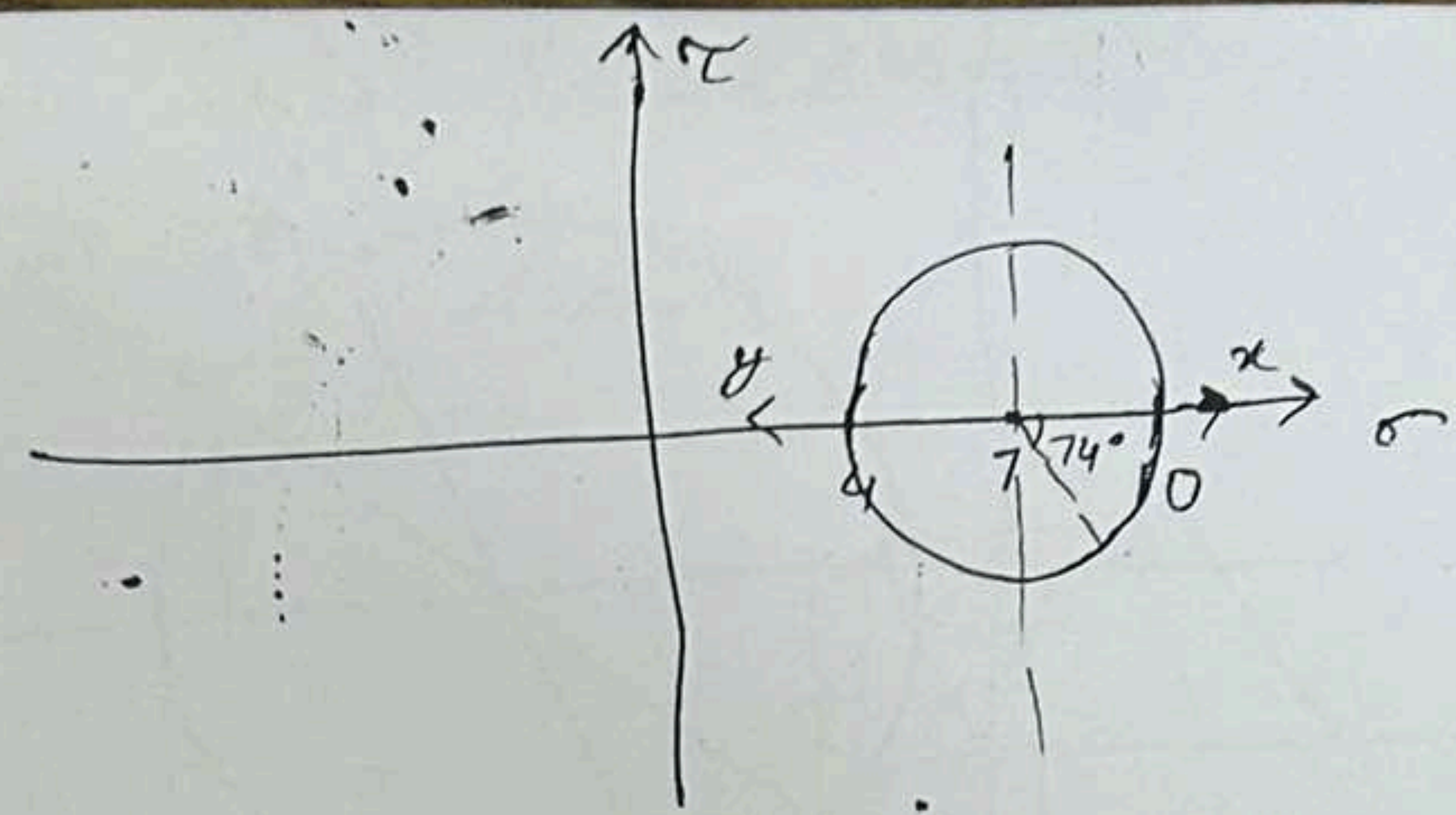
$\sigma_y = 4$ ksi

$\sigma_x = -8$
$\sigma_y = 8$

psi = -8 ksi

6

) = 37°



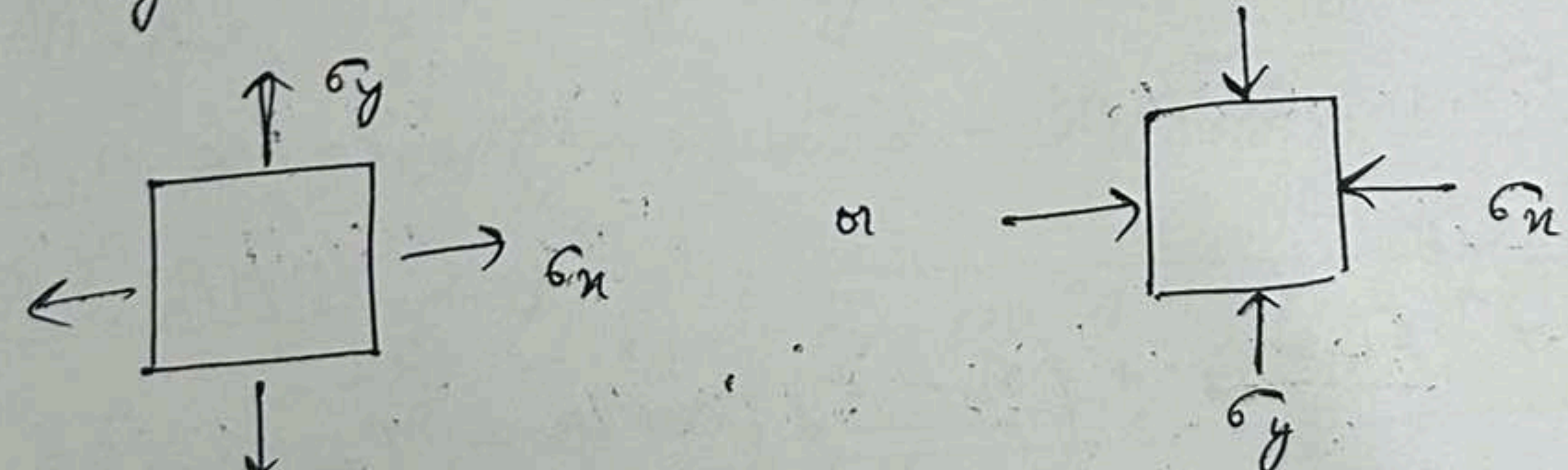
$$C_\sigma = \frac{\sigma_x + \sigma_y}{2} = 7$$

$$\sigma_\theta = C_\sigma + R_\sigma \cos 2\theta$$

$$\tau_\theta = R_\sigma \sin 2\theta$$

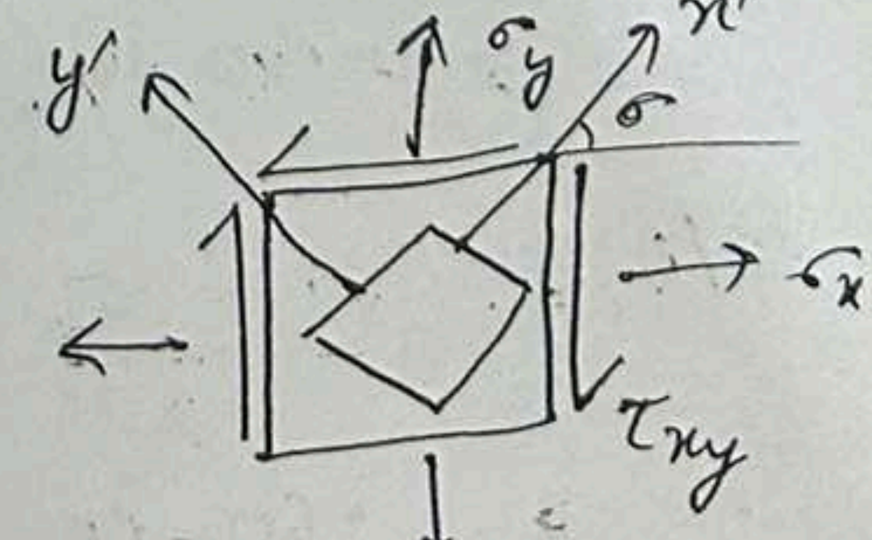
Q) Is it possible for Mohr's circle to have r = 0? If yes - illustrate corresponding state of stress with neat sketch.

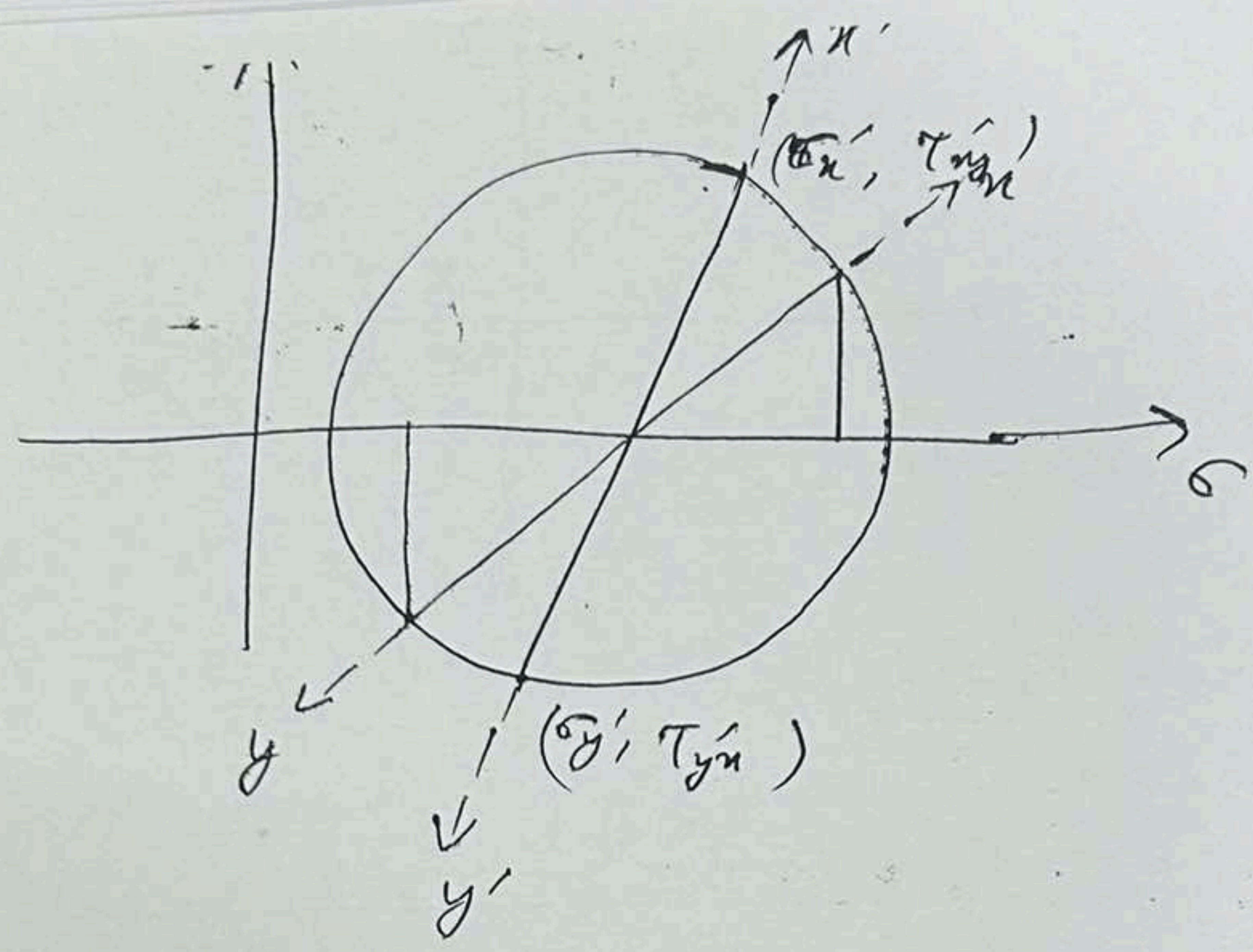
→ $\tau_{xy} = 0$ $\sigma_x = \sigma_y = \sigma$



Q) Prove that Mohr's circle r = 0 if material is subjected to non-zero hydraulic state of stress.

Q) Prove $\sigma_x' + \sigma_y' = \sigma_x + \sigma_y$





- Only 1 Mohr circle whatever be orientation of axes (for only 1 loading)
- Brittle material – shows poor resistance to tensile stress. In other words, if we apply tensile stress, shear & compressive stresses then material likely fails under tension

Q33) $\sigma_x = -60$ MPa $\qquad \sigma_y = 60$ MPa

At $\theta = 45°$ $\qquad \tau_{xy} = 40$ MPa $= \tau_{yx}$

$$\sigma(\theta) = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-60+60}{2} + \left(\frac{-60-60}{2}\right)\cos 90° - 40\sin 90°$$

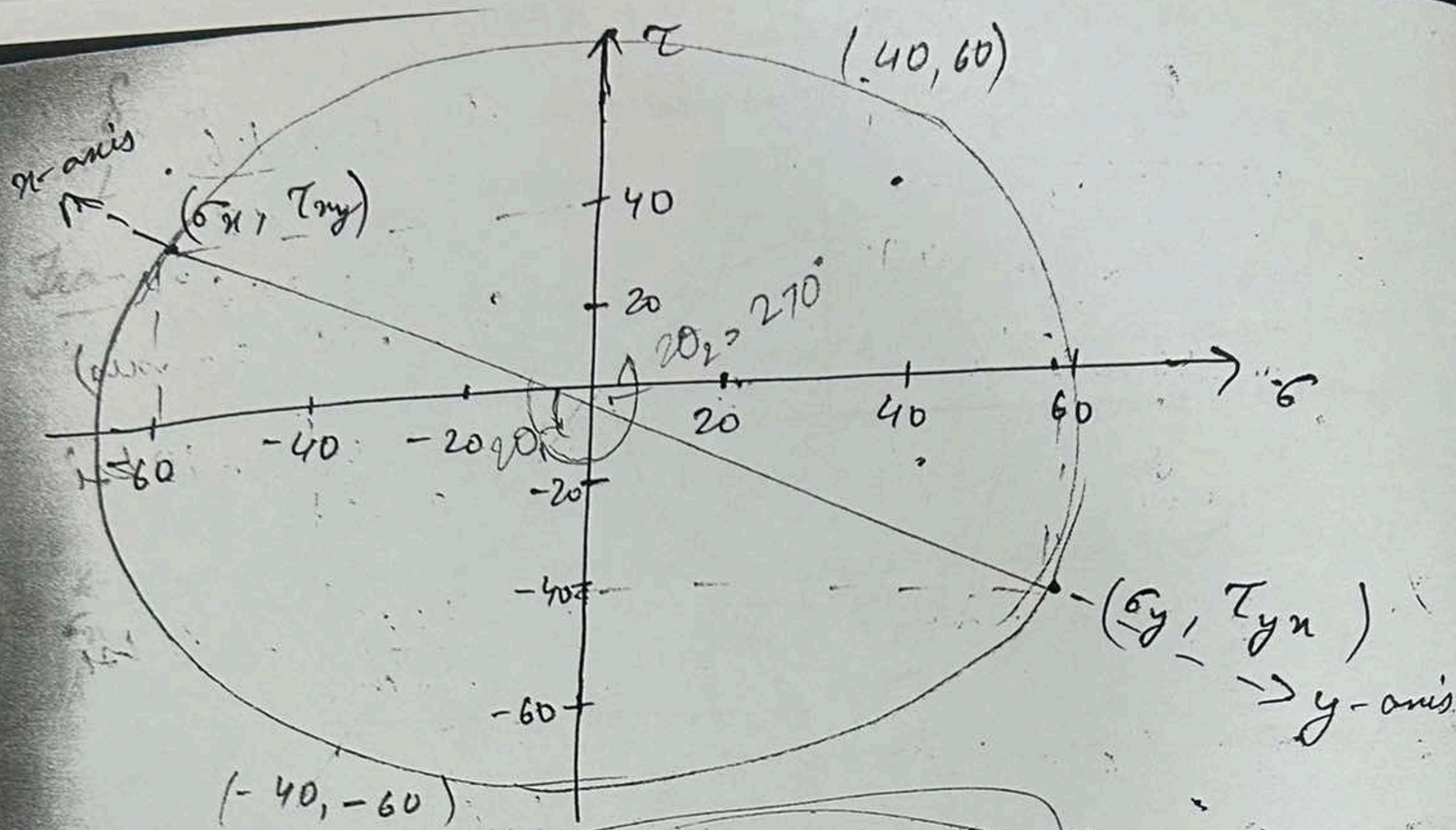
$$= 0 - 60 \times 0 - 40$$

$$= -40 \text{ MPa}$$

$$\tau(\theta) = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \frac{-60-60}{2} \times 1 + 0 = -60 \text{ MPa}$$

x-axis
-60
At $\theta =$
$\sigma(\theta)$
$\tau(\theta) =$
σ_1
σ_2
$\tau_{max} =$



At $\theta = 135°$

$\sigma(\theta) = 0 - 60\cos 270° - 40 \sin 270°$

$= 0 - 0 - 40 \times -1$

$= 40 \text{ MPa}$

$\tau(\theta) = -60 \times -1 + 0 = 60 \text{ MPa}$

$$\left.\begin{matrix}\sigma_1 \\ \sigma_2\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$= \cancel{60}\ 0 \pm \sqrt{60^2 + 40^2}$

$= 0 \pm \sqrt{3600 + 1600}$

$= 72.11 \text{ MPa}$

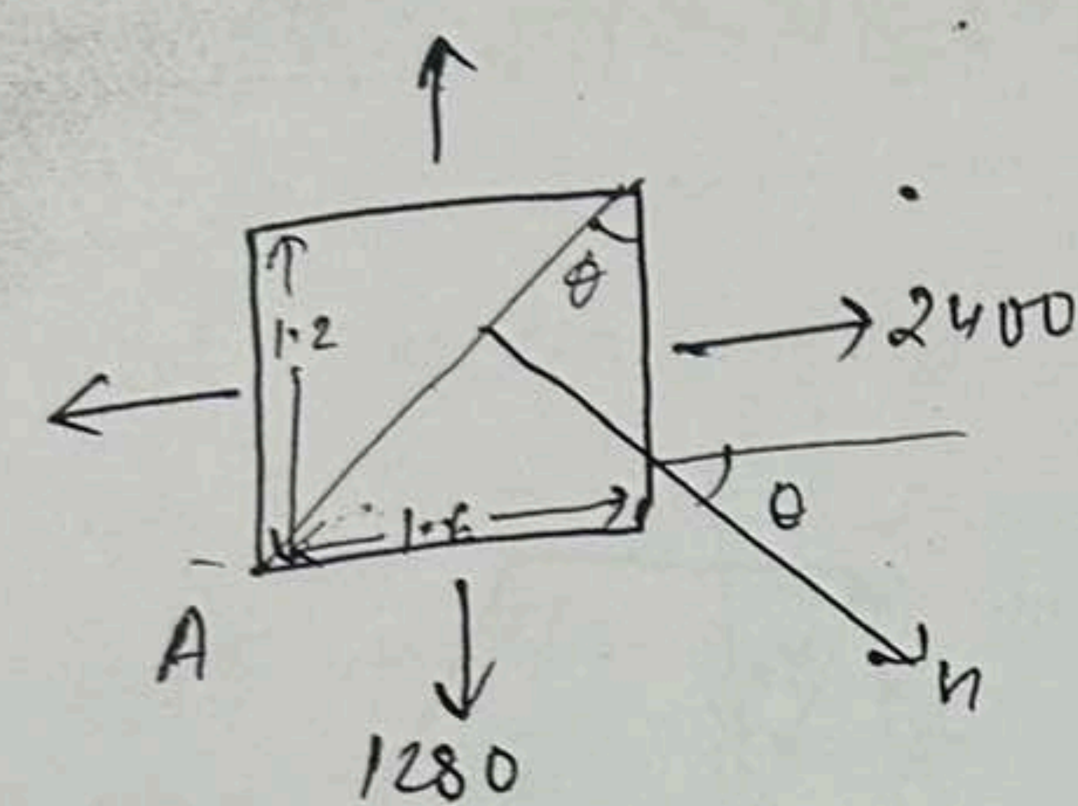
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 72.11 \text{ MPa}$$

2

A

BTECH/10348/24

934)



$\tan\theta = \frac{1.6}{1.2}$

$\theta = 53.6^\circ$

(CW with x-axis)

$\sigma_x = \frac{2400}{1.2 \times 0.2} = 10 \text{ ksi}$

$\sigma_y = \frac{1280}{1.6 \times 0.2} = 4 \text{ ksi}$

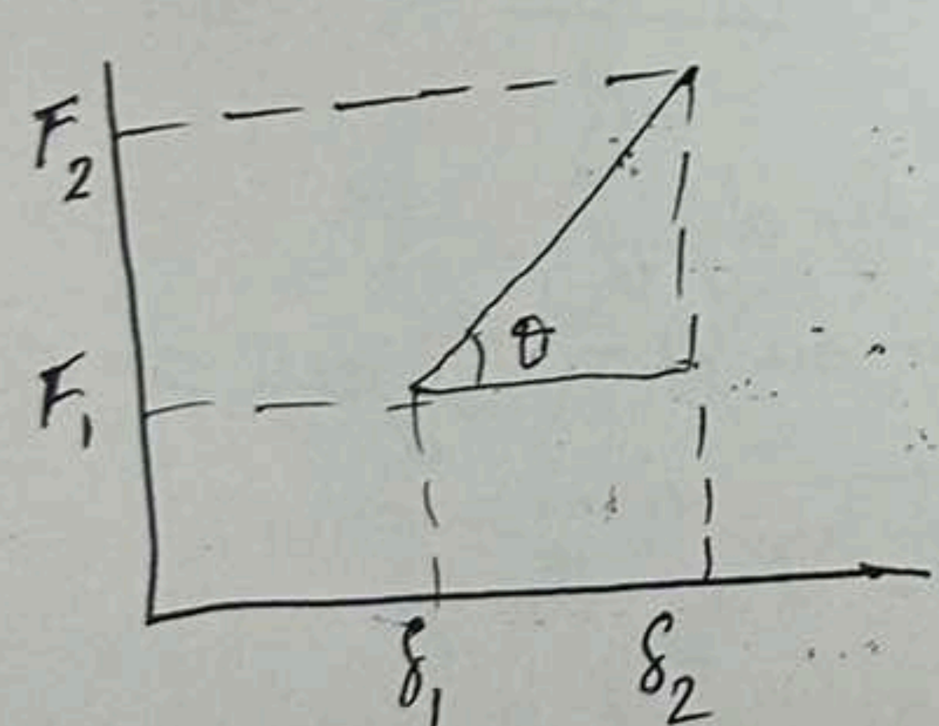
$C_\sigma = \frac{10+4}{2} = 7 \text{ ksi}$

$R_\sigma = \sigma_x - C_\sigma = 3 \text{ ksi}$

$\sigma_\theta = C_\sigma + R_\sigma \cos 2\theta = 6.72 \text{ ksi}$

$\tau_\theta = R_\sigma \sin 2\theta = -2.8 \text{ ksi}$

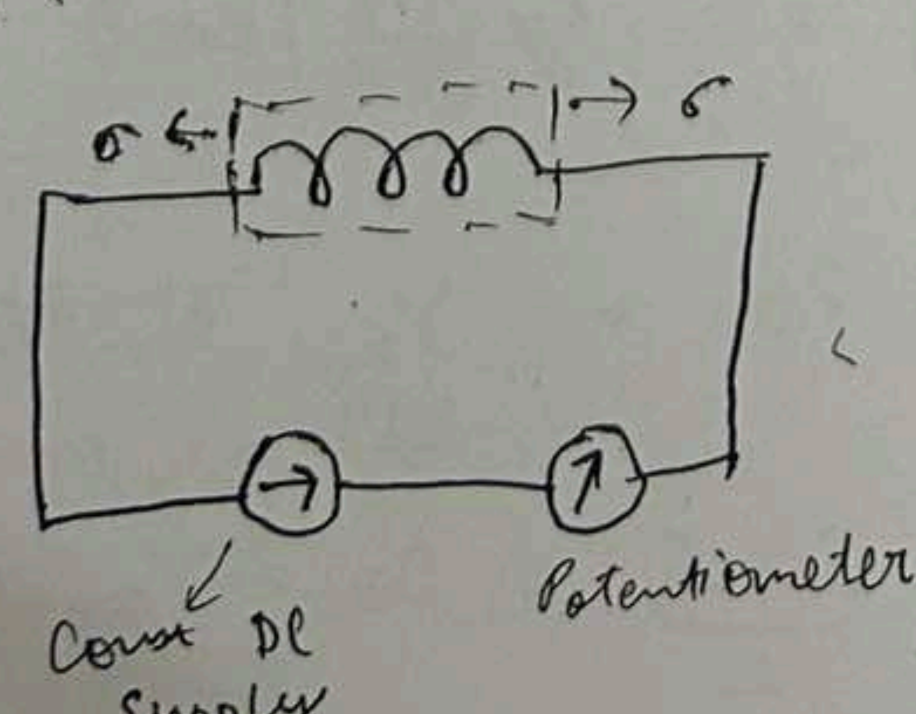
Strain Gauge



$\tan\theta = \frac{F_2 - F_1}{\delta_2 - \delta_1} = k$

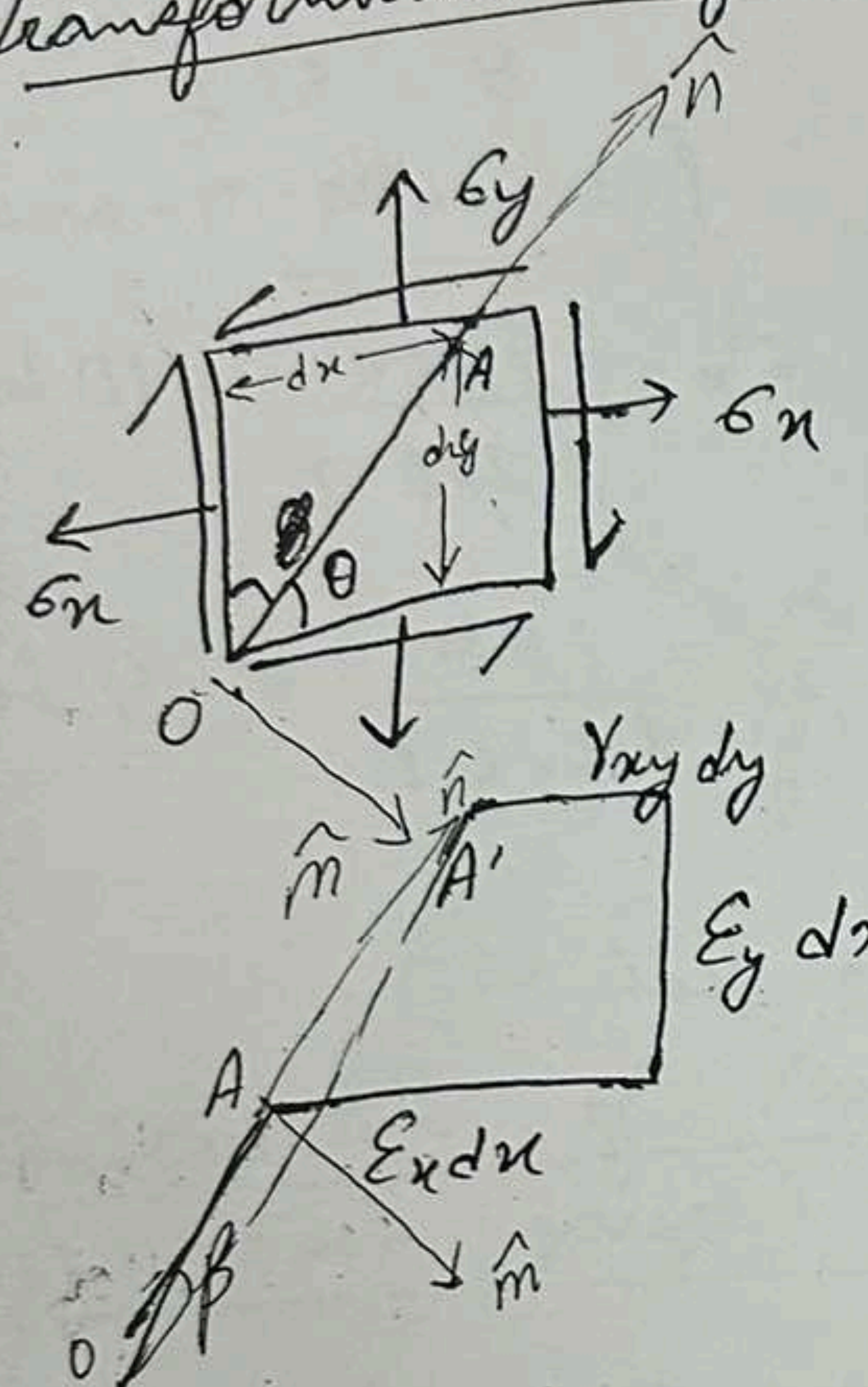
Stress is a mathematical concept which represents intensity of force on an area. It can't be measured directly. However, with help of strain gauge & by applying Hooke's law, it can be measured.

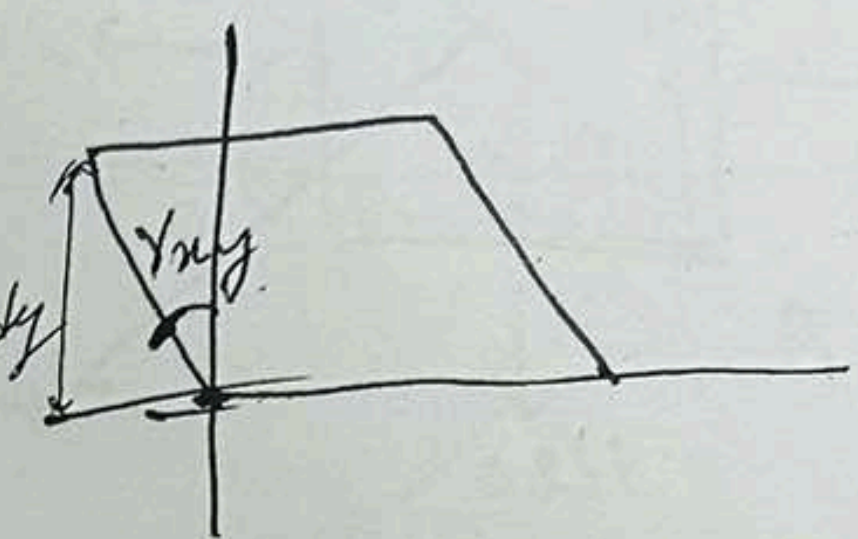
Working :



(ii) $\frac{\delta}{L} \propto \Delta V \Rightarrow \frac{\delta}{L} = k\Delta V + C$

Transformation of strain:





Elongation along $\vec{OA} = \vec{AA'}\cdot\hat{n}$

Deviation from $OA = \vec{AA'}\cdot\hat{m}$

$$\varepsilon_a = \frac{\vec{AA'}\cdot\hat{n}}{OA}$$

$$\beta = \frac{\vec{AA'}\cdot\hat{m}}{OA}$$

$$\vec{AA'} = (\varepsilon_x dx - \gamma_{xy} dy)\hat{i} + \varepsilon_y dy\,\hat{j}$$

$$\vec{AA'}\cdot\hat{n} = \varepsilon_x dx\cos\theta + \varepsilon_y dy\sin\theta - \gamma_{xy} dy\cos\theta$$

$$\frac{\vec{AA'}\cdot\hat{n}}{OA} = \varepsilon_x\cos^2\theta + \varepsilon_y\sin^2\theta - \gamma_{xy}\, s_\theta c_\theta$$

$$\therefore \boxed{\varepsilon_a = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta} \quad \text{— (i)}$$

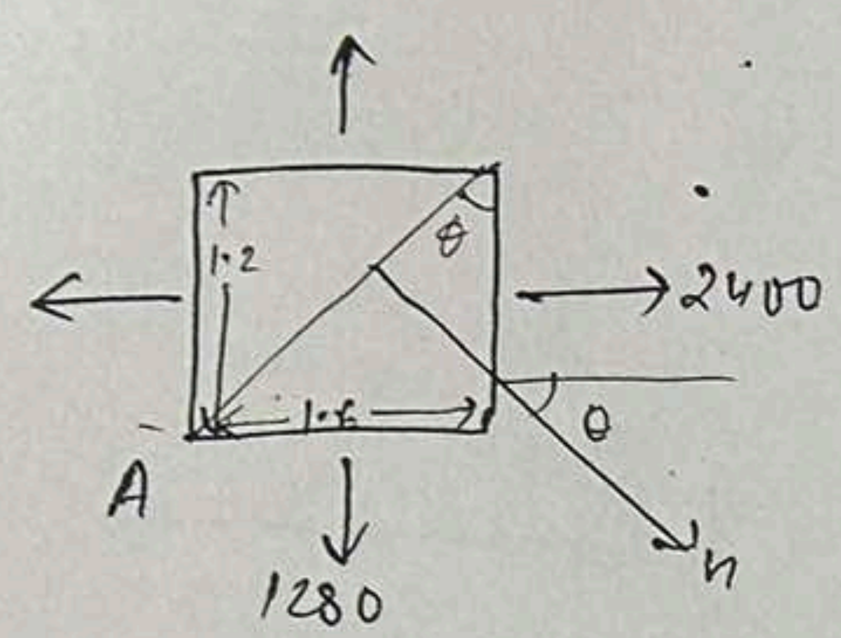
$$\vec{AA'}\cdot\hat{m} = \varepsilon_x dx\sin\theta - \varepsilon_y dy\cos\theta - \gamma_{xy} dy\sin\theta$$

$$\frac{\vec{AA'}\cdot\hat{m}}{OA} = \varepsilon_x\left(\frac{dx}{OA}\right)s_\theta - \varepsilon_y\left(\frac{dy}{OA}\right)\cos\theta - \gamma_{xy}\left(\frac{dy}{OA}\right)s_\theta$$

$$\therefore \beta = \frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta - \gamma_{xy}\sin^2\theta$$

$$\beta' = \beta(90+\theta) = -\frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta - \gamma_{xy}\cos^2\theta$$

934)



$\tan\theta = \frac{1.6}{1.2}$

$\theta = 53.6°$

(CW with x-axis)

$\sigma_x = \frac{2400}{1.2 \times 0.2} = 10$ ksi

$\sigma_y = \frac{1280}{1.6 \times 0.2} = 4$ ksi

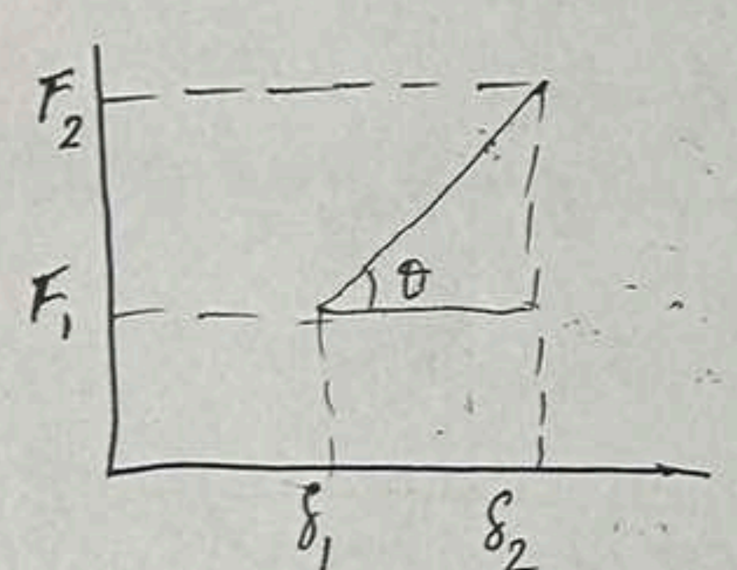
$C_\sigma = \frac{10+4}{2} = 7$ ksi

$R_\sigma = \sigma_x - C_\sigma = 3$ ksi

$\sigma_\theta = C_\sigma + R_\sigma \cos 2\theta = 6.72$ ksi

$\tau_\theta = R_\sigma \sin 2\theta = -2.8$ ksi

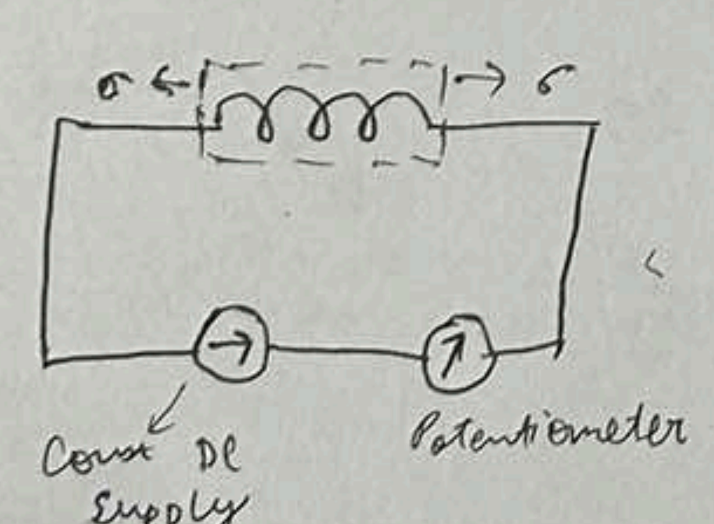
Strain Gauge



$\tan\theta = \frac{F_2 - F_1}{\delta_2 - \delta_1} = k$

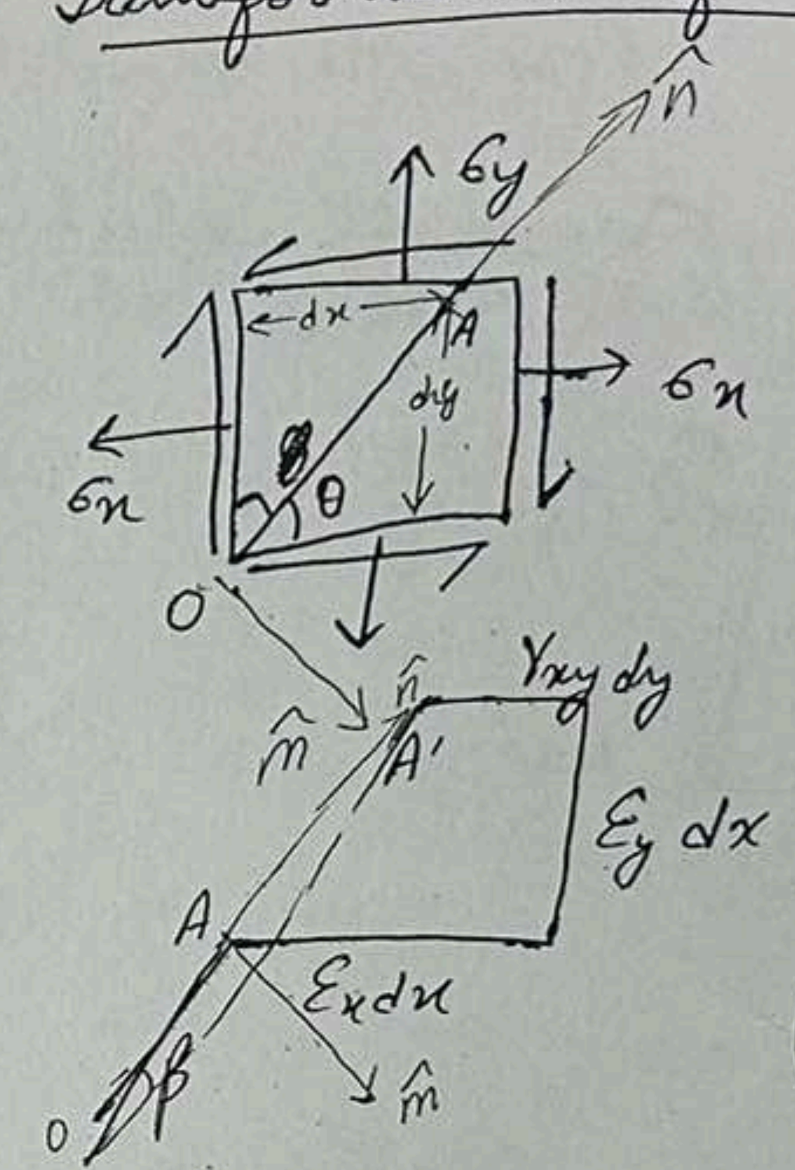
Stress is a mathematical concept which represents intensity of force on an area. It can't be measured directly. However, with help of strain gauge & by applying Hooke's Law, it can be measured.

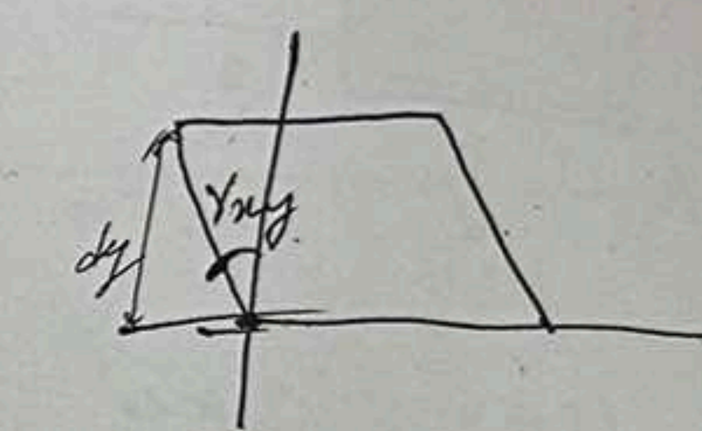
Working:



$\frac{\delta}{L} \propto \Delta V \Rightarrow \frac{\delta}{L} = k\Delta V + C$

Transformation of strain:





Elongation along $\vec{OA}$, $\vec{AA'}$

Deviation from OA = $\vec{AA'} \cdot \hat{m}$

$\varepsilon_a = \frac{\vec{AA'} \cdot \hat{n}}{OA}$

$\beta = \frac{\vec{AA'} \cdot \hat{m}}{OA}$

$\vec{AA'} = (\varepsilon_x dx - \gamma_{xy} dy)\hat{i} + \varepsilon_y dy \hat{j}$

$\vec{AA'} \cdot \hat{n} = \varepsilon_x dx \cos\theta + \varepsilon_y dy \sin\theta - \gamma_{xy} dy \cos\theta$

$\frac{\vec{AA'} \cdot \hat{n}}{OA} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta - \gamma_{xy} S_\theta C_\theta$

$$\therefore \boxed{\varepsilon_a = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta}$$

$\vec{AA'} \cdot \hat{m} = \varepsilon_x dx \sin\theta - \varepsilon_y dy \cos\theta - \gamma_{xy} dy \sin\theta$

$\frac{\vec{AA'} \cdot \hat{m}}{OA} = \varepsilon_x \left(\frac{dx}{OA}\right) S_\theta - \varepsilon_y \left(\frac{dy}{OA}\right)\cos\theta - \gamma_{xy}\left(\frac{dy}{OA}\right) S_\theta$

$\therefore \beta = \frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta - \frac{\gamma_{xy}}{2}\sin^2\theta$

$\beta' = \beta(90+\theta) = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta - \gamma_{xy}\cos^2\theta$

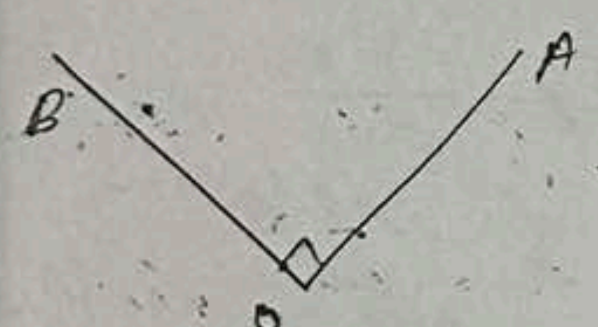
$$\boxed{\gamma_a = \frac{\beta - \beta'}{2} = (\epsilon_x - \epsilon_y)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta} \quad \text{—— (ii)}$$

$\hat{n}$ = unit vector along OA
$= \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\hat{m}$ = unit vector $\perp^r$ to OA = $\sin\theta\,\hat{i} - \cos\theta\,\hat{j}$

$\Delta OA = \vec{AA'} \cdot \hat{n}$

$\epsilon_a = \frac{\Delta OA}{OA}$



β': deviation of line element OB $\perp^r$ to OA

Since OA & OB rotate in opp directions, the absolute sum of deviation is their diff. In other words, shear strain is relative deviation of 2 non-parallel lines. $\Rightarrow \beta - \beta' = \gamma_{ab}$

Rewriting eq (i)

$$\boxed{\left(\epsilon_a - \frac{\epsilon_x + \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{ab}}{2}\right)^2 = \left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \text{ —— } (M_\epsilon)$$

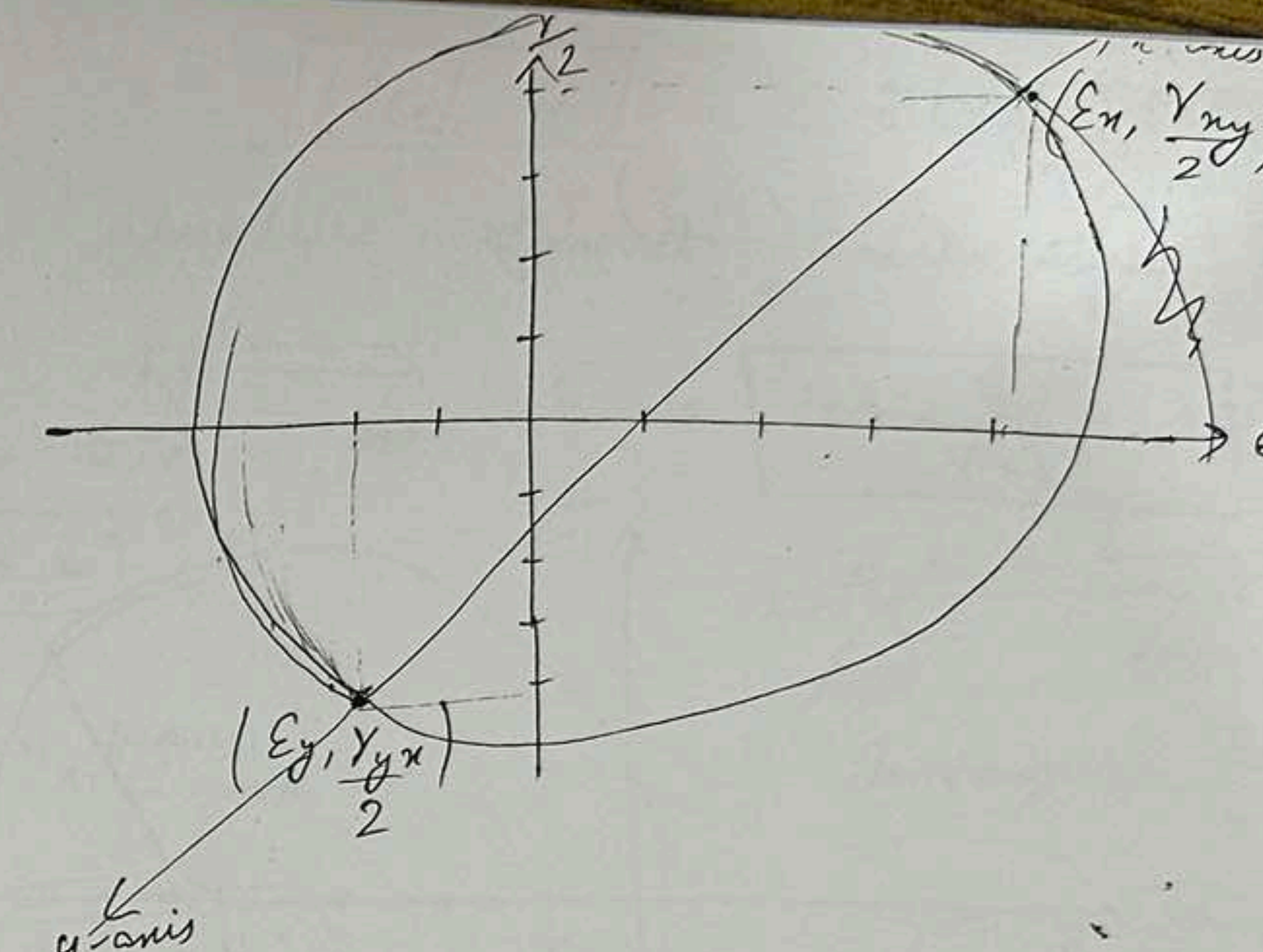
Mohr's circle of strain

$$\frac{\gamma_{ab}}{2} = \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$C_\epsilon = \frac{\epsilon_x + \epsilon_y}{2}$$

$$R_\epsilon = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Ex: $\epsilon_x = 400 \times 10^{-6}$
$\epsilon_y = -200 \times 10^{-6}$
$\gamma_{xy} = 800 \times 10^{-6}$



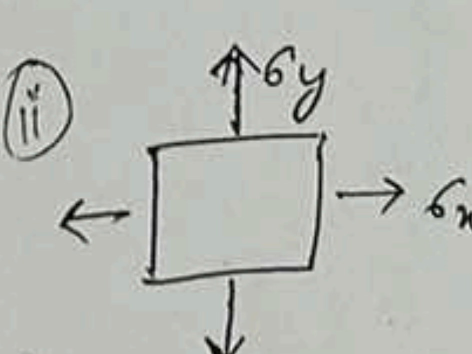
Construction of Mohr circle of stress with help of strain gauge's readings

$$C_\sigma = \frac{\sigma_x + \sigma_y}{2}$$

(i) $\epsilon_x = \frac{\sigma_x}{E}$ $\quad \epsilon_y = -\nu\epsilon_x = -\frac{\nu\sigma_x}{E}$

$\epsilon_y = \frac{\sigma_y}{E}$, $\epsilon_x = -\nu\epsilon_y \Rightarrow \epsilon_x = \frac{-\nu\sigma_y}{E}$

(ii)



$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E}$

$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E}$

$$\therefore C_\epsilon = \frac{\epsilon_x + \epsilon_y}{2} = \frac{1}{2}\left(\frac{\sigma_x + \sigma_y}{E} - \nu\left(\frac{\sigma_y + \sigma_x}{E}\right)\right)$$

$$= \frac{1-\nu}{2E}(\sigma_x + \sigma_y)$$

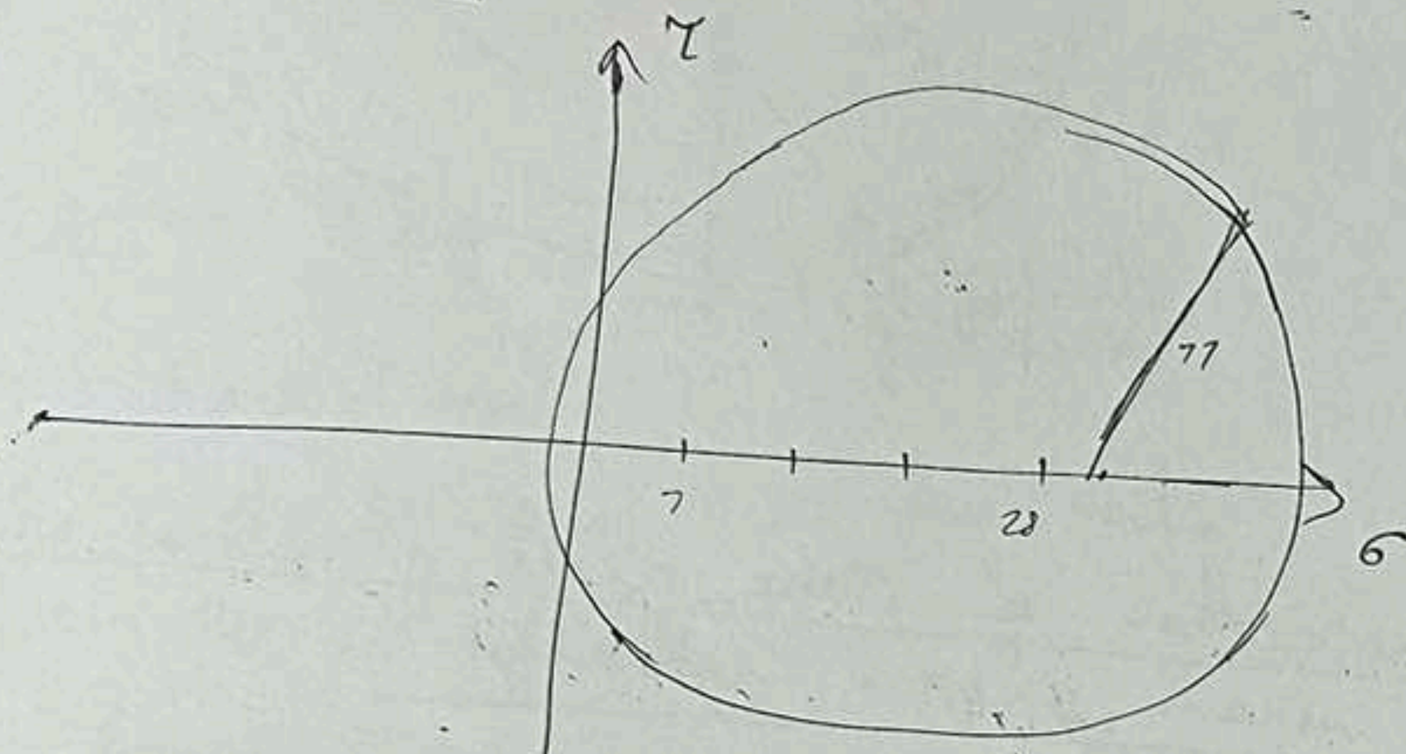
$$\boxed{C_\sigma = \frac{E}{1-\nu} C_\epsilon}$$

$E = 2(1+\nu)G$

$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2(1+\nu)}{E}\tau_{xy}$

$$\boxed{R_\sigma = \frac{E}{1+\nu} R_\epsilon}$$

Same Ex:



$R_\epsilon = \sqrt{300^2 + 400^2} = 500$

$E = 200\,GPa$
$\nu = 0.3$

$C_\epsilon = \frac{400-200}{2} = 100$

$C_\sigma = \frac{200 \times 10^3 \times 100 \times 10^{-6}}{1-0.3} = 28.5\,MPa$

$R_\sigma = \frac{200 \times 10^3}{1.3} \times 500 \times 10^{-6} \approx 77$

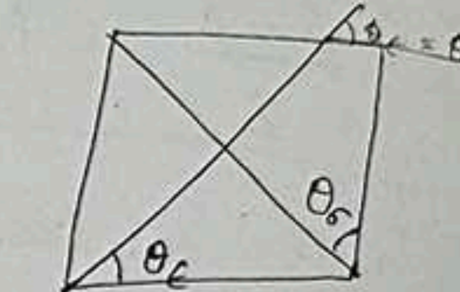
$\sigma_1 = C_\sigma + R_\sigma = 105.5\ MPa$

$\sigma_2 = C_\sigma - R_\sigma = -48.5\ MPa$

* Pg 362 - Proof b/w E & G

• Principle strains - Max & min normal/longitudinal strains
The line elements undergo principal strain are normal to principal planes.
To obtain principal strains, we set $\frac{d\epsilon}{d\theta} = 0$

$\Rightarrow \tan 2\theta = \frac{-\gamma_{xy}}{\epsilon_x - \epsilon_y}$



$\tan 2\theta_\epsilon = \tan 2\theta_\sigma$

$\frac{-\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-2\tau_{xy}}{\sigma_x - \sigma_y}$

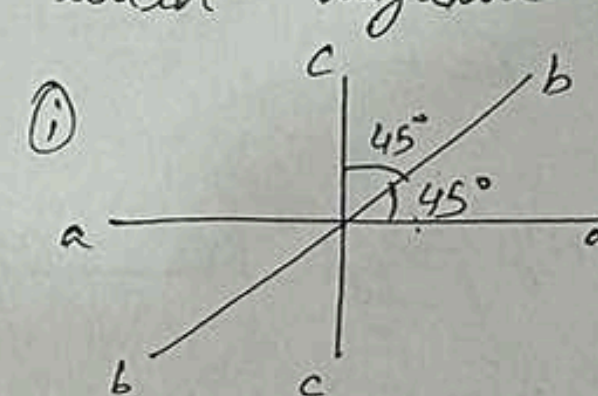
$\tau_{xy} = G\gamma_{xy}$ (Hooke's law).

$\epsilon_x = \frac{\sigma_x - \nu\sigma_y}{E}$ $\epsilon_y = \frac{\sigma_y - \nu\sigma_x}{E}$

$\therefore G = \frac{E}{2(1+\nu)}$ [Derivation of relation b/w E & G]

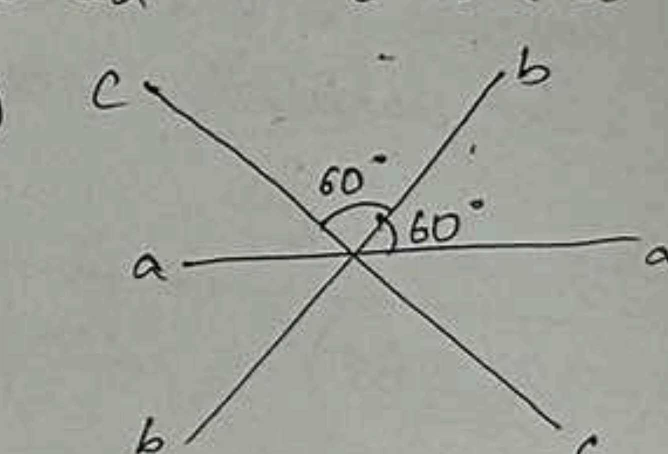
Strain Rosette - Combination of 3 strain gauges with angular offset oriented at $0° < \theta < 180°$

(i)



45° a rectangular strain rosette

(ii)



60° or equilateral

i) $\epsilon = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$

$\epsilon(0°) = \epsilon_x = \epsilon_a$

$\epsilon(45°) = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\gamma_{xy}}{2} = \epsilon_b$

$\epsilon(90°) = \epsilon_y = \epsilon_c$

So, $\frac{\gamma_{xy}}{2} = \frac{\epsilon_a + \epsilon_c}{2} - \epsilon_b$

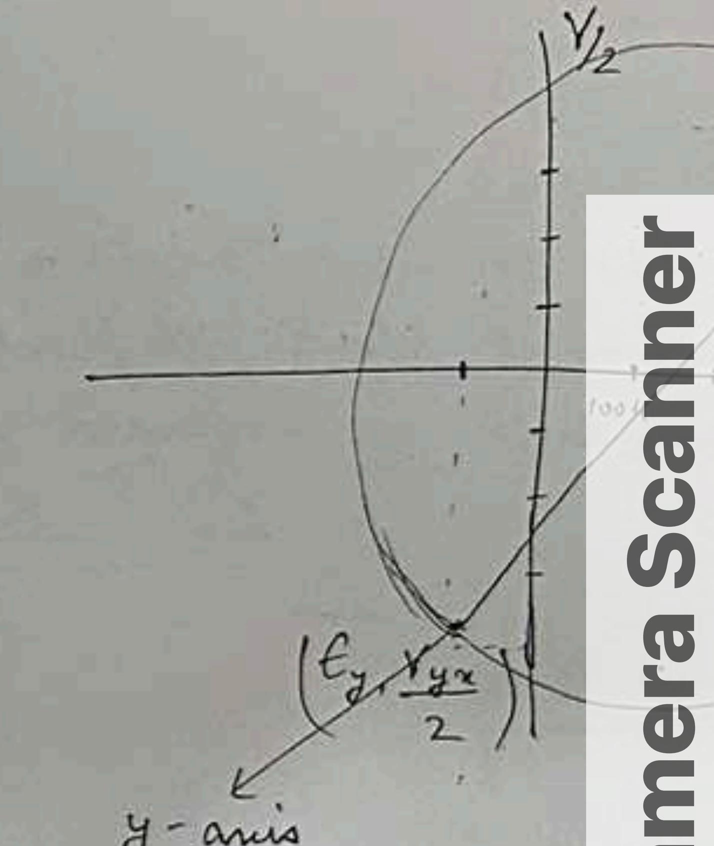
ii) Find for 60° rosette:

$\epsilon_x = \epsilon_a$

$\epsilon_y = \frac{1}{3}(2\epsilon_b + 2\epsilon_c - \epsilon_a)$

$\frac{\gamma_{xy}}{2} = \frac{1}{\sqrt{3}}(\epsilon_c - \epsilon_b)$

989) $\epsilon_a = 400\,\mu$
$E = 2 \times 10^5\ MPa$
For 45° rosette, $\epsilon_x = \epsilon_a$
$\epsilon_y = \epsilon$
$\frac{\gamma_{xy}}{2} =$



$C_\sigma = \frac{E}{1-\nu} C_\epsilon$
$= \frac{2 \times 10^5 \times 150}{1-0.3}$

$R_\sigma = \frac{E}{1+\nu} R_\epsilon$

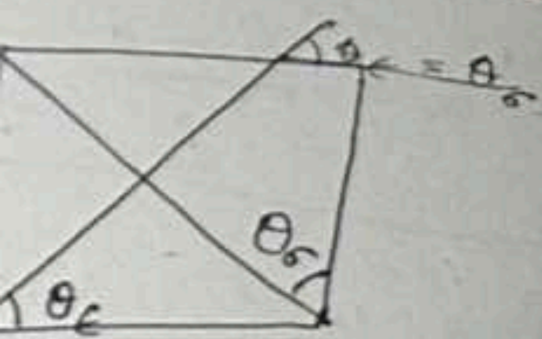
$\sigma_1 = C_\sigma + R_\sigma = 10$

$\tan 2\theta_\sigma = \frac{350}{250}$

$\Rightarrow \theta_\sigma = 27.2°$

Hw: 987 - 991

$\theta_c = \theta_6$

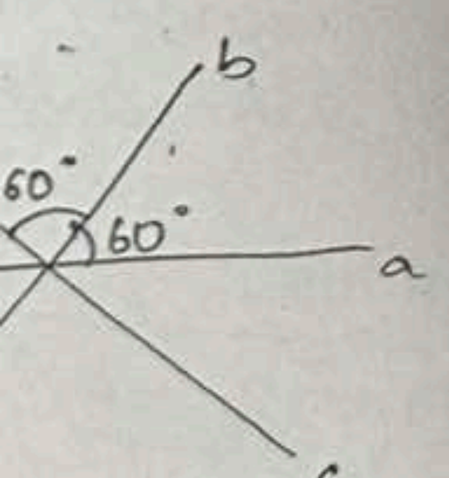


$\frac{\sigma_y - \nu\sigma_x}{E}$

of relation b/w E & σ]

strain gauges

$0^\circ < \theta < 180^\circ$



equilateral

sin 2θ

$2\varepsilon_b + 2\varepsilon_c - \varepsilon_a$)

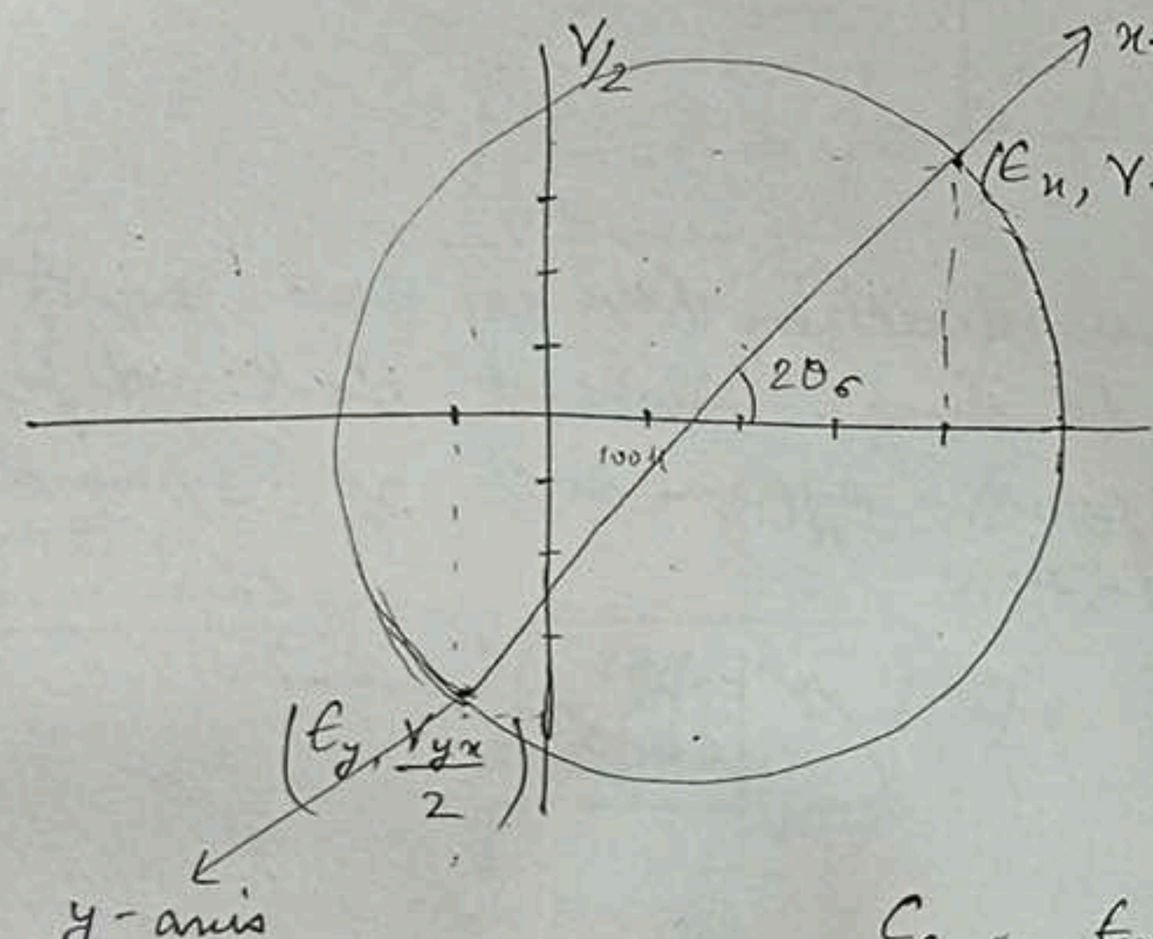
989) $\varepsilon_a = 400\,\mu$ $\quad \varepsilon_b = -200\,\mu$ $\quad \varepsilon_c = -100\,\mu$

$E = 2\times10^5$ MPa $\quad \nu = 0.3$

For 45° rosette, $\varepsilon_x = \varepsilon_a$

$\varepsilon_y = \varepsilon_c$

$\frac{\gamma_{xy}}{2} = \frac{400-100}{2} + 200 = 350\,\mu$



$C_\varepsilon = \frac{\varepsilon_x + \varepsilon_y}{2} = 150\,\mu$

$R_\varepsilon = \sqrt{250^2 + 350^2} = 430\,\mu$

$C_\sigma = \frac{E}{1-\nu} C_\varepsilon$

$= \frac{2\times10^5 \times 150\times10^{-6}}{1-0.3} = 42.85$ MPa

$R_\sigma = \frac{E}{1+\nu} R_\varepsilon = \frac{2\times10^5 \times 430\times10^{-6}}{1.3} = 66.15$ MPa

$\sigma_1 = C_\sigma + R_\sigma = 109$ MPa $\qquad \sigma_2 = C_\sigma - R_\sigma = -23$ MPa

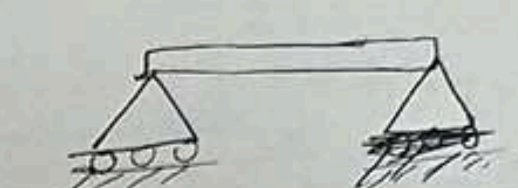
$\tan 2\theta_\sigma = \frac{350}{250}$

$\Rightarrow \theta_\sigma = 27.2^\circ$ (CW with x-axis)

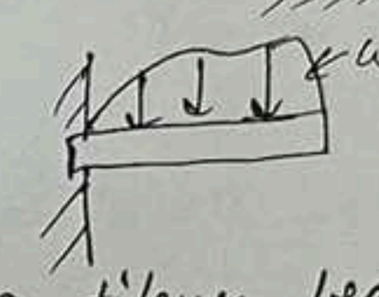
Hw: 987 – 991

Mod 2

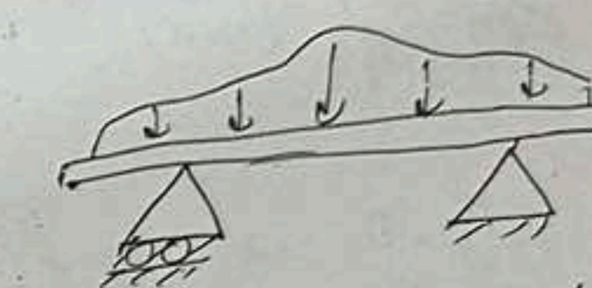
• **Beam** – Structure loaded transversely



Simple beam

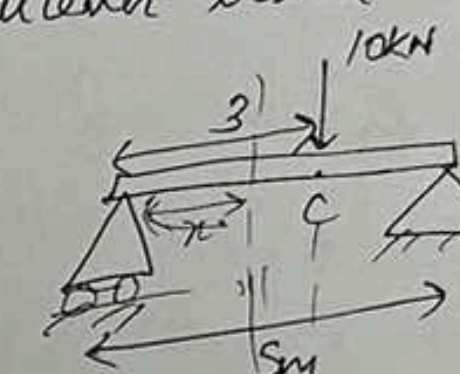


Cantilever beam

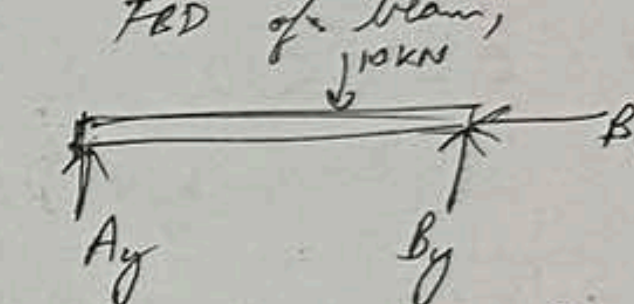


Overhanging beam

Ex:



FBD of beam,



$\Sigma F_x = 0 \Rightarrow B_x = 0$

$\Sigma F_y = 0$.

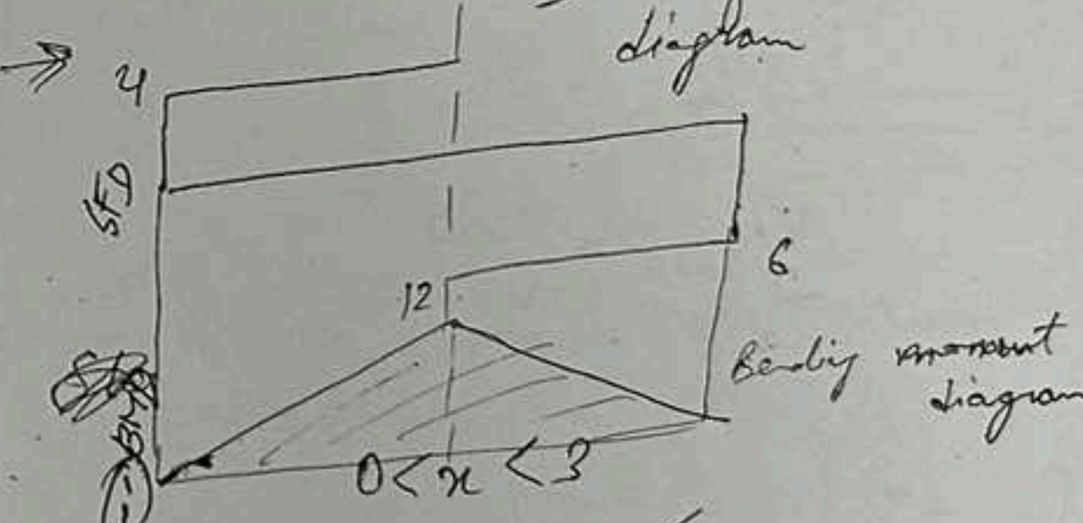
$\Rightarrow A_y + B_y = 10$

$\Sigma M_A = 0$

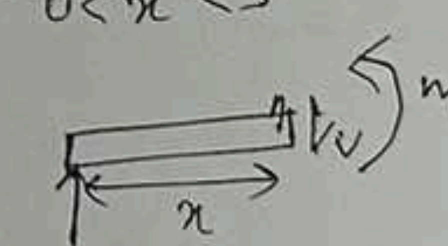
$\Rightarrow 5B_y = 10\times3$

$\Rightarrow B_y = 6$kN

$\therefore A_y = 4$kN



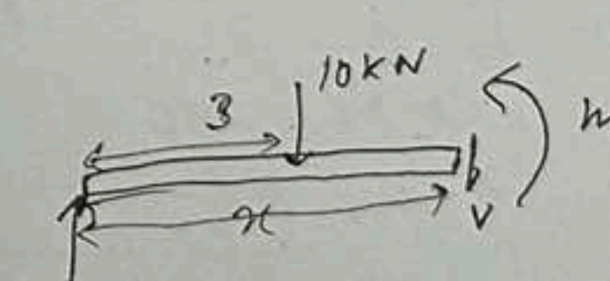
(i) $0 < x < 3$



$\Sigma F_y = 0 \Rightarrow 4 - V = 0$

$\Sigma M_n = 0 \Rightarrow m = 4x$

(ii) $3 < x < 5$



$\Sigma F_y = 0$

$\Rightarrow 4 - 10 - V = 0$

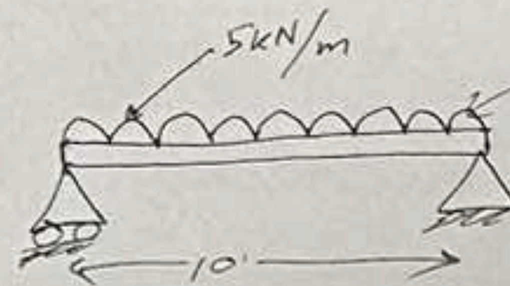
$V = -6$

$\Sigma M_n = 0$

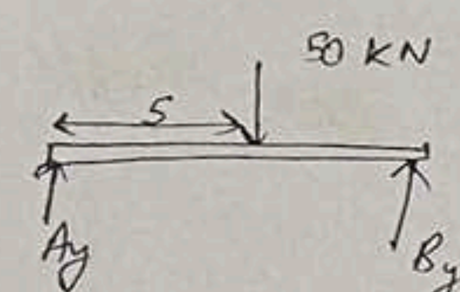
$\Rightarrow 4x - 10(x-3) - m = 0$

$\Rightarrow m = 30 - 6x$

Ex:



→ FBD of beam:

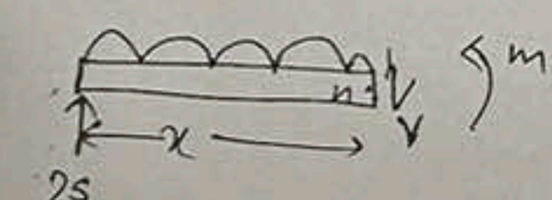


$\Sigma F_y = 0 \Rightarrow A_y + B_y = 50$

$\Sigma M_B = 0 \Rightarrow 10A_y = 50 \times 5$

$A_y = 25$ kN $B_y = 25$

i) $0 < x < 10$



$\Sigma F_y = 0$
$25 - 5x - V = 0$
$V = 25 - 5x$

$V|_{x=0} = 25$
$V|_{x=10} = -25$

Since $V|_{x=0}$ & $V|_{x=10}$ have diff signs, so V must be 0 at $0 < x < 10$

To find x, $25 - 5x = 0 \Rightarrow x = 5$m

$\Sigma M_n = 0$

$\Rightarrow 25x - 5x \times \frac{x}{2} - m = 0$

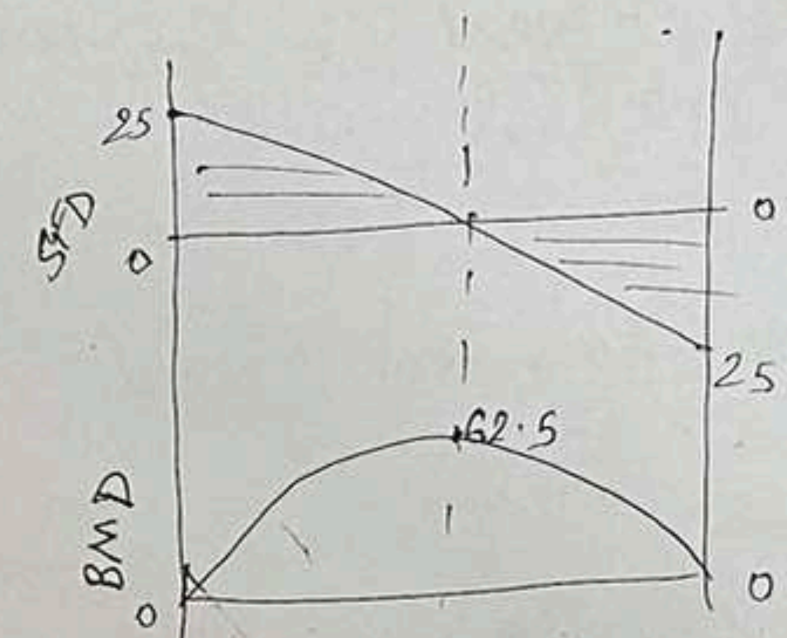
$\Rightarrow m = 25x - \frac{5x^2}{2}$ $m|_{x=0} = 0$ $m|_{x=10} = 0$

$\frac{dm}{dx}$ must be 0 at $0 < x < 10$

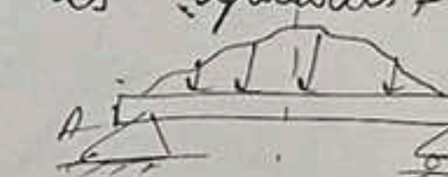
$\frac{dm}{dx} = 0 \Rightarrow x = 5$

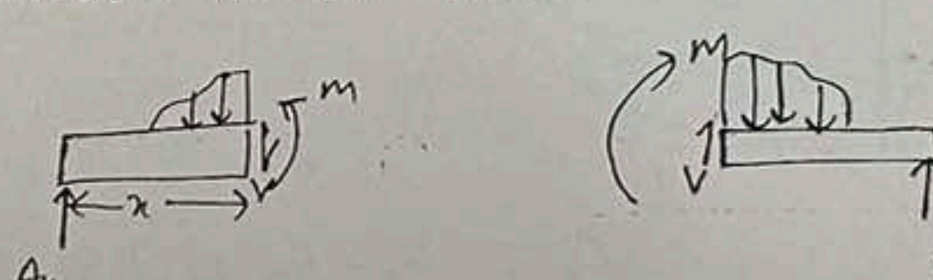
$\frac{d^2m}{dx^2} < 0$ ∴ m is max at $x = 5$

$M_{max} = m|_{x=5} = \frac{125}{2} = 62.5$ kN-m



• In beam problem, direction (+ve or -ve sign) of shear force & bending moment don't depend upon space or coord system such as upwards-downwards or CW-ACW



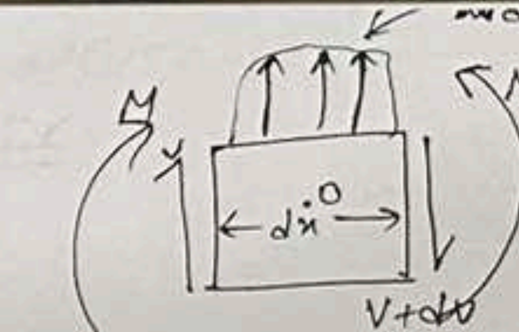


Bending moment creates +ve curvature is taken as +ve

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$ curvature $= \frac{1}{R}$

Relation b/w load, SF, BM

Consider an element of beam b/w 2 sections separated by distance 'dx'



$\Sigma F_y = 0 \Rightarrow w\,dx - (V + dV) + V = 0$

$\Rightarrow dV = w\,dx$

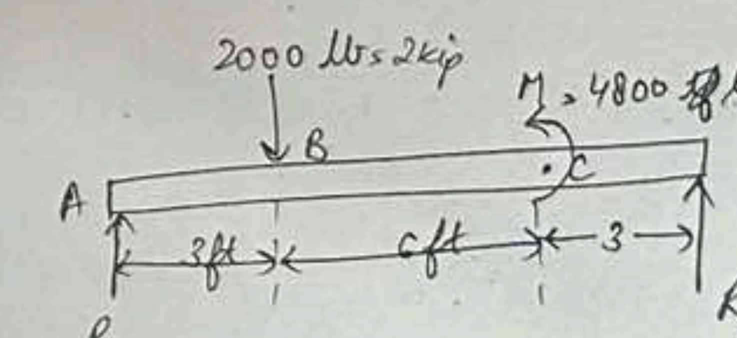
$\Rightarrow \boxed{\frac{dV}{dx} = w}$ slope of shear

$\Sigma M_O = 0 \Rightarrow M + dM - M - (V + dV)\frac{dx}{2}$

$\Rightarrow dM - V\,dx - \cancel{dV\frac{dx}{2}}$

∴ $\boxed{\frac{dM}{dx} = V}$ slope of

(404)

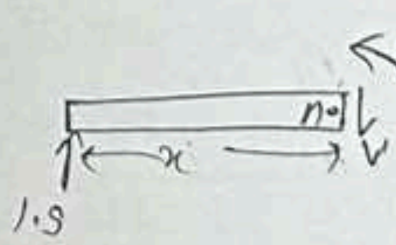


Salient pts: where loads applied

$\Sigma M_D = 0 \Rightarrow 12R_1 - 2 \times 9 - 4.8 = 0$

$R_1 = 1.9$ kip

i) $0 < x < 3$

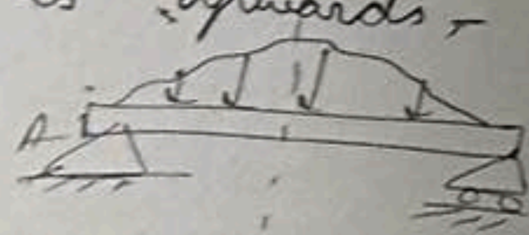


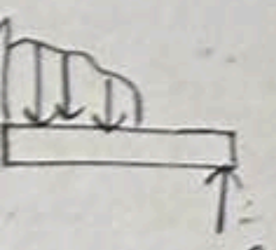
$\Sigma F_y = 0 \Rightarrow 1.9 - V = 0 \Rightarrow V = 1.9$

$\Sigma M_n = 0 \Rightarrow 1.9x - M = 0$

$M = 1.9x$

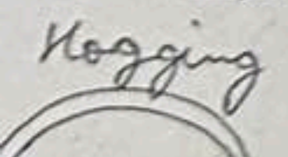
(+ve or -ve sign) of moment don't depend ... em such as upwards, -





B

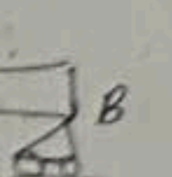
curvature is taken
Hogging



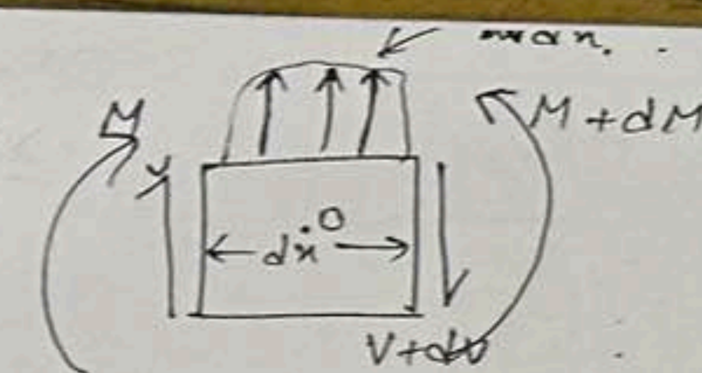
$-M$

curvature $= \frac{1}{R}$

→ b/w 2 sections



B



$\Sigma F_y = 0 \Rightarrow w\,dx - (V + dV) + V = 0$

$\Rightarrow dV = w\,dx$

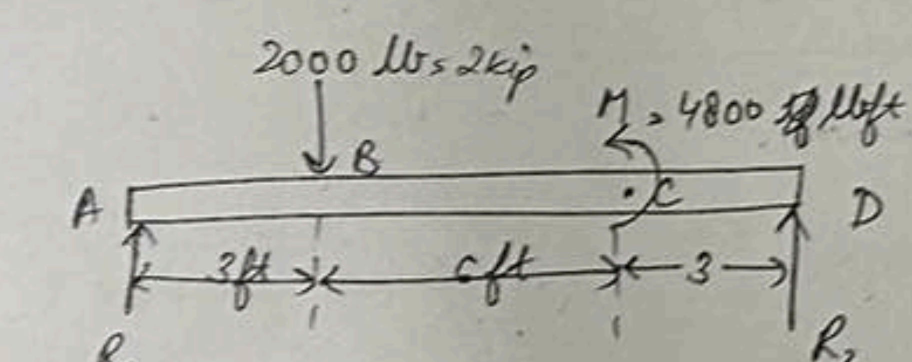
$\Rightarrow \boxed{\frac{dV}{dx} = w}$ slope of shear diagram

$\Sigma M_O = 0 \Rightarrow M + dM - M - (V+dV)\frac{dx}{2} - V\frac{dx}{2} = 0$

$\Rightarrow dM - V\,dx - dV\frac{dx}{2} = 0$

$\therefore \boxed{\frac{dM}{dx} = V}$ slope of moment diagram

(404)

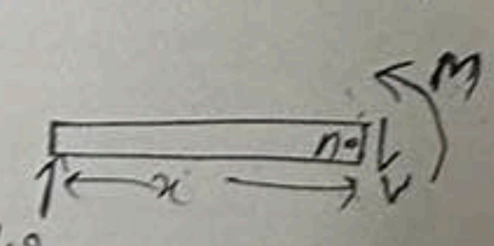


Salient pts: where loads applied or load function changes

$\Sigma M_D = 0 \Rightarrow 12R_1 - 2\times 9 - 4.8 = 0$

$R_1 = 1.9$ kip

i) $0 < x < 3$



$\Sigma F_y = 0 \Rightarrow 1.9 - V = 0 \Rightarrow V = 1.9$

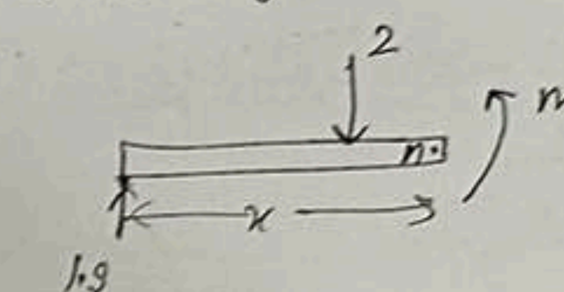
$\Sigma M_x = 0 \Rightarrow 1.9x - M = 0$

$M = 1.9x$

$V_A = V_B = 1.9$

$M_A = M|_{x=0} = 0$ $\qquad M_B = M|_{x=3} = 5.7$

ii) $3 < x < 9$



$\Sigma F_y = 0$
$\Rightarrow 1.9 - 2 - V = 0$
$\Rightarrow V = -0.1$

$\Sigma M_x = 0 \Rightarrow 1.9x - 2(x-3) - M = 0$

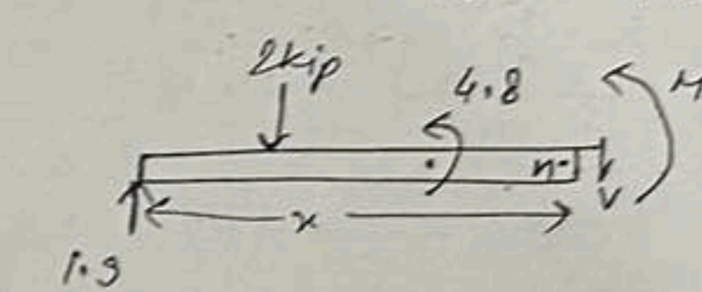
$M = 6 - 0.1x$

$V_B = V|_{x=3} = -0.1$

$V_C = V|_{x=9} = -0.1$

$M_B = M|_{x=3} = 5.7$ $\qquad M_C = M|_{x=9} = 5.1$

iii) $9 < x < 12$



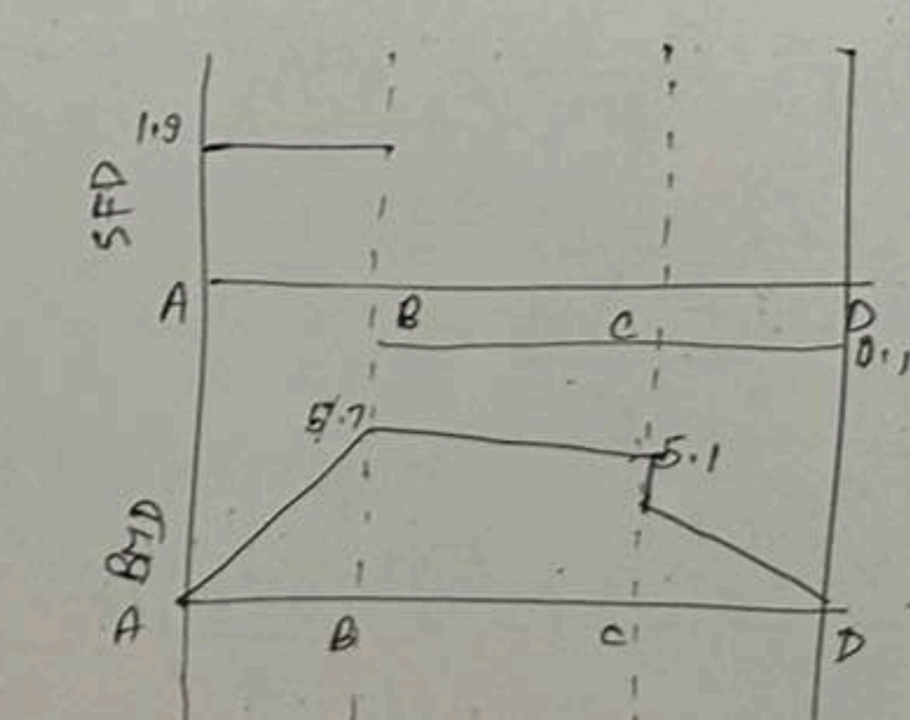
$\Sigma F_y = 0 \Rightarrow 1.9 = 2 - V \Rightarrow V = -0.1$

$\Sigma M_x = 0 \Rightarrow 1.9x - 2(x-3) - 4.8 - M = 0$

$M = 1.2 - 0.1x$

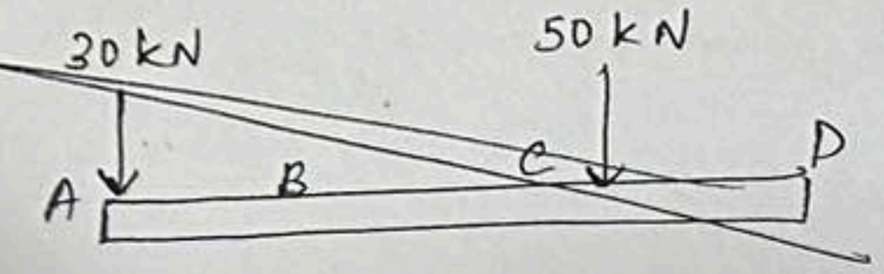
$M_C = M|_{x=9} = 0.3$

$M_D = M|_{x=12} = 0$



403

HW



935) $\sigma_1 = 2000$ psi $\qquad \sigma_2 = -8000$ psi

$$\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right\} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\sigma_1+\sigma_2 = \sigma_x+\sigma_y = -6000 \quad \text{——(i)}$$

$$\sigma_1-\sigma_2 = 2\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\Rightarrow \frac{2000+8000}{2} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+9000\times10^3}$$

$$\Rightarrow 25000\times10^3 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2+9\times10^6$$

$$\Rightarrow \frac{\sigma_x-\sigma_y}{2} = 4000 \quad \text{——(ii)}$$

$\therefore \sigma_x = 1000$ psi $\qquad \sigma_y = -7000$ psi

978) $$C_\sigma = \frac{\sigma_x+\sigma_y}{2} \quad \text{—— (i)}$$

$$C_\varepsilon = \frac{\varepsilon_x+\varepsilon_y}{2} \quad \text{——(ii)}$$

$$\varepsilon_x = \frac{\sigma_x-\nu\sigma_y}{E} \quad \text{——(iii)}. \qquad \varepsilon_y = \frac{\sigma_y-\nu\sigma_x}{E} \quad \text{——(iv)}$$

Substituting in (ii),

$$C_\varepsilon = \frac{\frac{\sigma_x-\nu\sigma_y}{E}+\frac{\sigma_y-\nu\sigma_x}{E}}{2}$$

$$= \frac{\sigma_x+\sigma_y-\nu(\sigma_x+\sigma_y)}{2E} = \left(\frac{1-\nu}{E}\right)\frac{(\sigma_x+\sigma_y)}{2}$$

$$= \frac{1-\nu}{E}\times C_\sigma$$

$$R_\sigma = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$R_\varepsilon = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$$

Also, $\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$ as $G = \frac{E}{2(1+\nu)}$

$$\therefore R_\varepsilon = \sqrt{\left(\frac{\frac{\sigma_x-\nu\sigma_y}{E}-\frac{\sigma_y-\nu\sigma_x}{E}}{2}\right)^2+\left(\frac{2(1+\nu)\tau_{xy}}{2E}\right)^2}$$

$$= \sqrt{\left(\frac{\sigma_x-\sigma_y+\nu(\sigma_x-\sigma_y)}{2E}\right)^2+\left(\frac{1+\nu}{E}\right)^2\tau_{xy}^2}$$

$$= \sqrt{\left(\frac{1+\nu}{E}\right)^2\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\left(\frac{1+\nu}{E}\right)^2\tau_{xy}^2}$$

$$= \frac{1+\nu}{E}\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \frac{1+\nu}{E}R_\sigma$$

Hence, proved

979) $\beta = (\varepsilon_x-\varepsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$

$\gamma_{ab} = 2(\varepsilon_x-\varepsilon_y)S_\theta C_\theta + \gamma_{xy}(C_\theta^2-S_\theta^2)$

For principal stress, $\tau_{xy}=0$ &

Using $\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$

$$\therefore \beta = (\varepsilon_x-\varepsilon_y)S_\theta C_\theta \quad \text{——(i)}$$

$$\gamma_{ab} = 2(\varepsilon_x-\varepsilon_y)C_\theta S_\theta \quad \text{——(ii)}$$

$$(i)\div(ii) \Rightarrow \frac{\beta}{\gamma_{ab}/2} = 1 \Rightarrow \boxed{\beta = \frac{\gamma_{ab}}{2}}$$

Hence, proved

980) $\varepsilon_x = -400\times10^{-6}$

$\varepsilon_y = 200\times10^{-6}$

$\varepsilon_x = \frac{\sigma_x-\nu\sigma_y}{2}$ $\qquad \varepsilon_y$

$$\therefore \sigma_x = \frac{E(\varepsilon_x+\nu\varepsilon_y)}{1-\nu^2}$$

$= 200\times10^9(-400\times10^{-6}$ ...

$= \frac{-340\times200\times10^3}{0.91}$

$$\sigma_y = \frac{E(\varepsilon_y+\nu\varepsilon_x)}{1-\nu^2}$$

$= 200\times10^9(200\times10$ ...

$= 17.6$ MPa

$\tau_{xy} = \frac{\gamma_{xy}\times E}{2(1+\nu)} = 800$ ...

Now, $\sigma(30^\circ) = \frac{\sigma_x+\sigma_y}{2} +$ ...

$= \frac{-74.7+17.6}{2}$ ...

$= -28.55 - 23.08$ ...

$= -104.89$ MPa

$\tau(30^\circ) = \frac{\sigma_x-\sigma_y}{2}\sin2\theta$ ...

$= -23.08 + 19.2$ ...

$\frac{E}{2(1+\nu)}$

$\left(\frac{2(1+\nu)\tau_{xy}}{2E}\right)^2$

$\left(\frac{1+\nu}{E}\right)^2 \tau_{xy}^2$

$)^2 \tau_{xy}^2$

, proved

$\sin^2\theta$

$- S_\theta^2)$

$\frac{\gamma_{ab}}{2}$ Hence, proved

980) $\varepsilon_x = -400 \times 10^{-6}$ $\quad \gamma_{xy} = 800 \times 10^{-6}$

$\varepsilon_y = 200 \times 10^{-6}$ $\quad E = 200$ GPa

$\nu = 0.3$

$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{2}$ $\quad \varepsilon_y = \frac{\sigma_y - \nu\sigma_x}{2}$

$\therefore \sigma_x = \frac{E(\varepsilon_x + \nu\varepsilon_y)}{1-\nu^2}$

$= \frac{200 \times 10^9(-400 \times 10^{-6} + 0.3 \times 200 \times 10^{-6})}{1 - 0.09}$

$= \frac{-340 \times 200 \times 10^3}{0.91} = -74.7$ MPa

$\sigma_y = \frac{E(\varepsilon_y + \nu\varepsilon_x)}{1-\nu^2}$

$= \frac{200 \times 10^9(200 \times 10^{-6} + 0.3 \times -400 \times 10^{-6})}{0.91}$

$= 17.6$ MPa

$\tau_{xy} = \frac{\gamma_{xy} \times E}{2(1+\nu)} = \frac{800 \times 10^{-6} \times 200 \times 10^9}{2 \times 1.3} = 61.5$ MPa

Now, $\sigma(30°) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

$= \frac{-74.7 + 17.6}{2} + \frac{-74.7 - 17.6}{2}\cos 60° - 61.5 \sin 60°$

$= -28.55 - 23.08 - 53.26$

$= -104.89$ MPa

$\tau(30°) = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= -23.08 + 13.24 = -7.84$ MPa

981) $\varepsilon_x = 600 \times 10^{-6}$ $\quad \varepsilon_y = -300 \times 10^{-6}$

$\gamma_{xy} = -400 \times 10^{-6}$ $\quad E = 30 \times 10^6$ psi

$\nu = 0.3$

$\sigma_x = \frac{30 \times 10^6(600 \times 10^{-6} + 0.3 \times -300 \times 10^{-6})}{1 - 0.09} = 16.81$ ksi

$\sigma_y = \frac{30 \times 10^6}{1 - 0.09}(-300 \times 10^{-6} + 0.3 \times 600 \times 10^{-6}) = -3.96$ ksi

$\tau_{xy} = \frac{E\gamma_{xy}}{2(1+\nu)}$

$= \frac{-30 \times 10^6 \times 400 \times 10^{-6}}{2(1+0.3)} = -4.62$ ksi

$\left.\begin{matrix}\sigma_1 \\ \sigma_2\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 6.425 \pm \sqrt{107.85 + 21.34}$

$\therefore \sigma_1 = 17.8$ ksi $\quad \sigma_2 = -4.95$ ksi

$\theta_\tau = \frac{1}{2}\tan^{-1}\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$ for τ_{max}

$= \frac{1}{2}\tan^{-1}\left(\frac{16.81 + 3.96}{-2 \times 4.62}\right)$

$= \frac{1}{2}\tan^{-1}(-2.25) = 33°$

$\tau_{max} = \tau(\theta_\tau) = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= \frac{16.81 + 2.96}{2}\sin 66° - 4.62\cos 66°$

$= 9.49 - 1.88 = 7.61$ ksi

982) $\varepsilon_x = -533 \times 10^{-6}$ $\quad \varepsilon_y = 67 \times 10^{-6}$

$\gamma_{xy} = -626 \times 10^{-6}$ $\quad E = 30 \times 10^6$ $\quad \nu = 0.3$

$\sigma_x = \frac{30\times10^6}{1-0.09}(-533\times10^{-6} + 0.3\times 67\times10^{-6})$

$= -16.91$ ksi

$\sigma_y = \frac{E(\varepsilon_y + \nu\varepsilon_x)}{1-\nu^2} = -3.06$ ksi

$\tau_{xy} = \frac{E\gamma_{xy}}{2(1+\nu)} = \frac{-626\times10^{-6}\times 30\times10^6}{2(1+0.3)} = -7.22$ ksi

$\therefore \sigma(45^\circ) = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

$= -2.77$ ksi

$\tau(45^\circ) = \frac{\sigma_x-\sigma_y}{2}\sin 90^\circ + \tau_{xy}\cos 90^\circ = -6.925$ ksi

983) $\varepsilon_x = -800\times10^{-6}$ $\varepsilon_y = 200\times10^{-6}$

$\gamma_{xy} = -800\times10^{-6}$ $E = 200\times10^9$ $\nu = 0.3$

$\sigma_x = \frac{200\times10^9}{1-0.09}(-800\times10^{-6} + 0.3\times 200\times10^{-6})$

$= -162.64$ MPa

$\sigma_y = \frac{200\times10^9}{1-0.09}(200\times10^{-6} - 0.3\times800\times10^{-6}) = -8.79$ MPa

$\tau_{xy} = \frac{-800\times10^{-6} * 200\times10^9}{2\times1.3} = -61.54$ MPa

$\therefore \sigma(20^\circ) = -105.1$ MPa

$\tau(20^\circ) = -96.59$ MPa

987) $\varepsilon_a = 100\times10^{-6}$ $\varepsilon_c = 400\times10^{-6}$

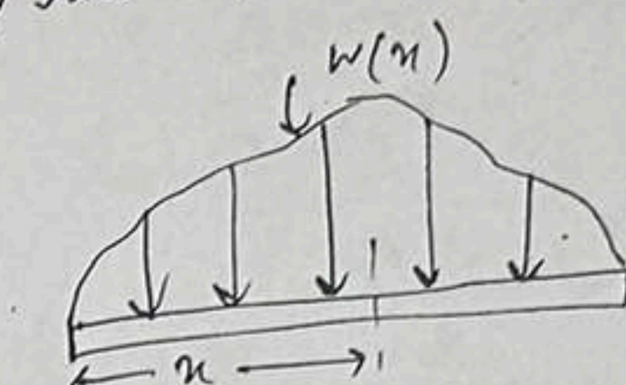
$\varepsilon_b = -200\times10^{-6}$ $E = 10^7$ psi

$\nu = \frac{1}{3}$

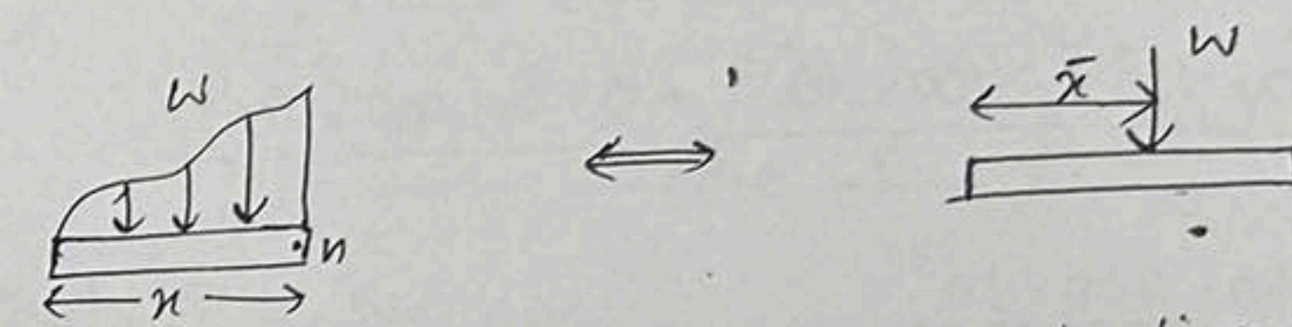
$\varepsilon_x = \varepsilon_a$

$\varepsilon_y =$

Q) 925 - 935, 978 - 983 $\longrightarrow$ Quiz



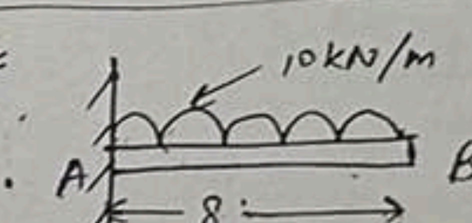
1st take section then draw equivalent system.



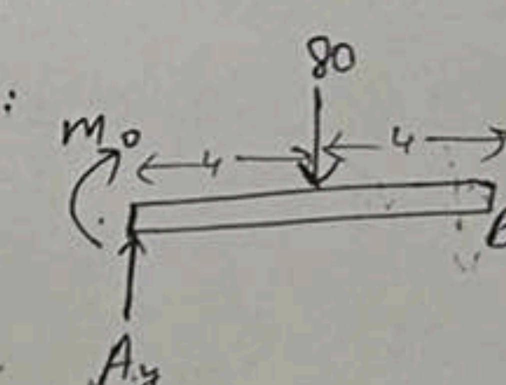
$W = \int_0^x \omega\,dx \rightarrow$ Area of load diagram

$W\bar{x} = \int_0^x \omega x\,dx$ or $\bar{x} = \frac{\int_0^x \omega x\,dx}{W}$

#



FBD:



$\Sigma F_y = 0$

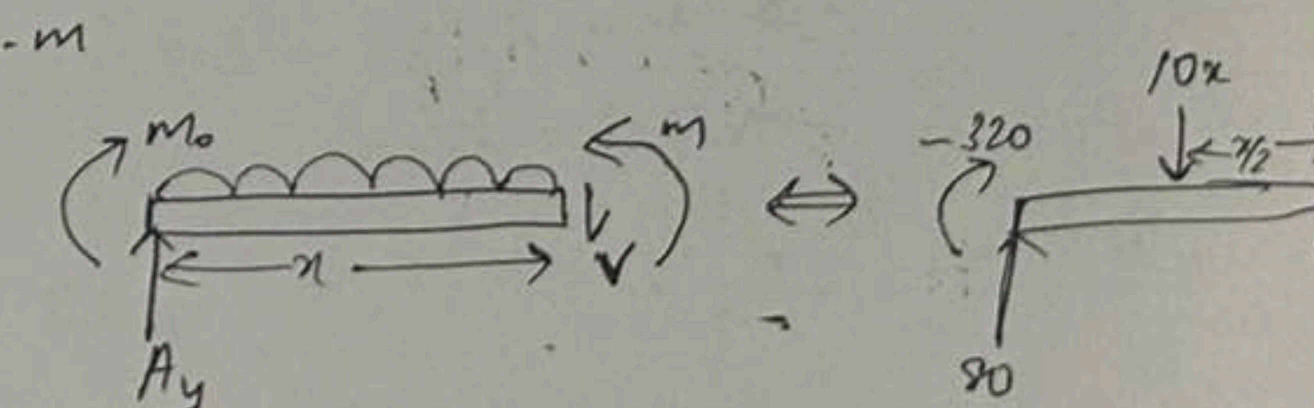
$A_y - 80 = 0$

$A_y = 80$

$\Sigma M_A = 0$

$M_0 + 4\times 80 = 0$

$M_0 = -320$ kN·m



$\Sigma F_y = 0$

$V + 10x - 80 = 0$

$\Sigma M_n = 0$

$\Rightarrow M + 10x\cdot\frac{x}{2} + 320 - 80x$

$\Rightarrow M = 80x - 5x^2 - 320$

$V_A = V|_{x=0} = 80$

$M_A = M|_{x=0} = -320$

$\bigstar \begin{cases} \frac{d^2M}{dx^2} < 0 \Rightarrow \text{max} \\ \frac{d^2M}{dx^2} > 0 \Rightarrow \text{min} \end{cases}$

SFD

80

0

BMD

320

OR (Invert the section
Taking

$V_A = -80$

$V_B = 0$

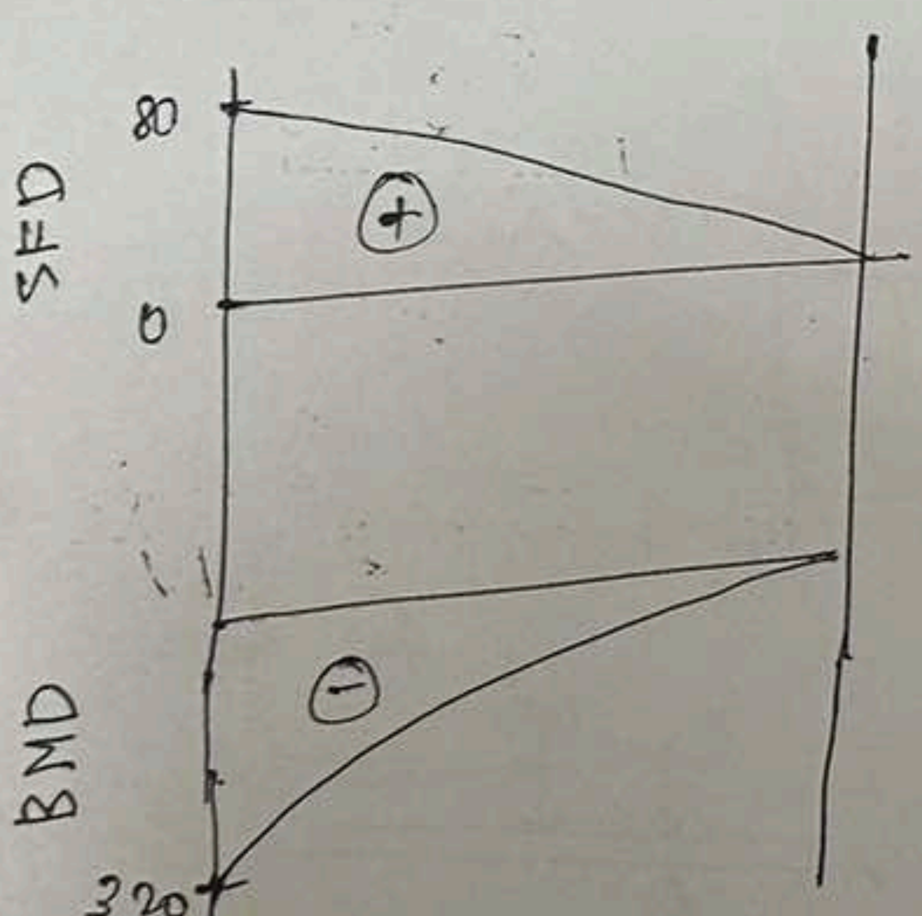
SFD

BMD

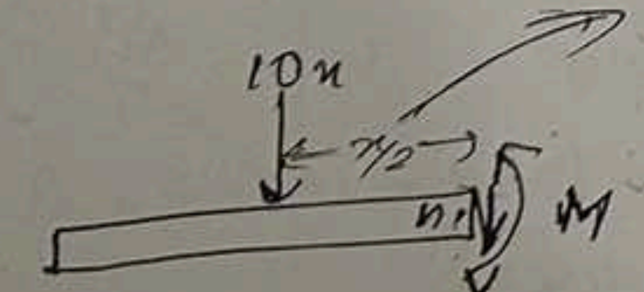
$V_A = V|_{x=0} = 80$ $\quad V_B = V|_{x=8} = 0$

$M_A = M|_{x=0} = -320$ $\quad M_B = M|_{x=8} = 0$

$$\left\{ \begin{array}{l} \frac{d^2M}{dx^2} < 0 \Rightarrow \text{maxima} \\ \frac{d^2M}{dx^2} > 0 \Rightarrow \text{min} \end{array} \right.$$



OR (Invert the beam)
Taking section:



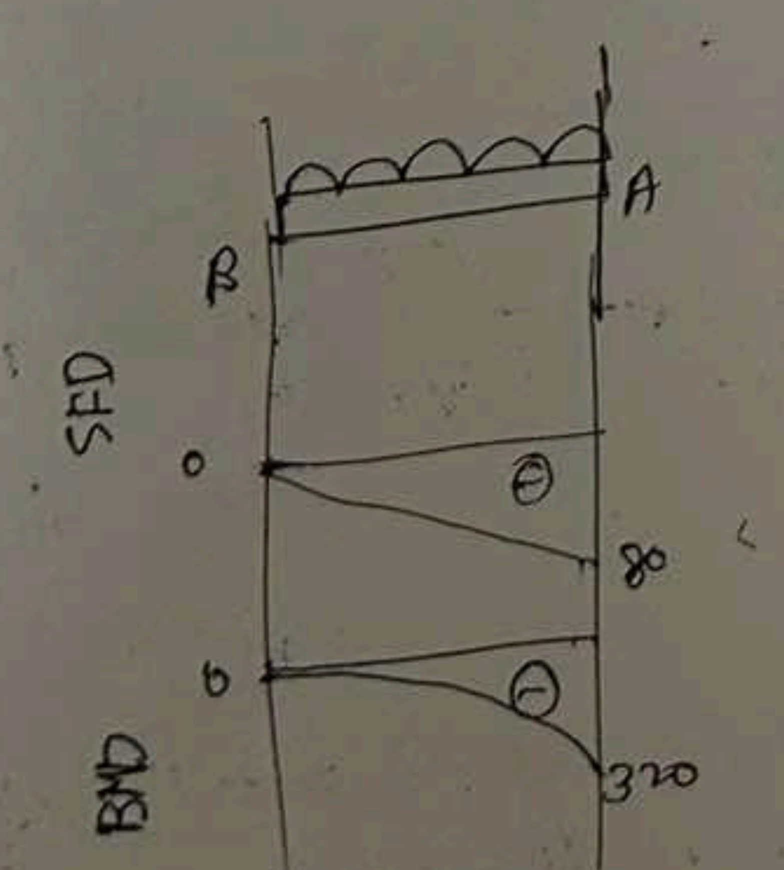
Horiz position of Centroid

$V = -10x$ $\quad M_A = -320$

$M = -5x^2$ $\quad M_B = 0$

$V_A = -80$

$V_B = 0$

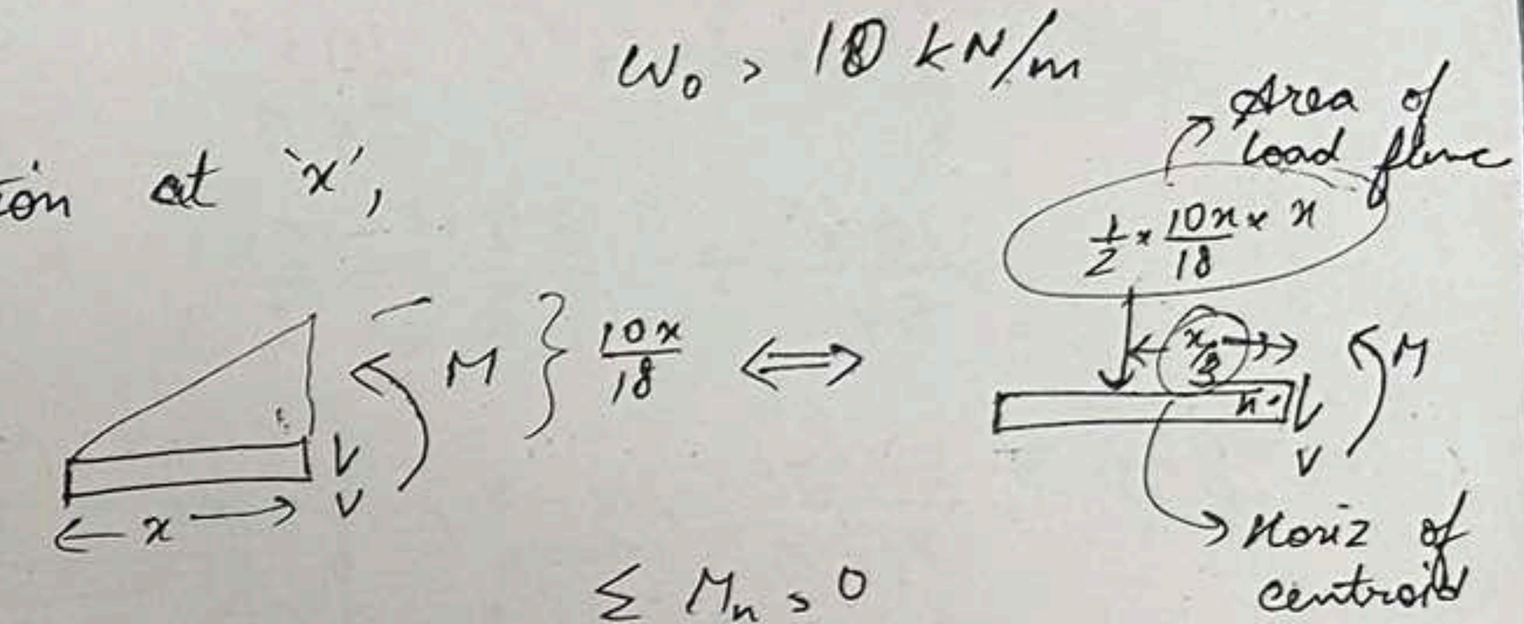


• While flipping cantilever beam, couple's direction changes, everything else is same

$\boxed{\text{HW: 403 - 411}}$

410) $L = 18$ $\quad W_0 = 10$ kN/m

Taking section at 'x',



$\Sigma F_y = 0$ $\quad \Sigma M_n = 0$

$V = -\frac{5x^2}{18}$ $\quad M + \frac{5x^2}{18} \times \frac{x}{3} = 0$

$M = -\frac{5x^3}{54}$

$M_A = M|_{x=0} = 0$

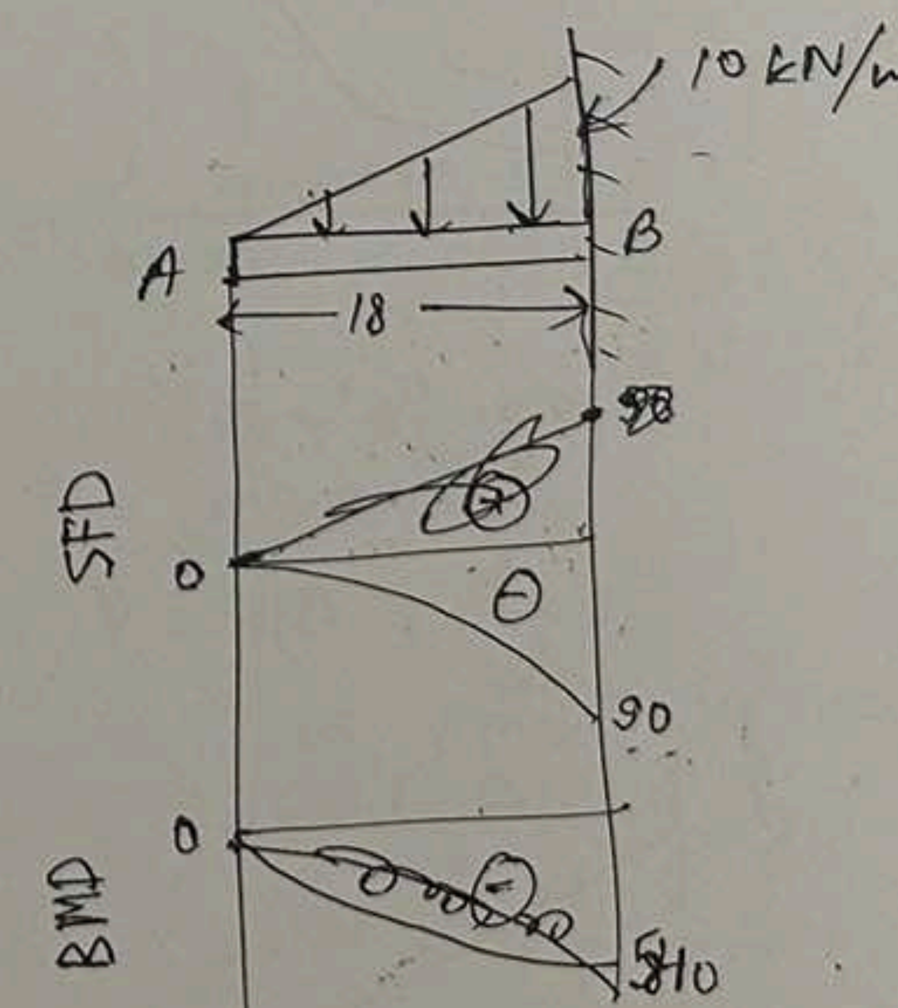
$M_B = M|_{x=18} = -\frac{5 \times 18 \times 18 \times 18}{54} = -540$

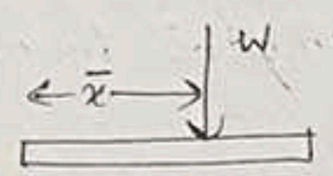
$V_A = V|_{x=0} = 0$

$V_B = V|_{x=18} = 5 \times 18 = -90$

$w = \frac{10x}{18}$

$W = \int_0^x w\,dx = \int_0^x \frac{10x\,dx}{18} = \frac{5x^2}{18}$





$W\bar{x} = \int_0^x wx\,dx = \frac{10x^3}{3\times 18}$

$\therefore \bar{x} = \frac{10x^3}{54} \div \frac{5x^2}{18} = \frac{2x}{3}$

(4ii) $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

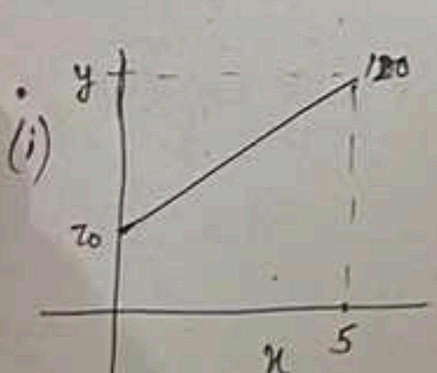
$W - W_0 = \frac{0 - W_0}{L - 0}(x - 0)$

$W = -\frac{W_0 x}{L} + W_0$

$W = \int_0^x w\,dx = -\frac{W_0 x^2}{2L} + W_0 x$

$\bar{x} = \frac{\int_0^x wx\,dx}{W} =$

(i)



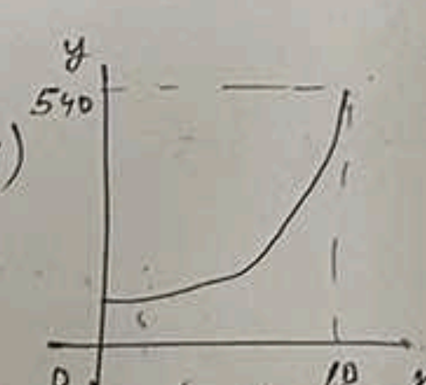
$y = 10x + 70$

$0 < x < 5$

$y|_{x=0} = 70$

$y|_{x=5} = 120$

(ii)



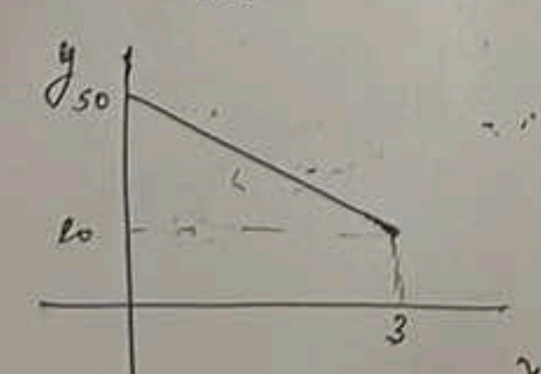
$y = 5x^2 + 3x + 10$

$0 < x < 10$

$\frac{dy}{dx} = 10x + 3$

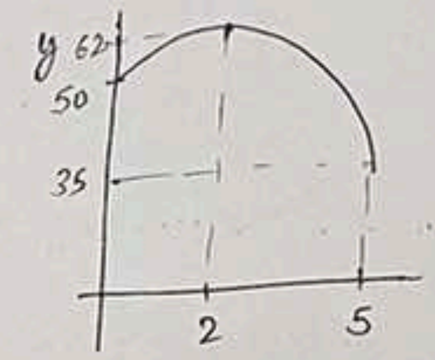
(iii) $y = 50 - 10x$

$0 < x < 3$



iv) $y = -3x^2 + 12x + 50$

$0 < x < 5$



$y|_{x=0} = 50$

$y|_{x=5} = 35$

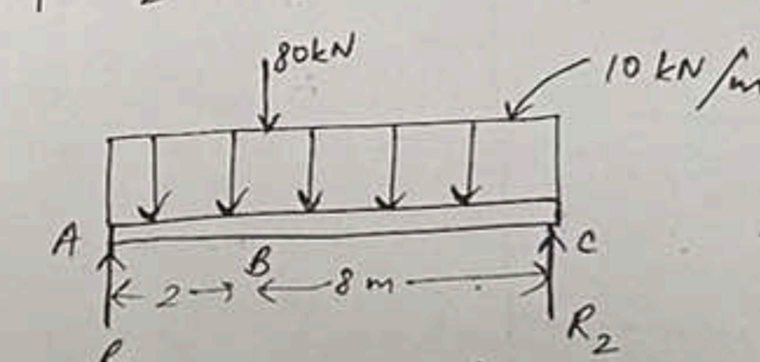
$y' = 12 - 6x$

at $x = 2$, $y' = 0$

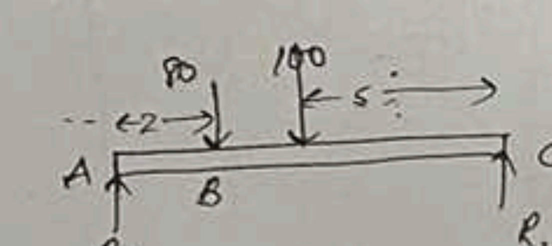
$y'' = -6$

$y_{max} = y|_{x=2} = 62$

Ex:



→ FBD of beam:

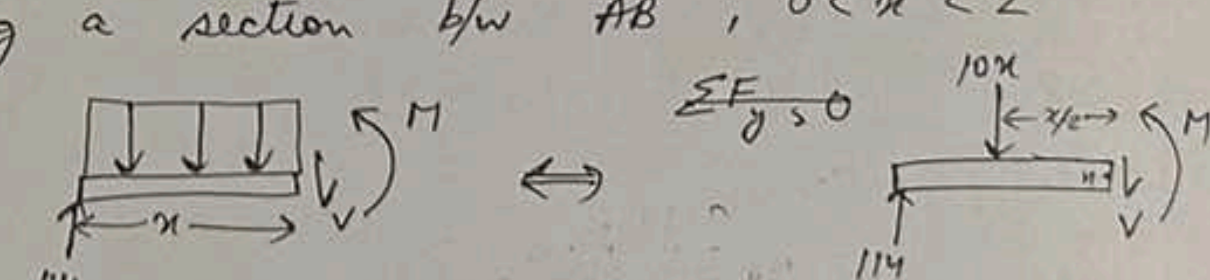


$\Sigma M_C = 0$

$\Rightarrow 10R_1 - 80\times 8 - 100\times 5 = 0$

$\Rightarrow R_1 = 114$

i) Taking a section b/w AB, $0 < x < 2$



$\Sigma F_y = 0 \Rightarrow V + 10x - 114 = 0$

$V = 114 - 10x$

$V|_{x=0} = 114$

$V|_{x=2} = 94$

$\Sigma F_y = 0$

$\Sigma M_x = 0$

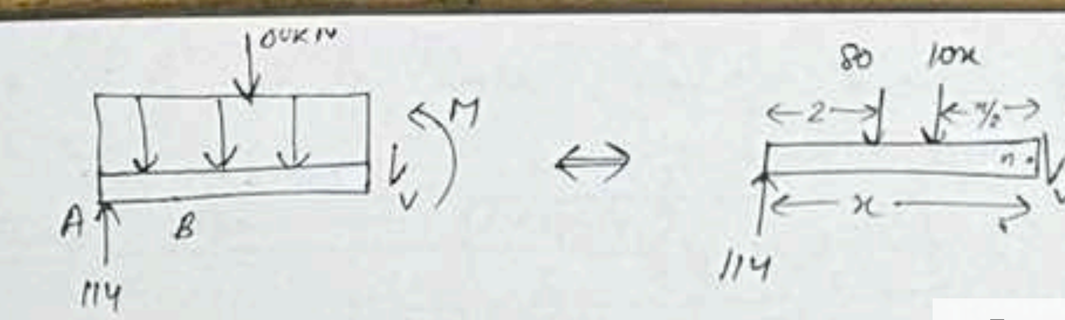
$M + 10x \times \frac{x}{2} - 114x = 0$

$M = 114x - 5x^2$

$M_A = 0$

$M_B = 114\times 2 - 20 = 208$

ii) Taking section b/w B & C at distance x from A



$\Sigma F_y = 0 \Rightarrow V = 34 - 10x$

$V_B = V|_{x=2} = 14$

$V_C = V|_{x=10} = -66$

$\Sigma M_x = 0$

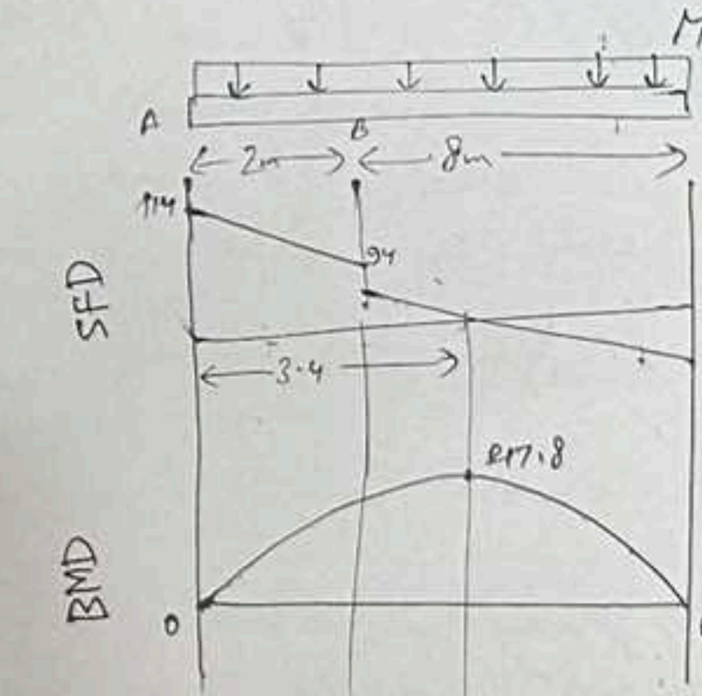
$\Rightarrow 80(x-2) \ldots$

$\Rightarrow M = \ldots$

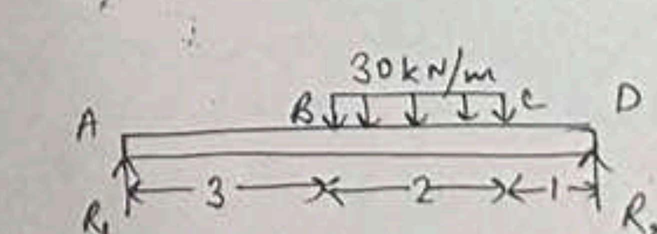
$M_B = \ldots$

$M_C = 0$

$M_{max} = M|_{x=\ldots}$



407)



$y/_{x=0} = 50$

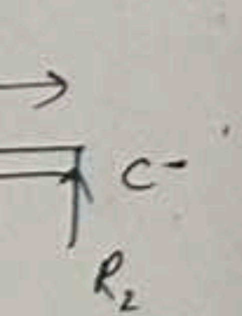
$y/_{x=5} = 35$

$y' = 12 - 6x$

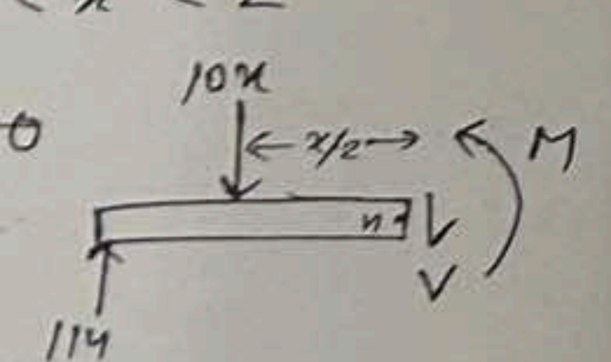
at $x = 2$, $y' = 0$

$y'' = -6$

$y_{max} = y/_{x=2} = 62$



$< x < 2$



$M_x = 0$

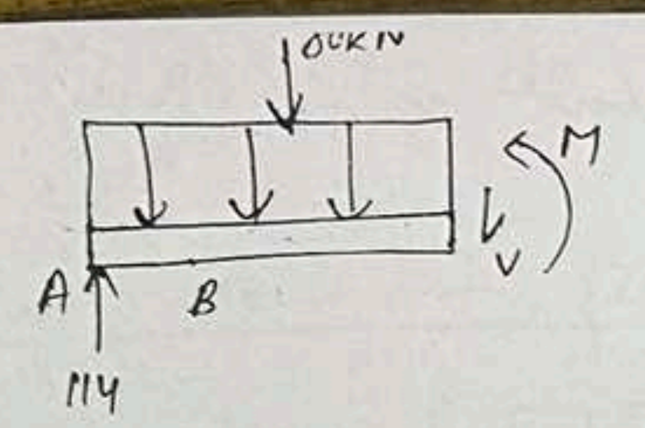
$10x \times \frac{x}{2} - 114x = 0$

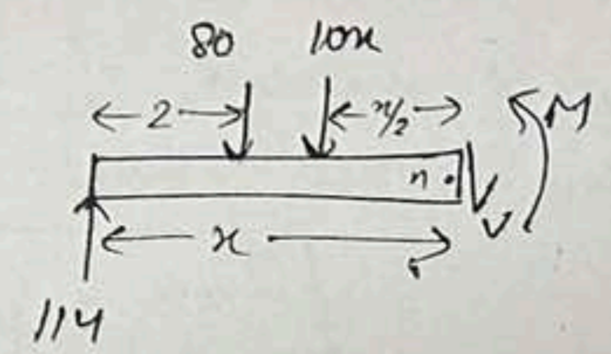
$= 114x - 5x^2$

$M_A = 0$

$M_B = 114 \times 2 - 20 = 208$

x from A





$\Sigma F_y = 0 \Rightarrow V = 34 - 10x$

$V_B = V/_{x=2} = 14$

$V_c = V/_{x=10} = -66$

$\Sigma M_x = 0$

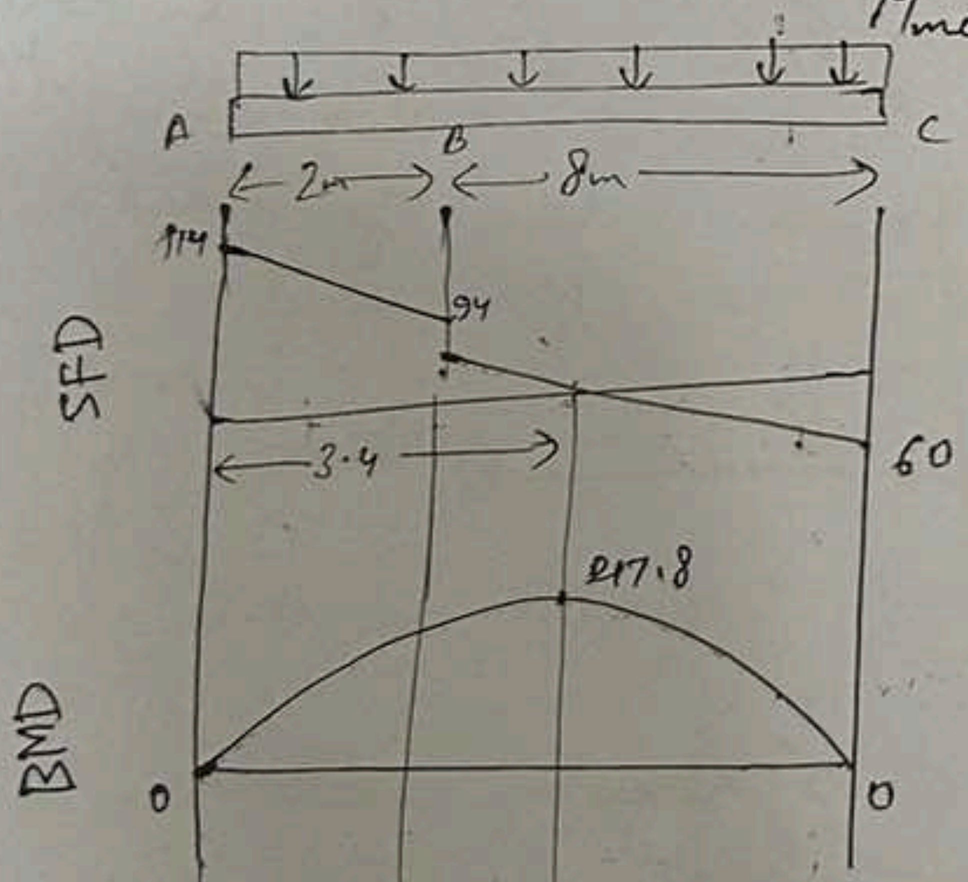
$\Rightarrow 80(x-2) - 114x + \frac{10x^2}{2} + M = 0$

$\Rightarrow M = -5x^2 + 34x + 160$

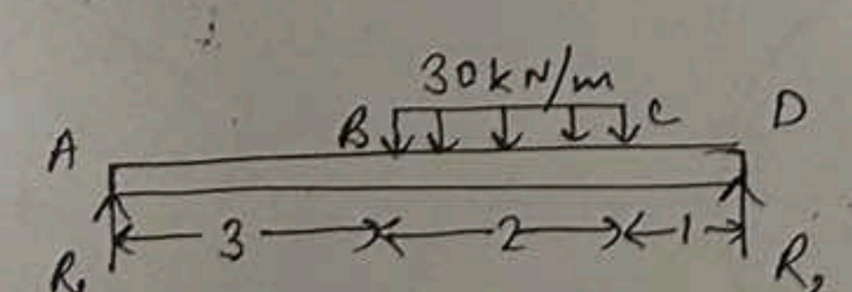
$M_B = 208$

$M_C = 0$

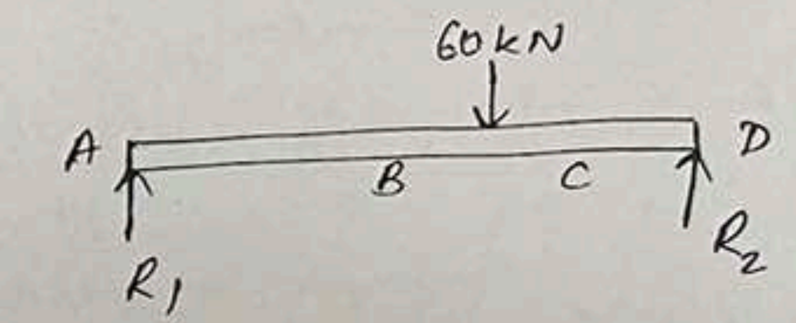
$M_{max} = M/_{x=3.4}$?



407)



FBD :

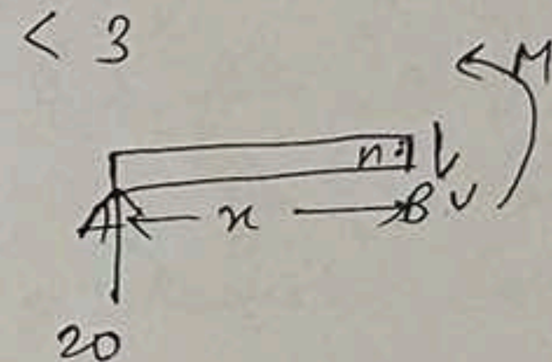


$R_1 + R_2 = 60$

$\Sigma M_D = 0 \Rightarrow 6R_1 = 60 \times 2$

$R_1 = 20$ kN $\quad R_2 = 40$ kN

i) $0 < x < 3$



$\Sigma F_y = 0$

$V = 20$ kN

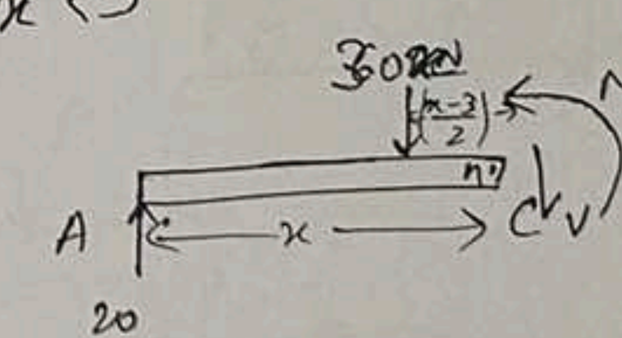
$\Sigma M_x = 0$

$\Rightarrow 20x = M$

$M_A = 0$

$M_B = 60$

ii) $3 < x < 5$



$60 + V = 20$

$V = -40$ kN

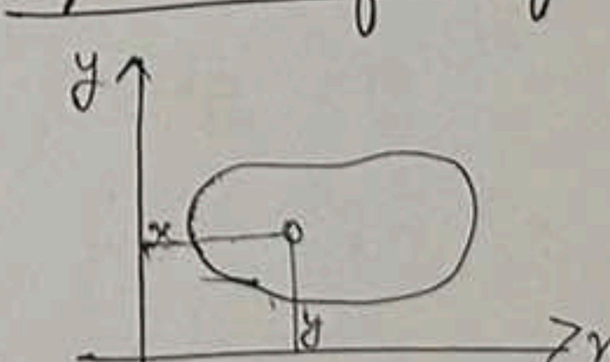
$\Sigma M_x = 0$

$\Rightarrow M + 30x\left(\frac{x-3}{2}\right) = 20x$

$\Rightarrow \cancel{M = 20x - 30x + 60}$

$M = -15x^2 + 65x$

Properties of surfaces



$dA = dx\,dy$

$A = \int dA = \iint dx\,dy$

In order to find position of centroid, we take area moment about x;

$M_x = \int_A y\,dA$

Area $\quad y, \; M_y = \int_A x\,dA$

$Ax_c = M_y$

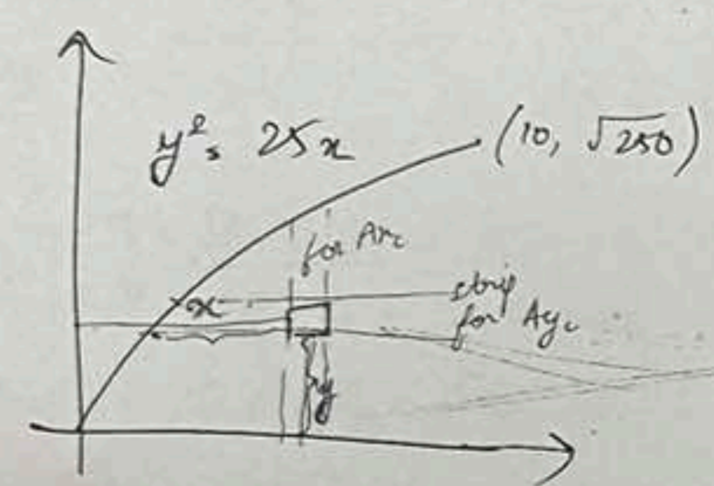
$\Rightarrow x_c = \dfrac{\int_A x\,dA}{A}$

Similarly,

$y_c = \dfrac{\int_A y\,dA}{A}$

We can concentrate entire area A at pt (x_c, y_c) called centroid.

Ex:



this x, y diff from $y^2 = 25x$ ka x, y

$\longrightarrow \quad A = \int_0^{10}\int_0^{y}(dy)\,dx$

$= \int_0^{10} y\,dx$

$= \int_0^{10} 5\sqrt{x}\,dx = 105.4$

$Ax_c = \int_A x\,dA$

$= \int_0^{10} xy\,dx = 5\int_0^{10} x^{3/2}\,dx$

$\Rightarrow x_c = 6$

$Ay_c = \int y\,dA = \iint y\,dx\,dy$

$= \int_0^{\sqrt{250}}\left(\int_x^{10} y\,dx\right)dy$

$= \int_0^{\sqrt{250}} y(10-x)\,dy$

$= \left[5y^2 - \dfrac{y^4}{100}\right]_0^{\sqrt{250}}$

$= 5 \times 250 - \dfrac{25\cancel{0} \times 25\cancel{0}}{1\cancel{0}\cancel{0}}$ ~~= 6~~

$y_c = \dfrac{625}{105.4} = 6$

- Second moment of area about x:

Area MOI about x,

$$\boxed{I_{xx} = \int_A y^2\,dA}$$

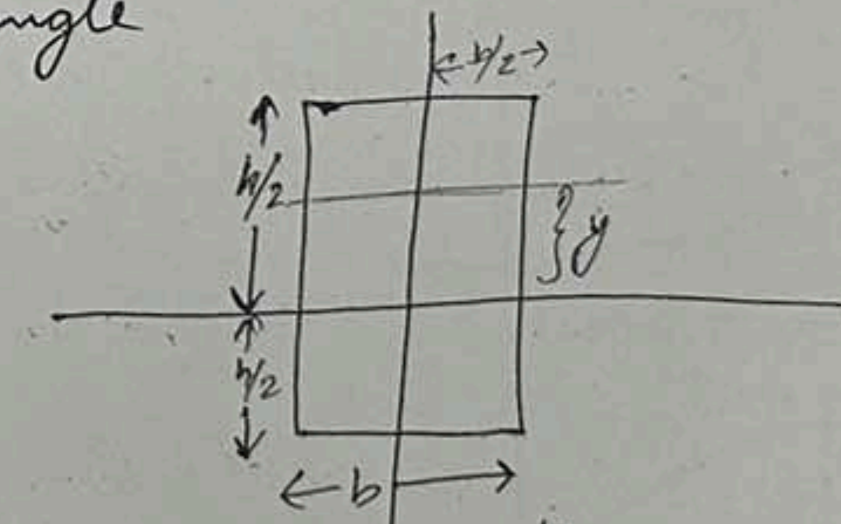
MI about y: $\boxed{I_{yy} = \int_A x^2\,dA}$

$\rightarrow I_{yy} = \int x^2\,dA = \int_0^{10} x^2 y\,dx = \int_0^{10} 5x^{5/2}\,dx = 45\ldots$

$I_{xx} = \int y^2\,dA = \int_0^{\sqrt{250}} y^2(10-x)\,dy$

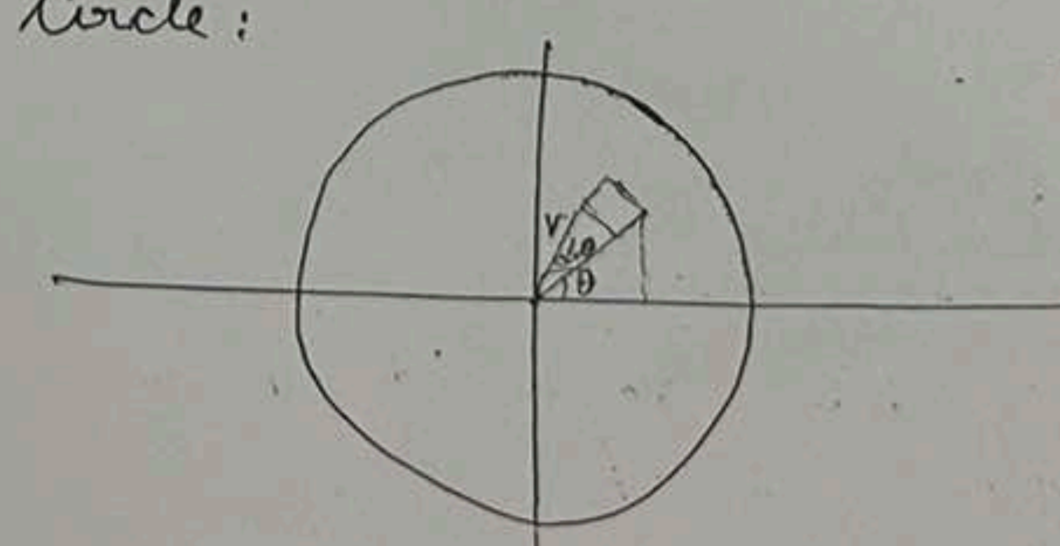
$= \int_0^{\sqrt{250}} y^2\left(10 - \dfrac{y^2}{25}\right)dy = 5270$

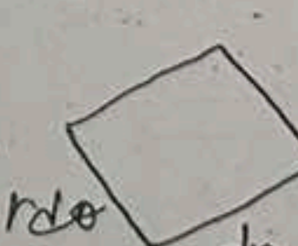
i) Rectangle



$I_{xx} = \int y^2\,dA = \int_{-h/2}^{h/2} y^2 b\,dy = \dfrac{bh^3}{12}$

ii) Circle:





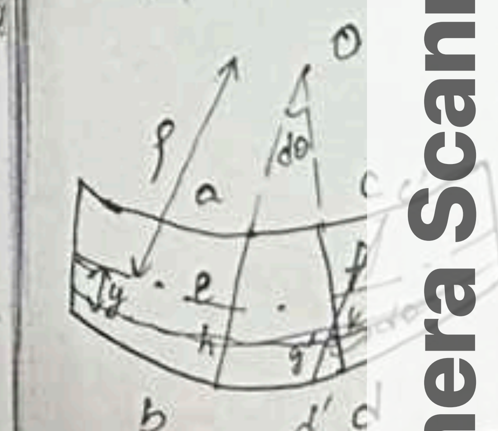
$dA = r\,d\theta\,dr$

$I_{xx} = \int y^2\,dA = \int (r\sin\theta)^2\,dA$

$= \int_0^{2\pi}\int_0^{R} r^2\sin^2\theta\, r\,d\theta\,dr$

$= \int_0^{2\pi} \dfrac{R}{4}\ldots$

Bending ...



i) Plane ...

ii) Material ... Hooke's ...

iii) Moduli ... are equ...

iv) Beam ... section.

v) Plane ... of beam ... beam's lon...

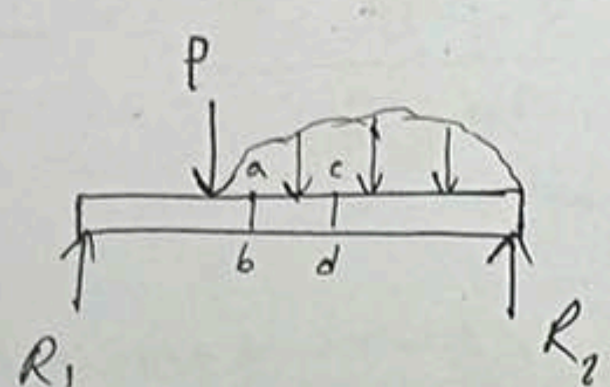
- Fiber ac ... Fiber bd ...

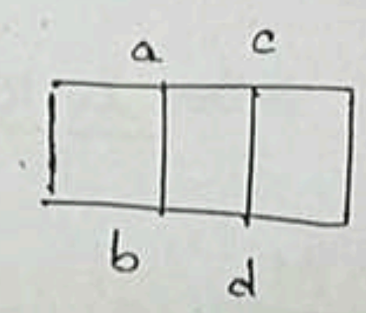
There must ... length rem... Surface contai... unstrained

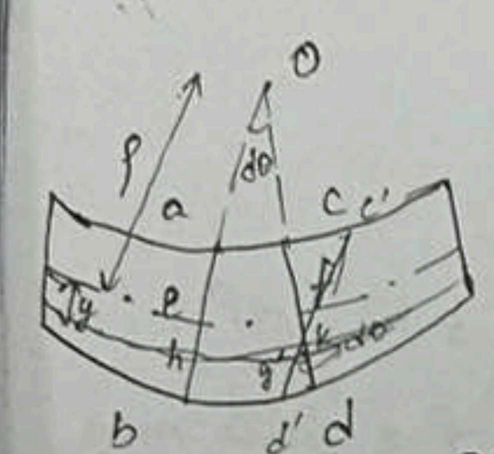
- Neutral ax... vertical cro...

$$= \int_0^{2\pi} \frac{R^4}{4} \sin^2\theta \, d\theta = \frac{\pi D^4}{64}$$

Bending stress in beams







ρ: radius

Assumptions:

i) Plane sections of beam remain plane
ii) Material in beam is homogeneous & obey Hooke's law.
iii) Moduli of elasticity for tension & compression are equal.
iv) Beam is initially straight & of const cross-section.
v) Plane of loading must contain principal axis of beam cross-section & load must be $\perp^r$ to beam's longitudinal axis.

- Fiber ac at top is shortened
Fiber bd at bottom is elongated.

There must be a fiber b/w ac & bd whose length remains unchanged $\rightarrow$ ef
~~Plane~~ Surface containing fiber ef is $\underbrace{\text{neutral surface}}$.
unstrained surface in beam

- Neutral axis (NA): Intersection of neutral surface with vertical cross-section.

$ab \parallel c'd'$

Elongation in fiber bd = $d'd$
Compression in " ac = $c'c$
(also draw all diagrams)
Now, consider fiber at distance y from neutral surface,

$g'k$ = elongation of that fiber hg'

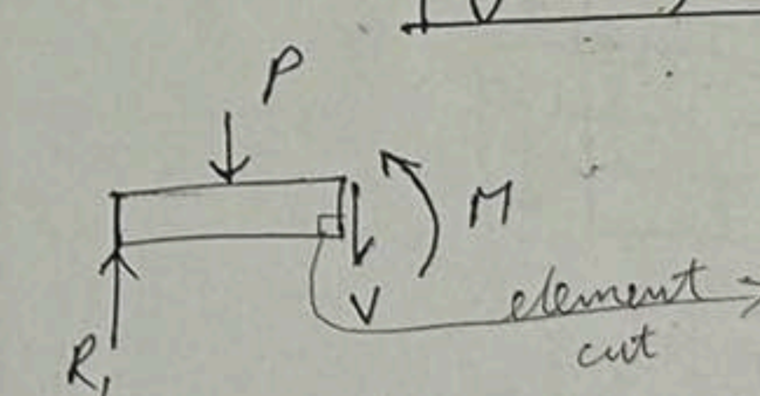
$$g'k = y\,d\theta$$

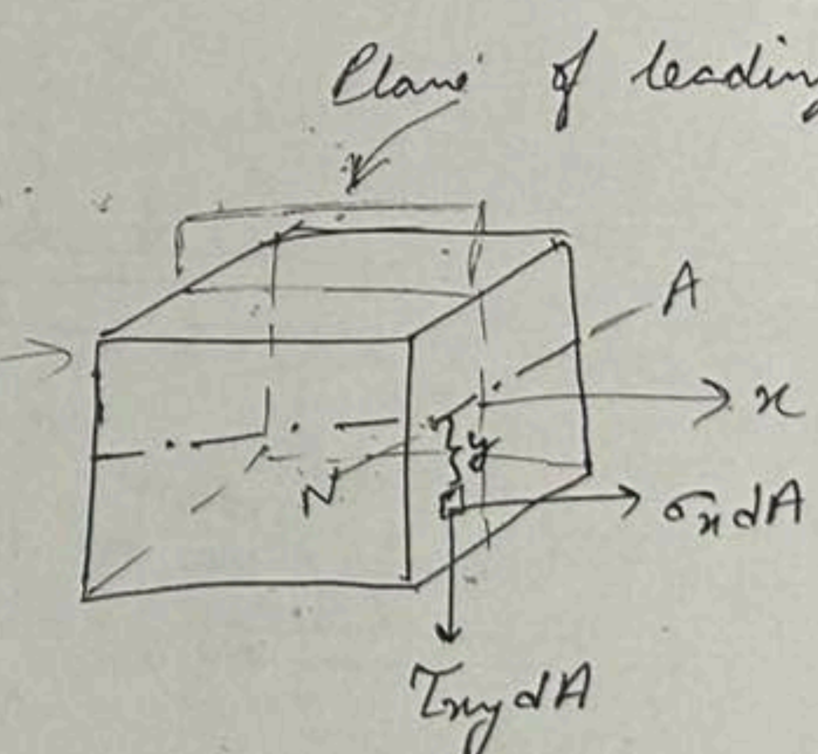
$$\varepsilon = \frac{g'k}{hg'} = \frac{y\,d\theta}{ef} = \frac{y\,d\theta}{\rho\,d\theta} = \frac{y}{\rho}$$

Applying Hooke's law,

$$\sigma = E\varepsilon$$
$$= \frac{Ey}{\rho}$$

$$\boxed{\frac{\sigma}{y} = \frac{E}{\rho}}$$





$$\Sigma F_x = 0$$

$$\int_A \sigma_x \, dA = 0$$

Here, $\sigma_x = \sigma = \frac{Ey}{\rho}$

$$\Rightarrow \int_A \frac{Ey}{\rho} dA = 0$$

$$\Rightarrow \frac{E}{\rho} \int_A y \, dA \Rightarrow \frac{E}{\rho} y_c A = 0$$

y_c is distance of centroidal axis from NA

$\therefore y_c = 0$ Thus NA coincides with CA.

Taking moment about NA:

$\int_A y\,\sigma_x\,dA = M$

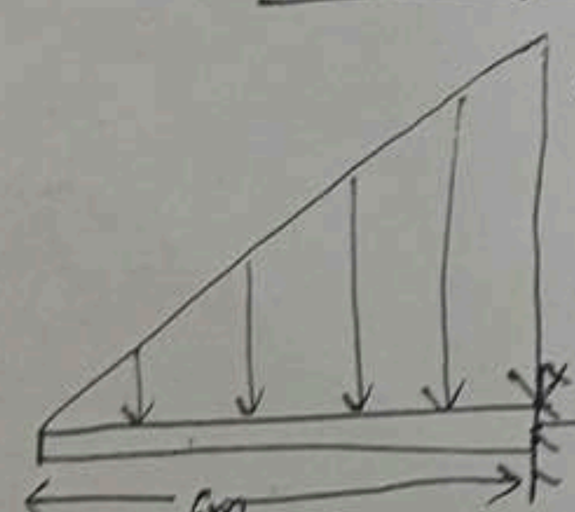
$\Rightarrow \int y \frac{Ey}{\rho} dA = M \quad \Rightarrow \frac{E}{\rho}\int y^2 dA = M$

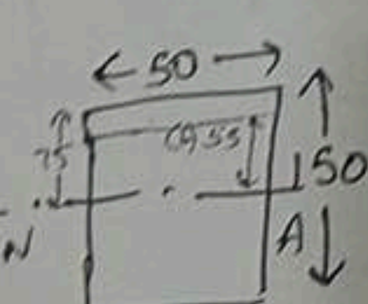
$\Rightarrow \frac{E}{\rho} I_{NA} = M$

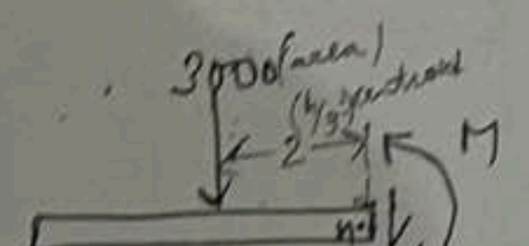
$\Rightarrow \boxed{\frac{E}{\rho} = \frac{M}{I}}$

Flexural formula: $\boxed{\frac{E}{\rho} = \frac{\sigma}{y} = \frac{M}{I}}$

503)







$\Sigma M_N = 0$

$M + 3000 \times 2 = 0$

$\Rightarrow M = 6000$ Nm (hogging)

$I = \frac{bh^3}{12} = \frac{50 \times 150^3}{12} = 14 \times 10^6 \ mm^4$

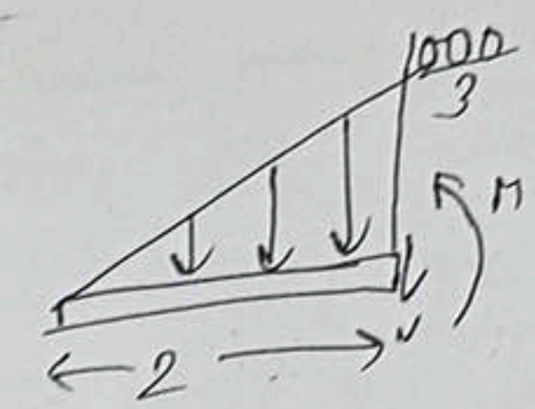
a) $\sigma_{max} = \frac{M_{max}\,y_{max}}{I}$

$= \frac{6 \times 10^6 \times 75}{14 \times 10^6} = 32 \ \cancel{N/mm^2} = 32$ MPa

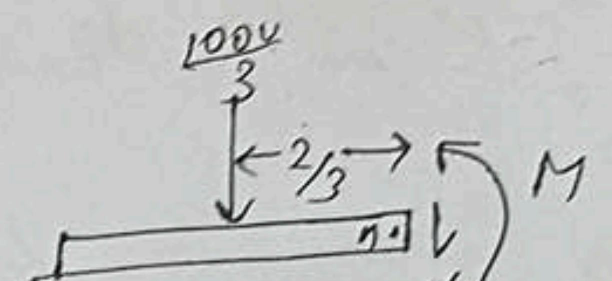
b) $w = \frac{1000x}{6}$

$\Sigma M_n = 0$

$\Rightarrow M + \frac{1000}{3} \times \frac{2}{3} = 0$



$\Longleftrightarrow$



$M = \cancel{200}\ \frac{2 \times 10^6}{9}$ Nmm

$\sigma = \frac{My}{I} = \frac{2 \times 10^6 \times 55}{9 \times 14 \times 10^6} = 870$ kPa

505) $\sigma = 400$ MPa $\qquad E = 200 \times 10^3$ MPa

a) $\sigma = \frac{Ey}{\rho} \qquad R = \rho = \frac{600}{2}$ mm

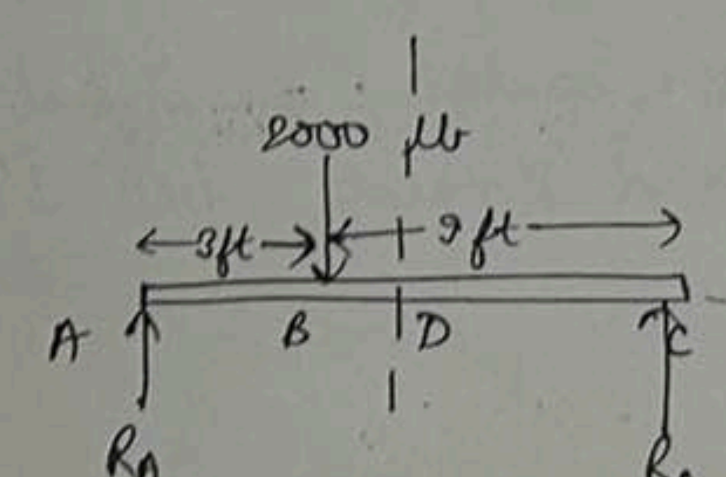
$= \frac{200 \times 10^3 \times 0.4}{300}$

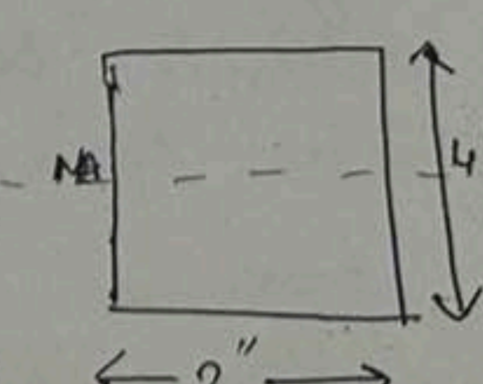
$= 267$ MPa

b) $400 = \frac{200 \times 10^3 \times 0.4}{\rho} \quad \Rightarrow \rho = 200$ mm

$\therefore D = 400$ mm

504)



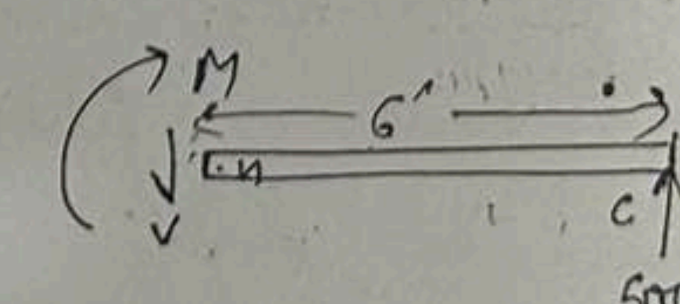


502-513

$\Sigma M_A = 0$

$12 R_C + 2000 \times 3 = 0$

$R_C = 500$ lb ~~(hogging)~~



$\Sigma M_n = 0$

$M_D = 500 \times 6$

$= 3000$ lb.ft

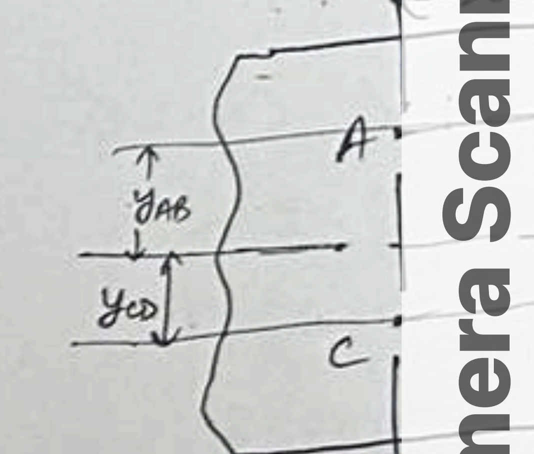
$= 36 \times 10^3$ lb.in

$\frac{\sigma}{y} = \frac{M}{I}$

At D,

$\sigma_{max} = \frac{36 \times 10^3 \ldots}{\frac{2 \times \ldots}{12}}$

507)



end couples = pure bendi...

$\varepsilon_{AB} = \frac{\Delta AB}{AB}$

$\varepsilon_{CD} = \frac{\Delta CD}{CD}$

$\sigma_{CD} = E\varepsilon_{CD}$

$\frac{E}{\rho} = \frac{\sigma}{y} \ldots$

$\Rightarrow \frac{\sigma_{AB}}{y_{AB}} = \frac{\sigma_{CD}}{y_{CD}}$

$\Rightarrow \frac{E\varepsilon_{AB}}{y_{AB}} = \frac{E\varepsilon_{CD}}{y_{CD}}$

$\Rightarrow y_{CD} = \frac{5}{3} y_{AB}$

$\therefore y_{top} = y_{AB} + 30$

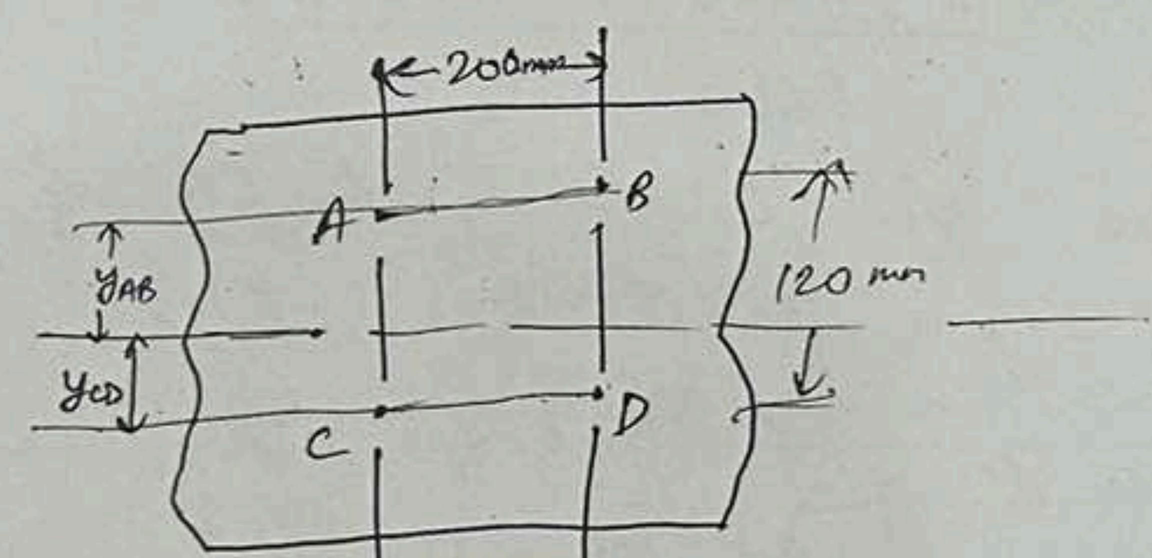
$y_{bottom} = y_{CD} + 75$

$$\frac{\sigma}{y} = \frac{M}{I}$$

At D,

$$\sigma_{max} = \frac{36 \times 10^3 \times 2}{\frac{2}{12} \times 4^3} = 6.74 \text{ ksi}$$

507)



end couples = pure bending

$$\varepsilon_{AB} = \frac{\Delta AB}{AB} = \frac{60 \times 10^{-3}}{200} = 300\,\mu \text{ (Tensile)}$$

$$\varepsilon_{CD} = \frac{\Delta CD}{CD} = \frac{100 \times 10^{-3}}{200} = 500\,\mu \text{ (compression)}$$

$$\sigma_{CD} = E\varepsilon_{CD} \qquad \sigma_{AB} = E\varepsilon_{AB}$$

$$\frac{E}{\rho} = \frac{\sigma}{y} = \frac{M}{I}$$

$$\Rightarrow \frac{\sigma_{AB}}{y_{AB}} = \frac{\sigma_{CD}}{y_{CD}} \qquad \left(\because \frac{E}{\rho} = \text{const}\right)$$

$$\Rightarrow \frac{E\varepsilon_{AB}}{y_{AB}} = \frac{E\varepsilon_{CD}}{y_{CD}} \qquad y_{AB} + y_{CD} = 120$$

$$\Rightarrow y_{CD} = \frac{5}{3} y_{AB} \qquad \Rightarrow \frac{8}{3} y_{AB} = 120$$

$$y_{AB} = 45$$

$$y_{CD} = 75$$

$$\therefore y_{top} = y_{AB} + 30 = 75$$

$$y_{bottom} = y_{CD} + 75 = 150$$

$$\therefore \frac{\sigma_{top}}{y_{top}} = \frac{\sigma_{AB}}{y_{AB}}$$

$$\Rightarrow \sigma_{top} = \frac{75}{45} \times 70 \times 10^3 \times 300 \times 10^{-6} \text{ MPa}$$

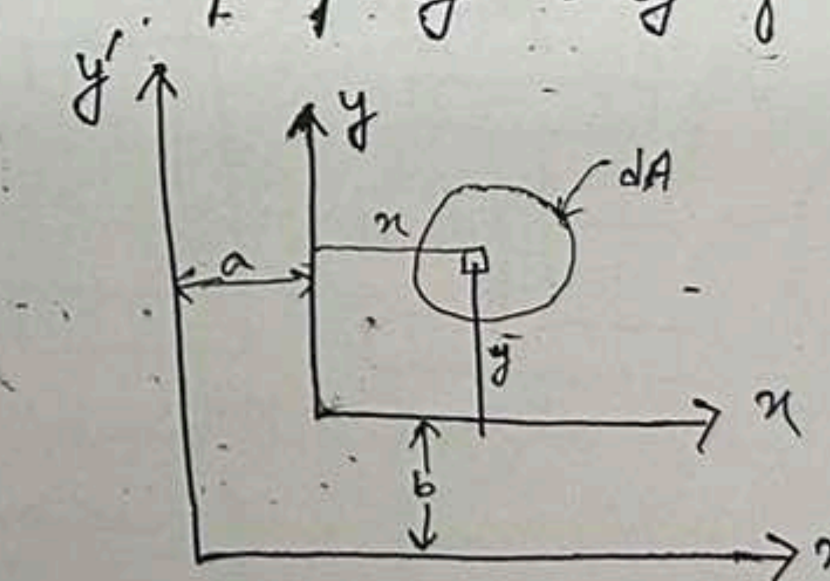
$$= 35 \text{ MPa}$$

$$\therefore \frac{\sigma_{bottom}}{150} = \frac{E\varepsilon_{CD}}{y_{CD}}$$

$$\Rightarrow \sigma_b = 70 \times 10^3 \times 500 \times 10^{-6} \times \frac{150}{75}$$

Q) Show that centroid of area 'A' is same pt. for x & y- axes & x', y'. Thus, position of centroid of an area is a property only of area.

→



$$x' = x + a \qquad y' = y + b$$

We have to prove that: $x'_c = x_c + a$

$y'_c = y_c + b$

$$A = \int_A dx\,dy = \int_A dx'\,dy'$$

$$Ax_c = \int_A x\,dx\,dy$$

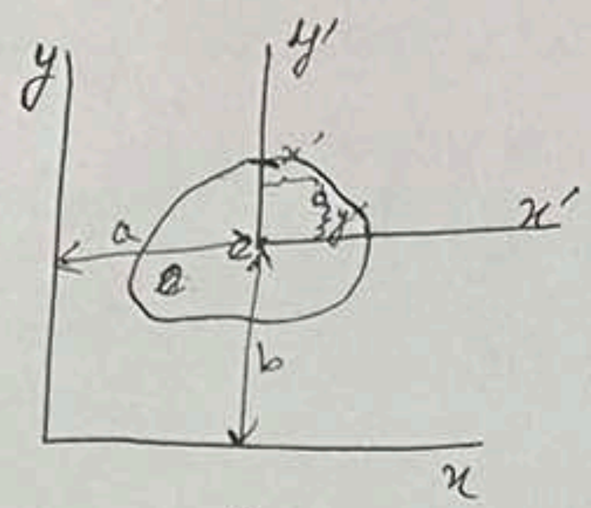
$$Ax'_c = \int_A x'\,dx'\,dy' = \int_A (x+a)\,dx'\,dy'$$

$$= \int x\,dx'\,dy' + \int a\,dx'\,dy'$$

$$= Ax_c + aA$$

$$\Rightarrow x'_c = x_c + a$$

Second moment of area



$\int_A x' dx' dy' = 0$

$\int_A y' dx' dy' = 0$

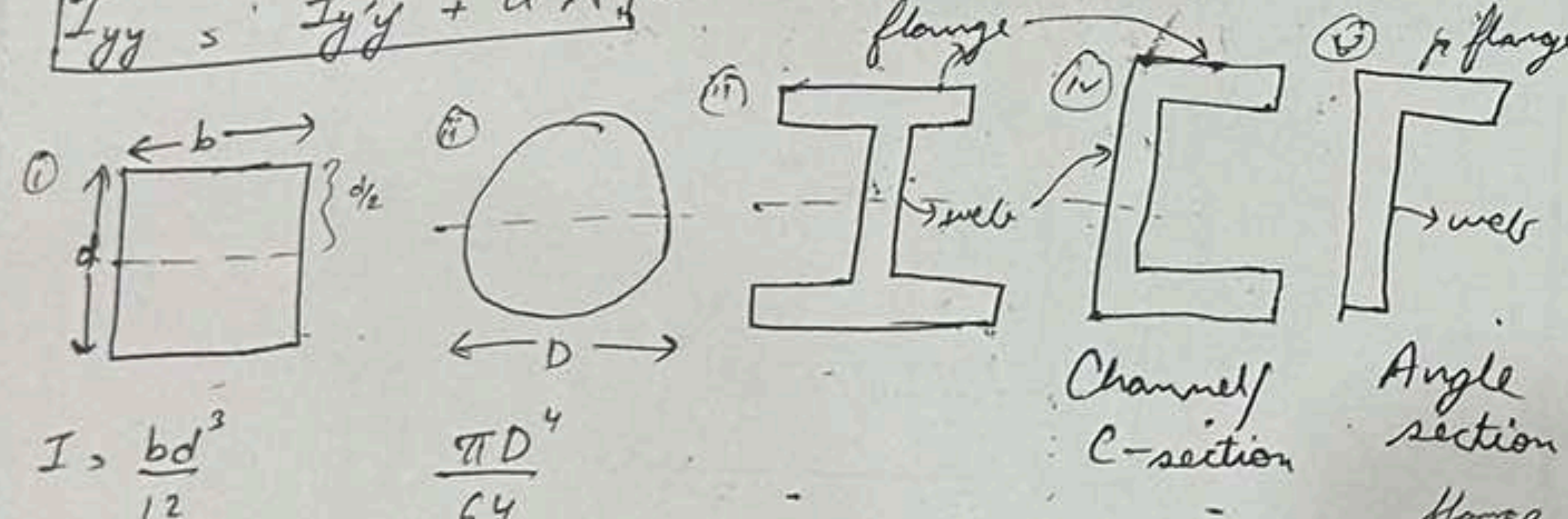
$I_{xx} = \int_A y^2 dA$

$I_{x'x'} = \int_A (y')^2 dA$

$I_{xx} = \int_A (y'+b)^2 dA$

$= \int_A (y')^2 dA + b^2 \int_A dA + \cancel{2b \int_A y' dA} \to 0$

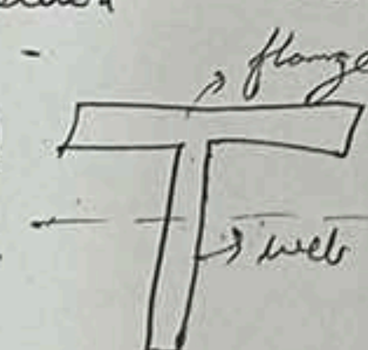
$\boxed{I_{xx} = I_{x'x'} + b^2 A}$
$\boxed{I_{yy} = I_{y'y'} + a^2 A}$ } Parallel axis theorem



$I = \frac{bd^3}{12}$ $\qquad \frac{\pi D^4}{64}$

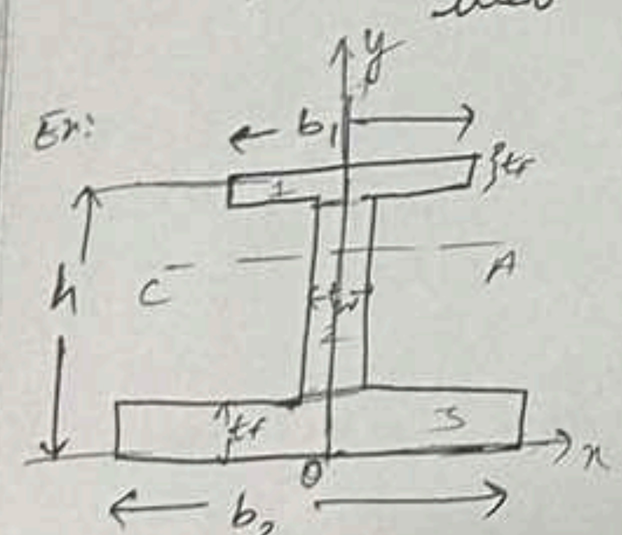
Channel/ C-section $\qquad$ Angle section

iii) If wider flange ⇒ W-section
 web ⇒ I-section



T-section

Ex:



$b_1 = 60$ mm
$b_2 = 100$
$t_f = 10$
$t_w = 20$
$h = 120$

Find I_{CA}

→ $A_1 = 60 \times 10 = 600$ mm²
$A_2 = 2000$ mm²
$A_3 = 1000$ mm²

$\bar{y}_1 = 10 + 100 + 5 = 115$ mm
$\bar{y}_2 = 10 + 50 = 60$ mm
$\bar{y}_3 = 5$ mm

$y_c = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3}{A_1 + A_2 + A_3}$

$= \frac{600 \times 115 + 2000 \times 60 + 1000 \times 5}{3600} = 53.8$ mm

y_{c_1} = dist b/w CA of A_1 & CA of A

$= \bar{y}_1 - y_c = 115 - 53.8 = 61.2$

$y_{c_2} = \bar{y}_2 - y_c = 60 - 53.8 = 6.2$

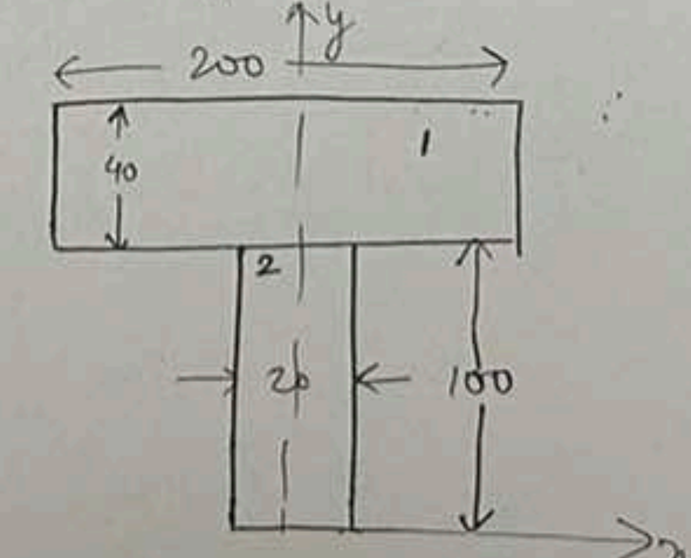
$y_{c_3} = \bar{y}_3 - y_c = 5 - 53.8 = -48.8$

$I_{CA} = I_1 + A_1 y_{c_1}^2 + I_2 + A_2 y_{c_2}^2 + I_3 + A_3 y_{c_3}^2$

$= \underbrace{\frac{b_1 t_f^3}{12}}_{I_1} + \underbrace{\frac{(h - 2t_f)^3 t_w}{12}}_{I_2} + \underbrace{\frac{b_2 t_f^3}{12}}_{I_3} + 600 \times 61.2^2 + 2000 \times 6.2^2 + 1000 \times 48.8^2$

$= 1.7 \times 10^6$ mm⁴

Ex: $\qquad I_{CA}$?



→ $A_1 = 200 \times 40 = 8000$
$A_2 = 100 \times 20 = 2000$

$\bar{y}_1 = 100 + 20 = 120 \qquad \bar{y}_2 = 50$

$y_c = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2} = \frac{8000 \times 120 + 2000 \times 50}{10000} = 106$

$y_{c_1} = 120 - 106 =$
$y_{c_2} = 50 - 106 = -$

$I_{CA} = I_1 + A_1 y_{c_1}^2 + I$

$= \frac{200 \times 40^3}{12} + 8000$

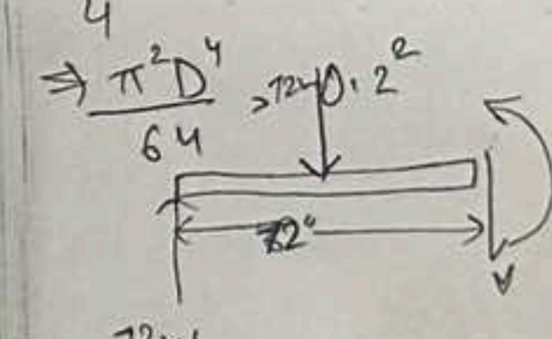
$= 10.6 \times 10^6$ mm

(509)

$A = 0.2$

$\frac{\pi D^2}{4} = 0.2$

$\Rightarrow \frac{\pi^2 D^4}{64} = 0.2^2$



$I_{NA} = 2(A_1 \times 3^2$

$= 2(2 \times 0.2($

$10 \times 10^3 =$

⇒ $w = 13$

$R_1 + R_2 = 144w$

$(R_1 = R_2)$

$y_{c_1} = 120 - 106 = 14$

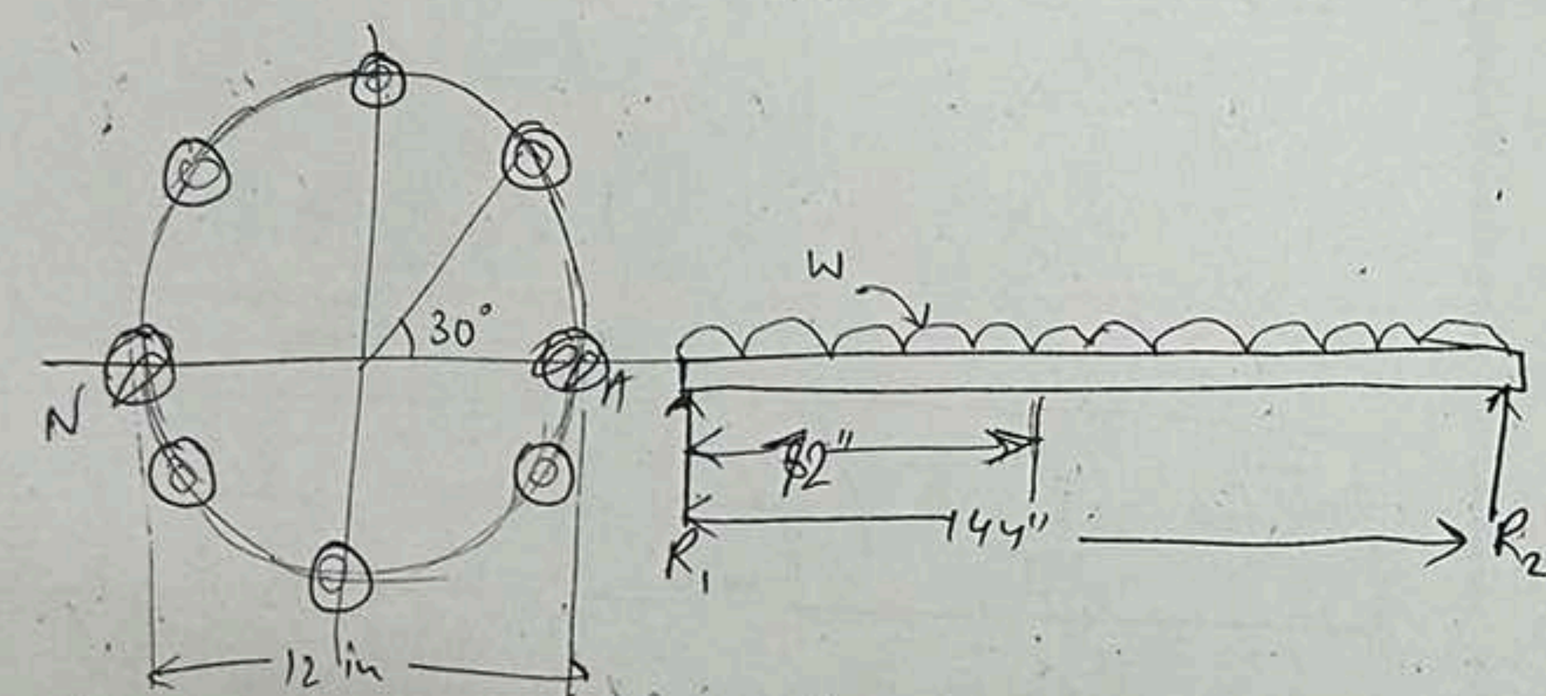
$y_{c_2} = 50 - 106 = -56$

$I_{CA} = I_1 + A_1 y_{c_1}^2 + I_2 + A_2 y_{c_2}^2 + \cancel{I_3 + A_3 y_c}$

$= \frac{200 \times 40^3}{12} + 8000 \times 14^2 + \frac{20 \times 100^3}{12} + 2000 \times 56^2$

$= 10.6 \times 10^6 \ mm^4$

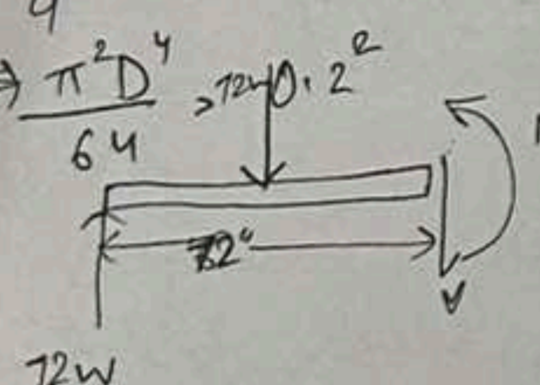
(509)



$A = 0.2$

$\frac{\pi D^2}{4} = 0.2$

$\Rightarrow \frac{\pi^2 D^4}{64} = 0.2^2$



$M_{max} = \cancel{\frac{72 \times 6 w}{2}}$ $72w \times 72 - 72w \times \frac{72}{2}$

$\frac{72^2 w}{2}$

$I_{NA} = 2\left(A_1 \times 3^2 + A_2 \times 6^2 + A_3 \times 3^2\right) + \frac{\pi D^4}{64} \times 6 = 21.6 \ in^4$

$= 2(2 \times 0.2(9 + 36 + 9)) + \frac{0.2^2}{4\pi} \times 6$

$\sigma = \frac{M_{max} \times 6}{I_{NA}}$

$10 \times 10^3 = \frac{\frac{72^2}{2} w \times 6}{21.6}$

$\Rightarrow w = 13.89 \ lb/in$

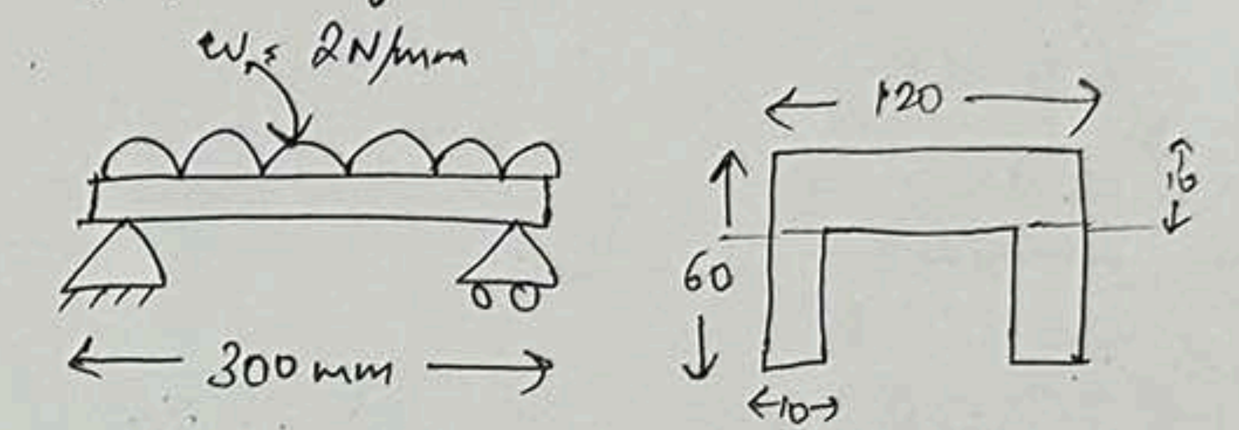
(522)

$R_1 + R_2 = 144w$

$(R_1 = R_2)$ ...

- M_{max} is at midspan $= \frac{WL^2}{8}$

★ Design of cart & axle



$\bar{y}_1 = 55$

$\bar{y}_2 = \bar{y}_3 = 25$

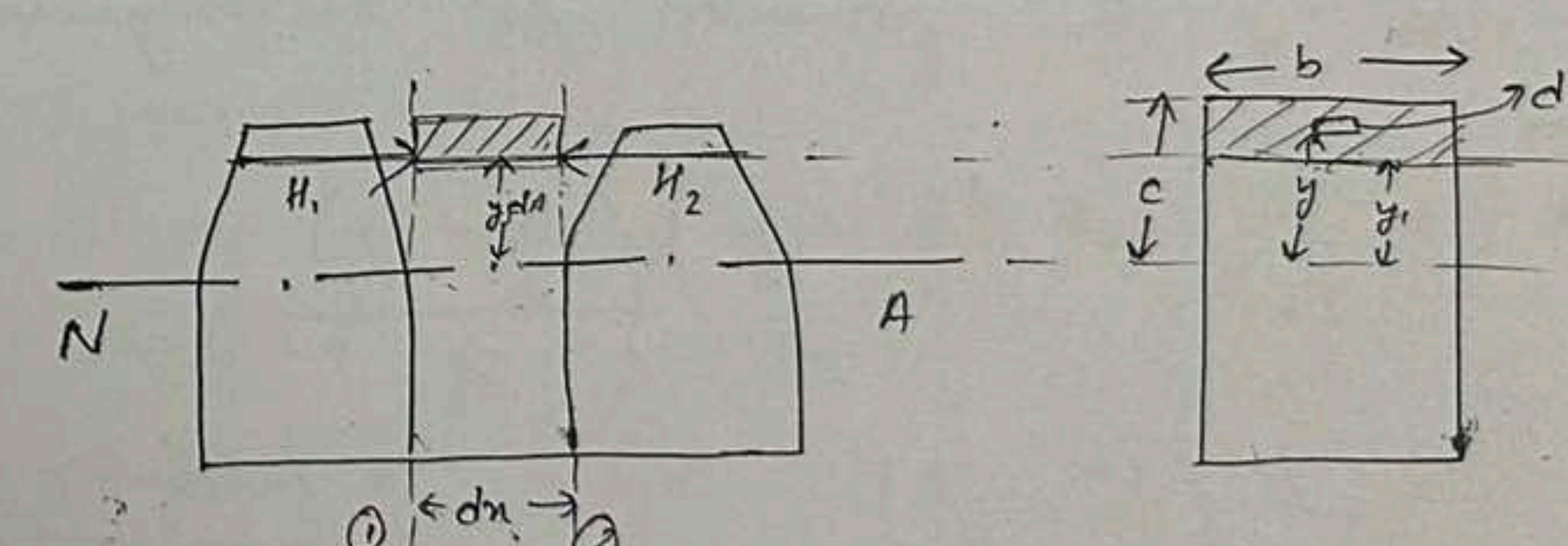
$y_{c_1} = \bar{y}_1 - y_c$

$y_{c_2} = \bar{y}_2 - y_c$

$(\sigma_T)_{max} = \left(\frac{M_{max}}{I}\right) y_{c_2}$

$(\sigma_C)_{max} = \left(\frac{M_{max}}{I}\right) y_{c_1}$

Formula for Horizontal shear stress



For given loaded beam, consider 2 section dx apart, also consider layer at distance y_1 from NA. dF is force to keep it in static condition.

Let $H_2 > H_1$

For eqlb,

$H_1 + dF = H_2$

$dF = H_2 - H_1$

$= \int_{y_1}^{c} \sigma_2 \, dA - \int_{y_1}^{c} \sigma_1 \, dA$

$$= \int_{y_1}^{c} \frac{M_2}{I} y \, dA - \int_{y_1}^{c} \frac{M_1}{I} y \, dA$$

$$= \frac{M_2 - M_1}{I} \int_{y_1}^{c} y \, dA$$

$\because dx$ is very small

$$dF = \frac{dM}{I} \int_{y_1}^{c} y \, dA$$

$$\Rightarrow \tau b \, dx = \frac{dM}{I} \int_{y_1}^{c} y \, dA \qquad \frac{dM}{dx} = V \text{ (vertical shear)}$$

$$\tau = \frac{dM}{dx} \times \frac{1}{Ib} \int_{y_1}^{c} y \, dA = \frac{V}{Ib} A' \bar{y}$$

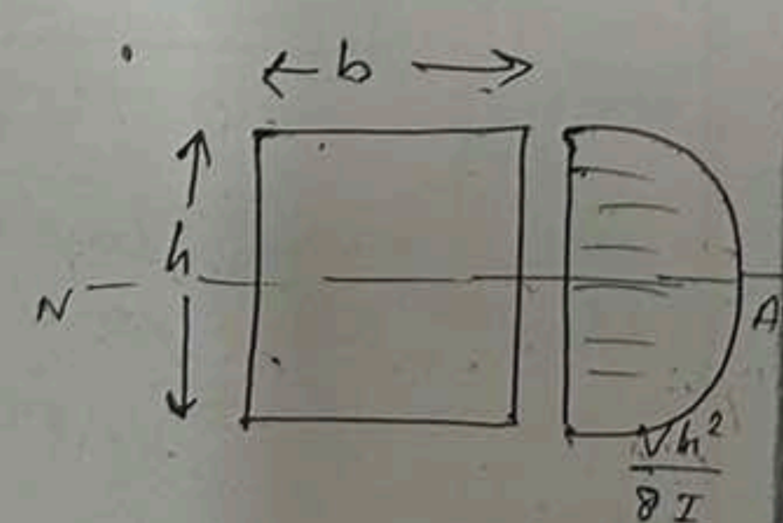
$$\boxed{\tau = \frac{VQ}{Ib}}$$

$A' \bar{y}_1 = Q =$ static moment of area

Application of this formula for rectangular section :

$$\tau = \frac{V}{Ib} \times b \left(\frac{h}{2} - y\right) \frac{\left(\frac{h}{2} + y\right)}{2}$$

$$= \frac{V}{2I} \left(\frac{h^2}{4} - y^2\right)$$



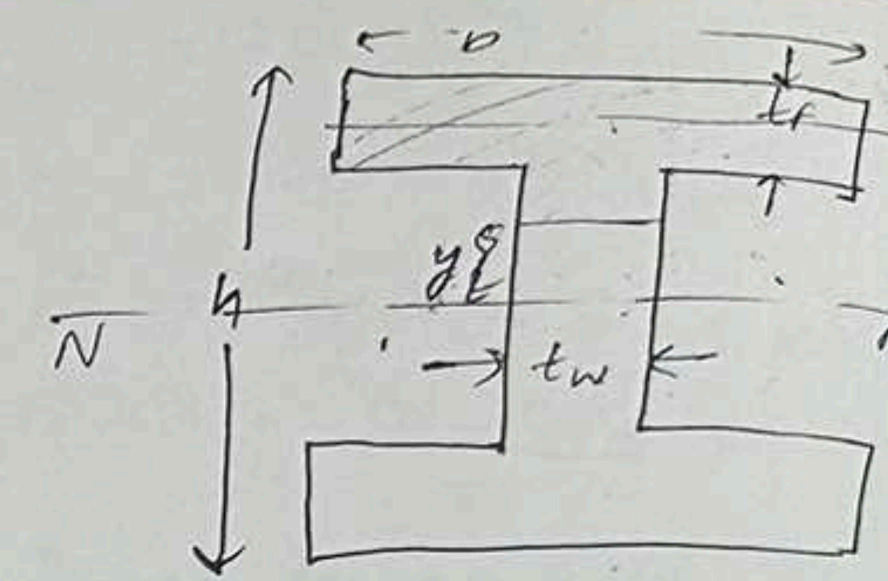
$$\tau_{min} = \tau \Big|_{y = \frac{h}{2}} = 0.$$

$$\tau_{max} = \tau \Big|_{y=0} = \frac{V}{8I} h^2$$

Application to I section

$$\tau = \frac{V}{Ib} A' \bar{y} = \frac{V}{Ib} \times b \left(\frac{h}{2} - y\right) \frac{\left(\frac{h}{2} + y\right)}{2}$$

$$= \frac{V}{2I} \left(\frac{h^2}{4} - y^2\right)$$



$$\frac{h}{2} - t_f < y < \frac{h}{2}$$

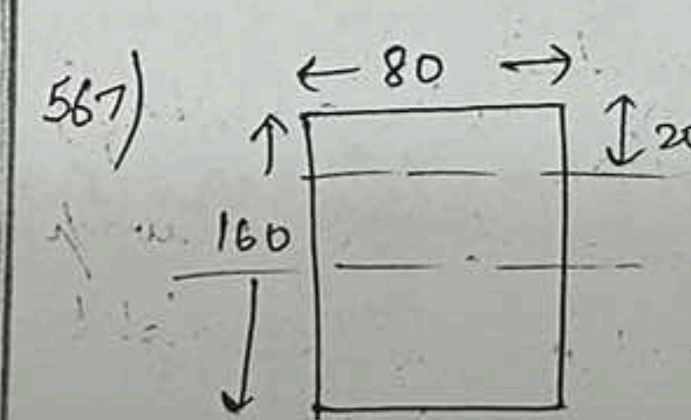
In web,

$$\tau = \frac{V}{I t_w} \times b t_f \left(\frac{\frac{h}{2} + \frac{h}{2} - t_f}{2}\right) + \frac{V}{I t_w} \times t_w$$

$$\left(\frac{h}{2} - t_f - y\right) \left(\cancel{\frac{h}{2} - t_f} \; y \right) + \frac{1}{2} \left(\frac{h}{2} - t_f - y\right)$$

$$\boxed{\tau = \frac{V}{2I} b \left(\frac{t_f}{t_w}\right) \frac{h}{2} + \frac{V}{2I} \left(\frac{h^2}{2} - y^2\right)}$$

567)



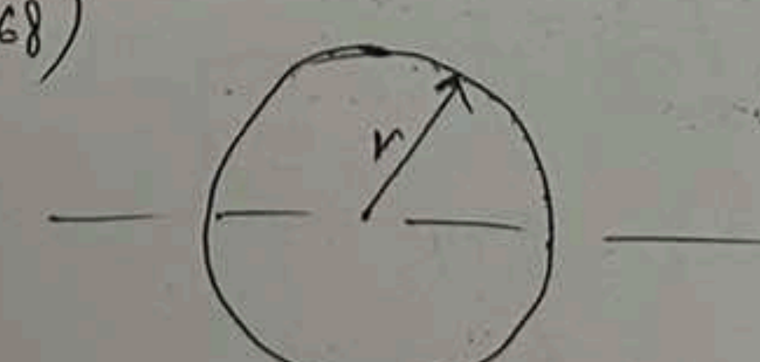
$$\tau = \frac{V}{2I} \left(\frac{h^2}{4} - y^2\right)$$

$$= \frac{80 \times 160^3}{12} = 27.31 \times 10^6$$

$V = 40$ kN, $h = 160$, $y = 20$

$\therefore \tau = 4.4$ N/mm² (upon plugging values)

568)



$$\tau = \frac{V}{Ib} A' \bar{y}$$

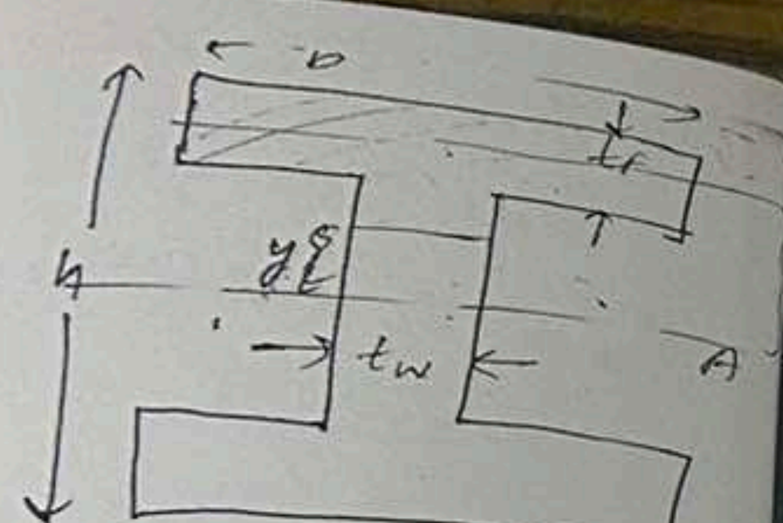
$$= \frac{V}{I \times 2r} \times \frac{\pi}{2} r^2 \times \frac{4r}{3\pi}$$

$$= \frac{V}{\frac{\pi r^4}{4} \times 2r} \times \frac{\pi}{2} r^2 \times \frac{4r}{3\pi}$$

$$= \frac{4}{3} \frac{V}{\pi r^2}$$

$\frac{h^2}{4} - y^2)$

$y < \frac{h}{2}$



$t_f\left(\frac{\frac{h}{2}+\frac{h}{2}-t_f}{2}\right) + \frac{V}{It_w} \times t_w$

$_f - y)\left(\cancel{\frac{h}{2}} \cancel{+ t_f}\right) \left[y\right] + \frac{1}{2}\left(\frac{h}{2} - t_f - y\right.$

$\boxed{\left.\frac{t_f}{t_w}\right)\frac{h}{2} + \frac{V}{2I}\left(\frac{h^2}{2} - y^2\right)}$

$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right)$

$= \frac{80 \times 160^3}{12} = 27.31 \times 10^6$

$V = 40$ kN, $h = 160$, $y = 20$

$/mm^2$ (upon plugging values)

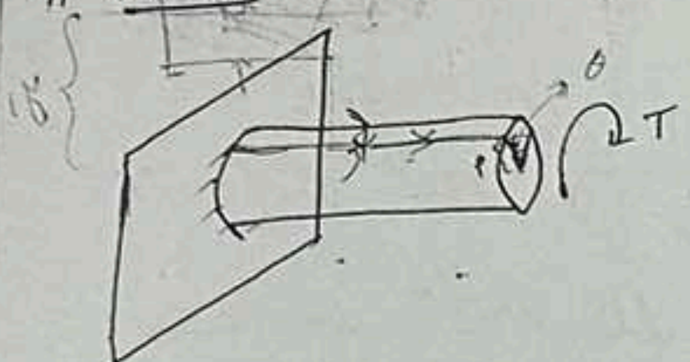
$\tau = \frac{V}{Ib} A'\bar{y}$

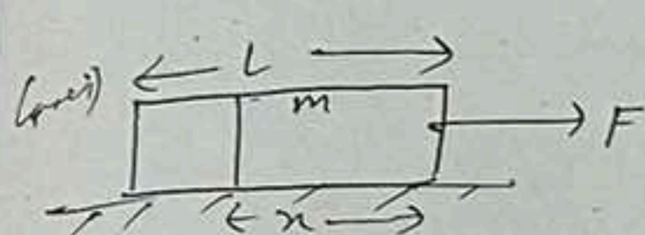
$= \frac{V}{I \times 2r} \times \frac{\pi}{2} r^2 \times \frac{4r}{3\pi}$

$= \frac{V}{\frac{\pi r^4}{4} \times 2r} \times \frac{\pi}{2} r^2 \times \frac{4r}{3\pi}$

$= \frac{4}{3} \frac{V}{\pi r^2}$

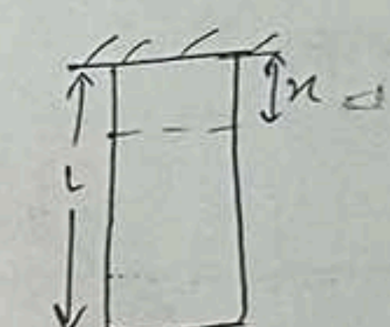
Torsion



(a) 

Stress at 'x' distance $= \frac{mx}{L}\left(\frac{F}{m}\right)$

$\frac{mx}{L}\left(\frac{F}{m}\right) \leftarrow \square \rightarrow \frac{mx}{L}\left(\frac{F}{m}\right)$

ii) 

$\Rightarrow \frac{m}{L}(L - x)g$

Assumptions: (to derive Torsion formula for circular shaft)

i) Circular sections remain circular

ii) Plane " " plane & don't warp

iii) Projection upon a transverse section of straight radial lines in section remains straight.

iv) Shaft is loaded by twisting couples in planes that are $\perp^r$ to axis of shaft

v) Stresses don't exceed proportional limit.

Shear strain $= \phi$

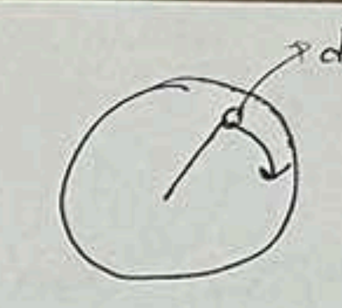
$\tau = G\phi$

From geometry,

$L\phi = \rho\theta$

$\Rightarrow \tau = G\frac{\rho\theta}{L}$

$\Rightarrow \frac{\tau}{\rho} = \frac{G\theta}{L}$ — (i)



$dF = \tau\, dA$

$T = \int_A dF\, \rho$

$= \int_A \rho\, \tau\, dA$

$= \int_A \frac{G\theta}{L} \rho^2\, dA$

$= \frac{G\theta}{L} \left(\int \rho^2\, dA\right) \rightarrow$ Polar MOI

$= \frac{G\theta}{L} J$

$\Rightarrow \frac{T}{J} = \frac{G\theta}{L}$ — (ii)

$\boxed{\frac{T}{J} = \frac{\tau}{\rho} = \frac{G\theta}{L}}$ Flexure formula in torsion

T = Torque

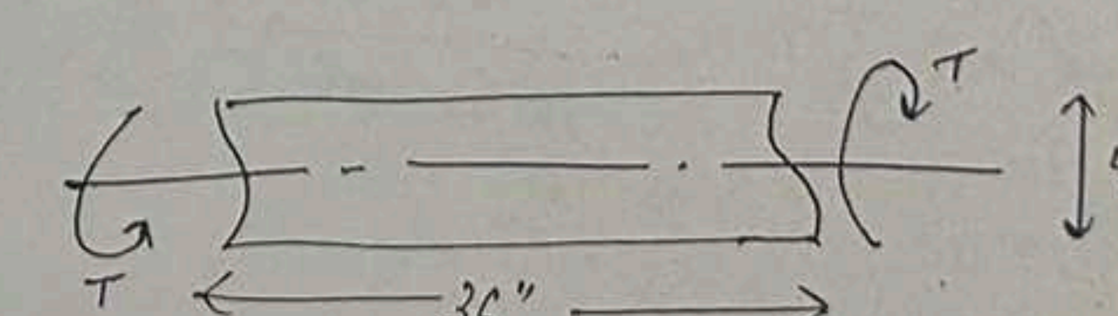
τ = Shear stress at a pt at radial dist. ρ

G = Modulus of rigidity

L = Length of shaft

θ = angle of twist in radian

$J = \frac{\pi D^4}{32}$ in circular section

304) 

$T = 15 \times 10^3$ lb. ft $= 180 \times 10^3$ lb. in

$J = \frac{\pi D^4}{32} = \frac{\pi \times 4^4}{32} = 25.133$ in^4

$\tau_{max} = \left(\frac{T}{J}\right)\rho_{max}$

$= \frac{180 \times 10^3}{25.133} \times 2 = 14.32$ ksi

$\theta = \frac{T}{J} \times \frac{L}{G}$

$= \frac{180 \times 10^3}{25.133} \times \frac{36}{12 \times 10^6} = 21.5 \times 10^{-3}$ rad

$= 21.5 \times 10^{-3} \times \frac{180}{\pi}$

$= 1.23°$

305) $\theta = 3° = 0.052$ rad | $T = 12$ kN.m $= 12 \times 10^3 \times 10^3$ Nmm

$L = 6$ m $= 6000$ mm | $G = 83$ GPa $= 83 \times 10^3$ N/mm²

$\frac{T}{J} = \frac{G\theta}{L}$

$\rightarrow J = \frac{12 \times 10^6 \times 6 \times 10^3}{83 \times 10^9 \times 3 \times \frac{\pi}{180}} = 16.57 \times 10^6$

$D^4 = \frac{32}{\pi} \times 16.57 \times 10^6$ ~~$= 168.32 \times 10^{-6}$ m⁴~~

$D = 114$ mm

$\rho_{max} = 57$ mm

$\tau_{max} = \frac{T}{J}\rho_{max} = \frac{12 \times 10^6}{16.6 \times 10^6} \times 57 = 41.2$ N/mm²

306) $d = 14$ in

$L = 18$ ft $= 18 \times 12$ in

$\omega = 189$ rpm $= 189 \times \frac{2\pi}{60} = 19.8$ rad/s

$G = 12 \times 10^6$ psi

$J = \frac{\pi \times 14^4}{32} = 3.77 \times 10^3$ in⁴

$P = 5000$ HP $= 5000 \times 550$ lb. ft/s

$= 275 \times 10^4 \times 12$ lb. in/s

$= 33 \times 10^6$ lb. in/s

$\boxed{P = T\omega}$

$T = \frac{P}{\omega} = 1.67 \times 10^6$ lb. in

$\therefore \tau_{max} = \left(\frac{T}{J}\right)\rho_{max}$

$= \frac{1.67 \times 10^6}{3.77 \times 10^3} \times 7 = 3.1$ ksi

H.W. {304-311, 314, 316}

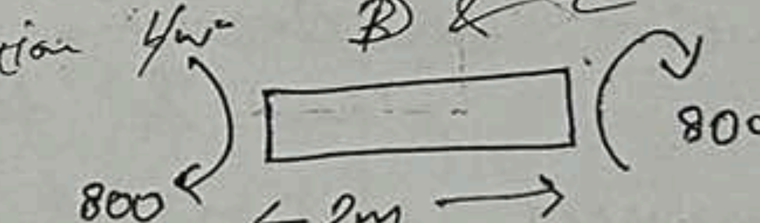
311) $G = 28$ GPa $= 28 \times 10^3$ N/mm²

$d = 50$ mm $\quad L = 2000$ mm.

$T = 800 \times 10^3$ N. mm

Section b/w B & C

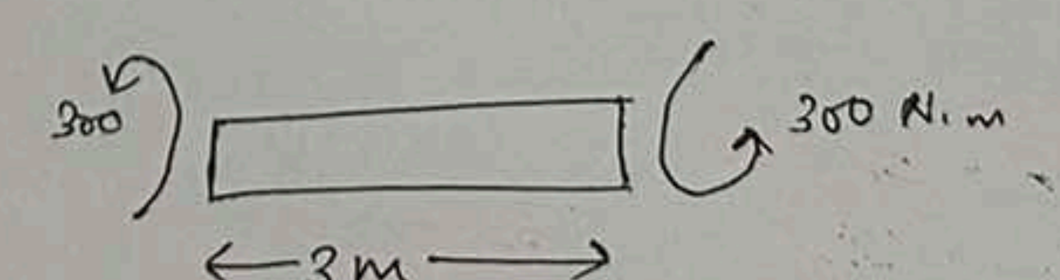
i)



$J = \frac{\pi d^4}{32} = 613.6 \times 10^3$ mm⁴

$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow \theta_{D/C} = \frac{T \times 2000}{28 \times 10^3 \times J} = 93.24 \times 10^3$

ii)



$\theta_{C/B} = \frac{300 \times 10^3 \times 3000}{28 \times 10^3 \times 613.6 \times 10^3} =$ (CCW)

iii) $\theta_{B/A} =$ (CW)

$\therefore \theta_{D/A} = \theta_{D/C} + (-\theta_{C/B}) + \theta_{B/A}$

(i)

Lines that ... are called ...

(ii)

M

$\sigma = \left(\frac{M}{I}\right)y$

Shear stre...

Tensile

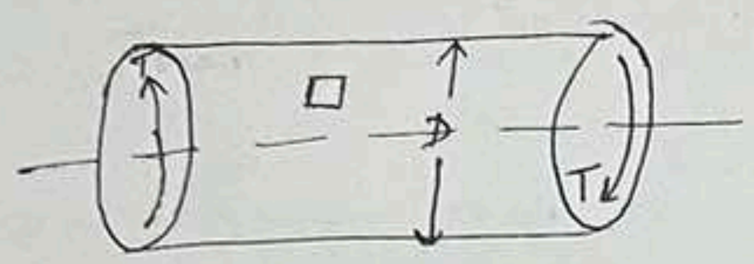
936)

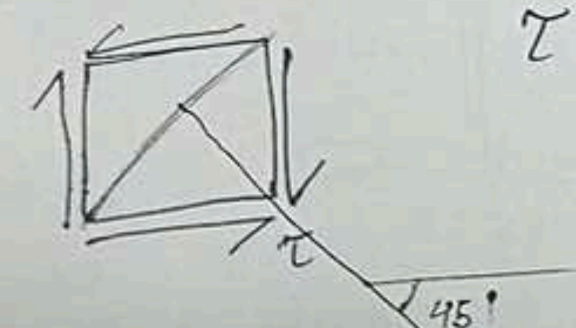
$D_o = 150$ mm

$D_i = 150 - 10$

40 N/mm

\# (i)

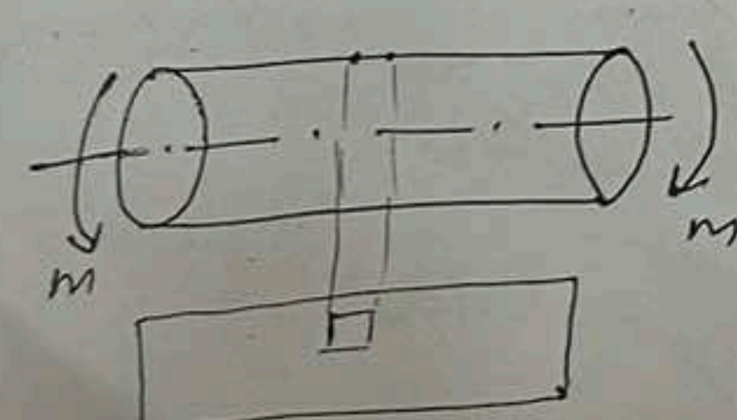




$\tau = \left(\frac{T}{J}\right)\left(\frac{D}{2}\right)$

Lines that show direction of principal stress are called stress trajectories.

(ii)

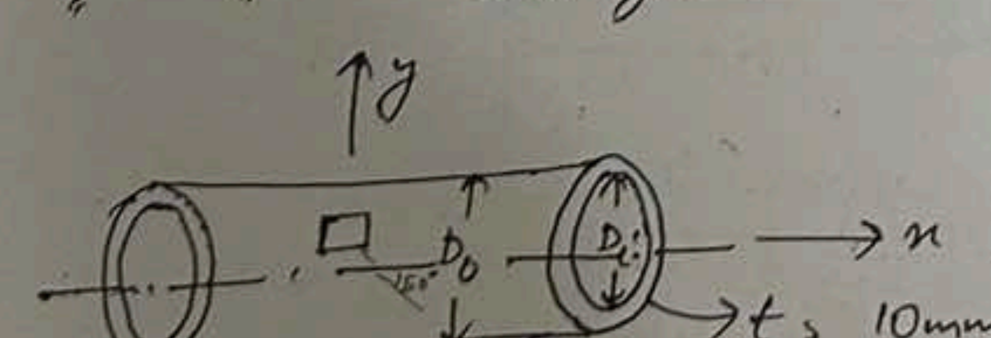


$\sigma = \left(\frac{M}{I}\right) y = \left(\frac{M}{I}\right)\left(\frac{D}{2}\right)$

Shear stress results distortion

Tensile " " elongation

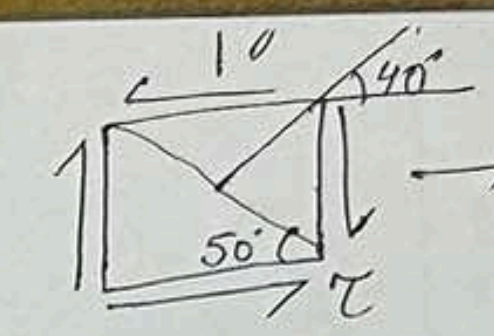
936)

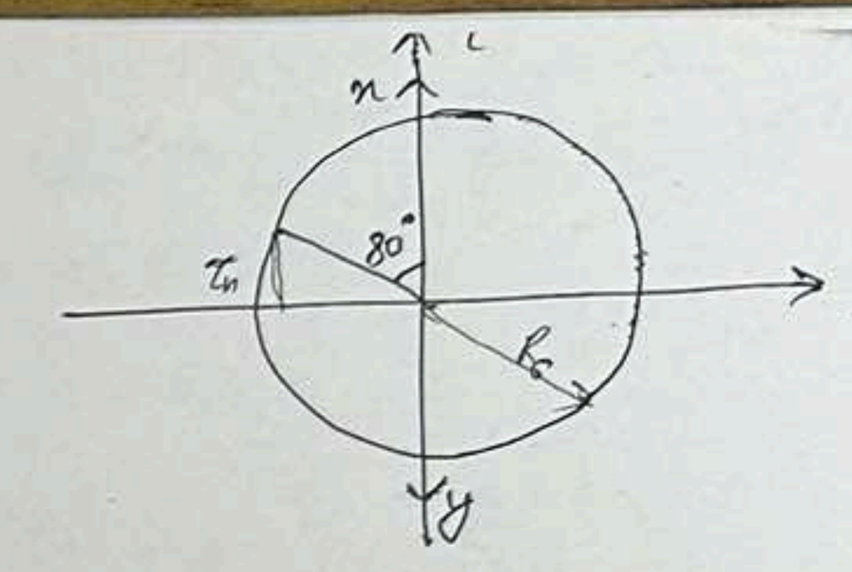


$D_o = 150$ mm

$D_i = 150 - 10 - 10 = 130$ mm

$\tau_n = 40\ N/mm^2$





$R_\sigma \sin 10^\circ = 40$ — (i)

$J = \frac{\pi}{32}(D_o^4 - D_i^4)$

$= 21.67 \times 10^6\ mm^4$

$\tau_{max} = \frac{T}{J} \times \frac{D_o}{2} = R_\sigma$

$\Rightarrow \tau = \frac{T \times 75}{21.67 \times 10^6}$

$\Rightarrow 40 = T \times 3.46 \times 10^{-6}$

$\Rightarrow T_{max} = 66.65 \times 10^6$ N-mm

\# Macaulay's / Singularity Method, Operation calculus

Macaulay's bracket: < >

For $a \geq 0$ & $n \geq 0$

i) $\langle x-a \rangle^n = (x-a)^n \quad \forall\ x > a$

$\qquad = 0 \quad \forall\ x \leq a$

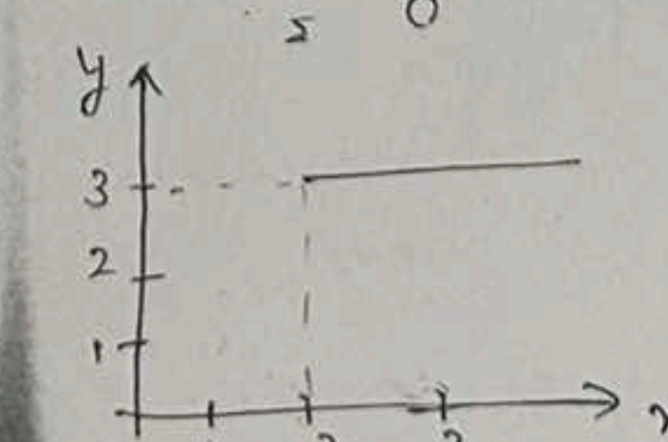
ii) $\langle x-a \rangle^{-n} = 0 \quad \forall\ x \in R$

iii) $\int \langle x-a \rangle^n dx = \frac{\langle x-a \rangle^{n+1}}{n+1} + c_1$

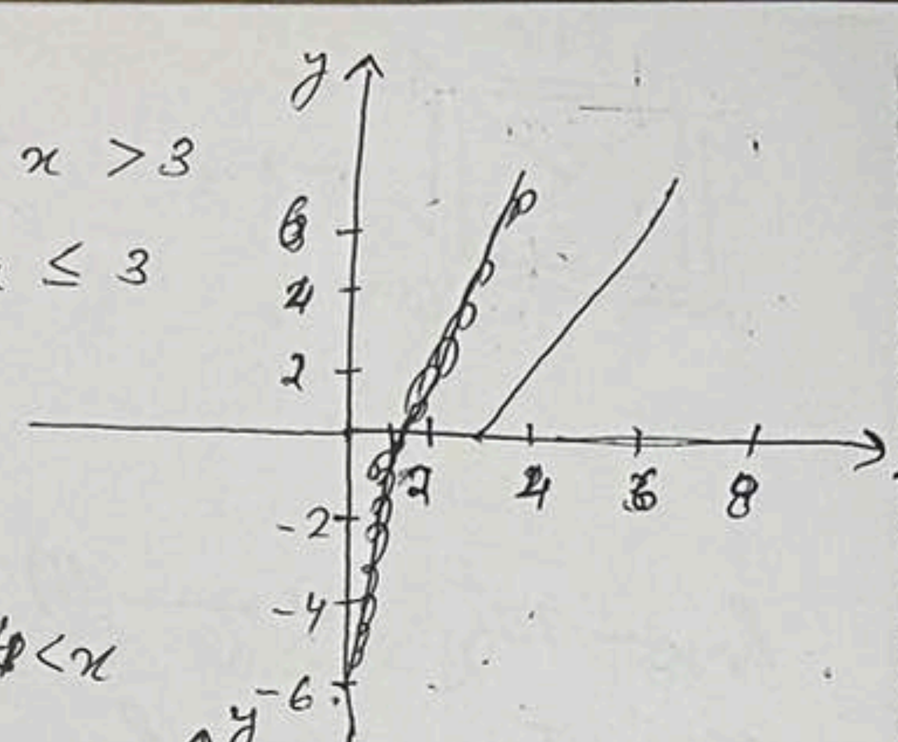
$\int \langle x-a \rangle^{-n} dx = \langle x-a \rangle^{-n+1} + c_2$

Ex: $y = 3\langle x-2 \rangle^0 = 3(x-2)^0 = 3 \quad \forall\ x > 2$

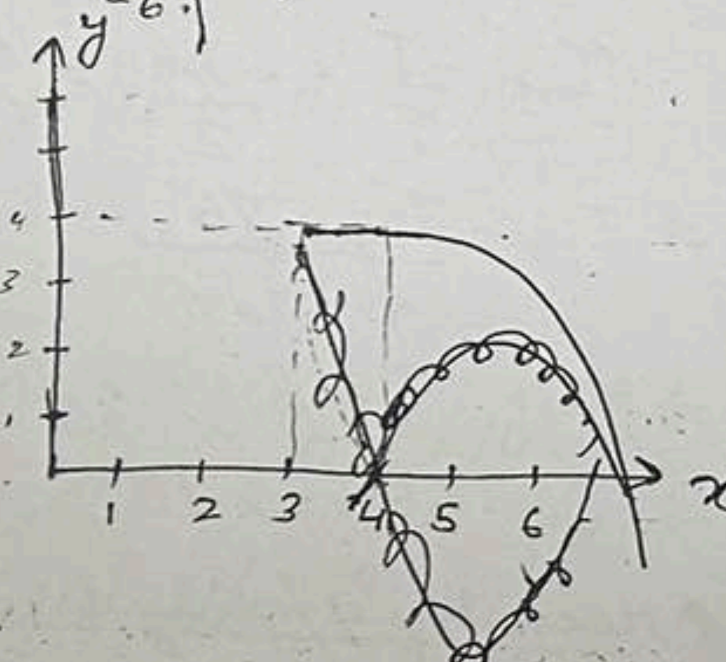
$= 0 \quad \forall\ x \leq 2$



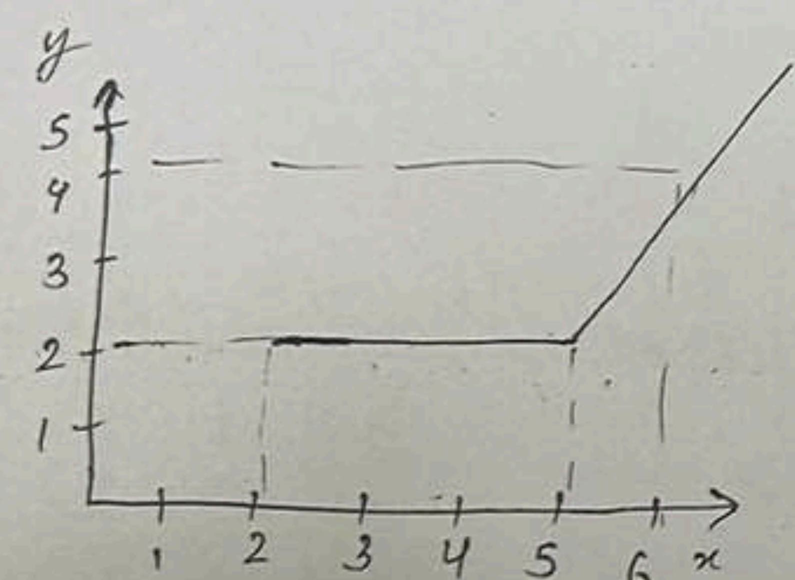
Ex: $y = 2\langle x-3\rangle^1$
$= 2(x-3) \quad \forall\, x > 3$
$0 \quad \forall\, x \le 3$



Ex: $y = 4\langle x-3\rangle^0 - 5\langle x-4\rangle^2$
$= 4(x-3)^0 - 5(x-4)^2;\ 4 < x$
$= 4 - 5(x-4)^2$
$4,\ 3 < x < 4$

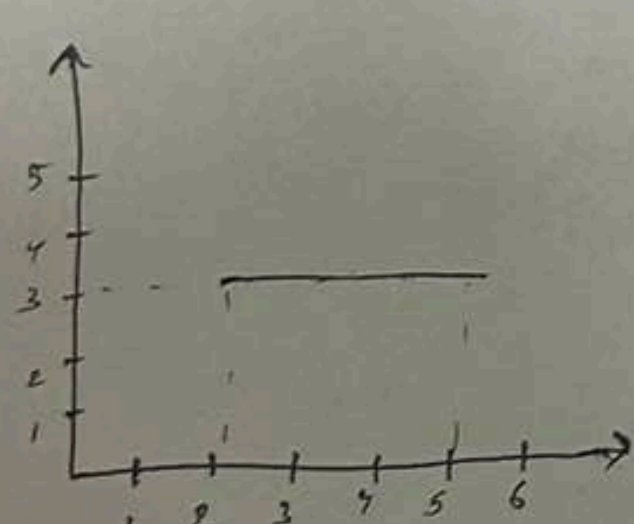


Ex:



$\Rightarrow 2\langle x-2\rangle^0 + 2\langle x-5\rangle^1$

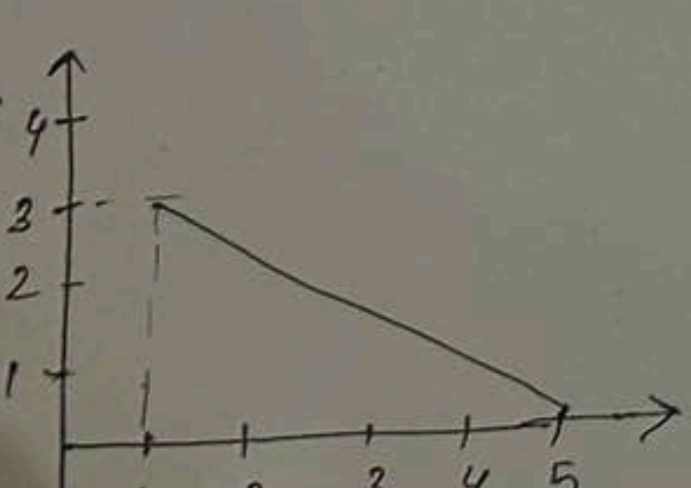
Ex:



$\Rightarrow 3\langle x-2\rangle^0 - 3\langle x-5\rangle^0$

x doesn't exist after 5
$\Rightarrow x \le 5$

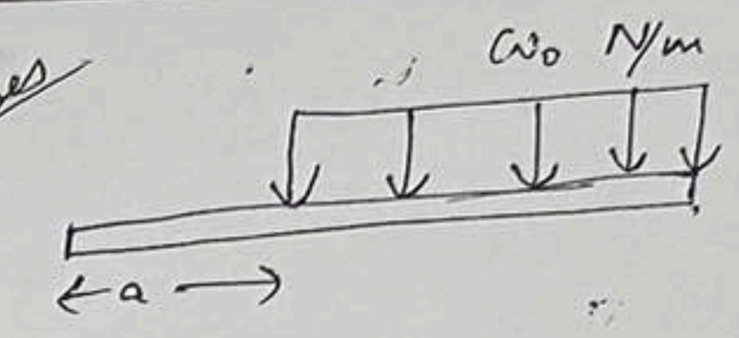
Ex:



$\Rightarrow 3\langle x-1\rangle^0 - \frac{3}{4}\langle x-5\rangle^1$

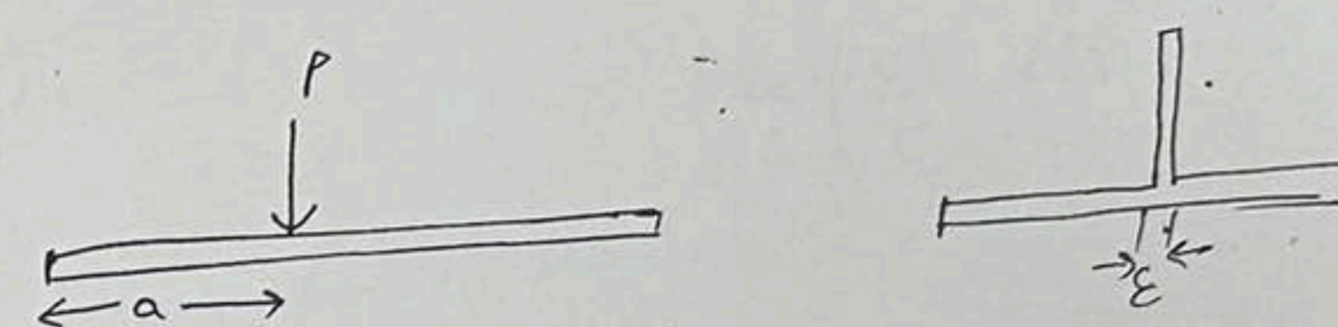
Cases

i)



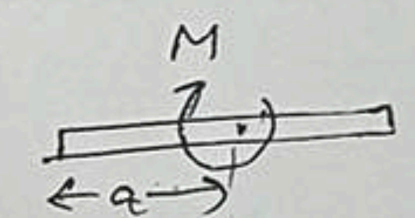
$\omega = -\omega_0 \langle x-a\rangle^0$

ii)



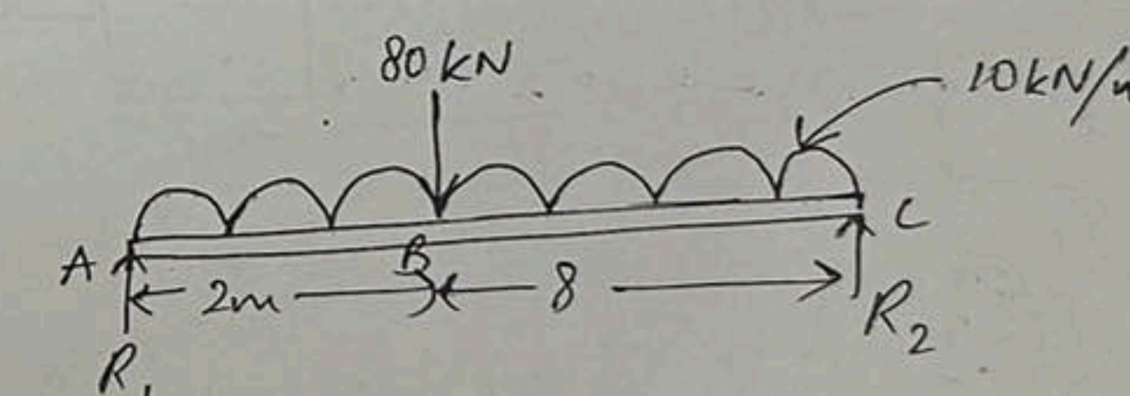
$\omega = \lim_{\varepsilon \to 0} \frac{P}{\varepsilon} = \lim_{x \to a} \frac{P}{x-a} = -P\langle x-a\rangle^{-1}$

iii)

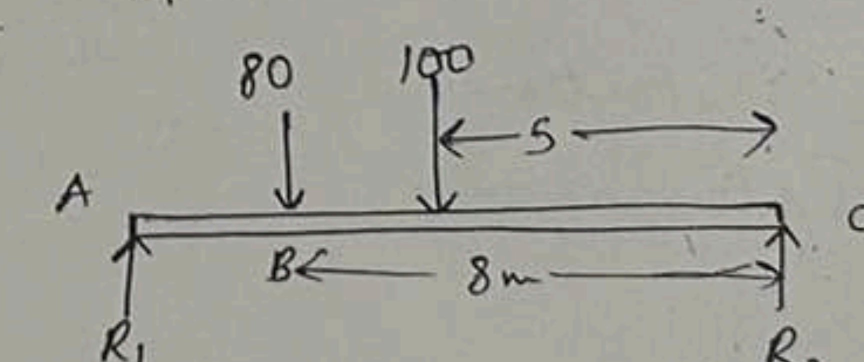


$\omega = M_0 \langle x-a\rangle^{-2}$
CW moment $\Rightarrow$ +ve
Upward force $\Rightarrow$ +ve

Q)



$\longrightarrow$



$\Sigma M_C = 0 \Rightarrow 10R_1 - 80\times 8 - 100\times 5 = 0$
$R_1 = 114\ kN$
$\omega = 114\langle x-0\rangle^{-1} - 80\langle x-2\rangle^{-1} - 10\langle x-0\rangle^0$

$\frac{dV}{dx} = \omega \Rightarrow V = \int \omega\, dx$
$= 114\langle x-0\rangle^0 - 10\langle x-0\rangle^1 - 80\langle x-2\rangle^0$

$0 < x < 2 \Rightarrow V = 114 - 10x \quad ,\ V_B = 94$
$2 < x < 10 \Rightarrow V = 114 - 10x - 80 = 34 - 10x$

$\frac{dM}{dx} = V$
$\Rightarrow M = \int V dx$

$0 < x < 2$

407)

$\Sigma M_D = 0$

$\omega = 20\langle x-0$
$V = 20\langle x-$
$M = 20\langle x-$

408) $\Sigma M_D = 0$

$\omega = -\omega_0 \langle x-a\rangle^0$

$= \lim_{x \to a} \frac{P}{x-a} = -P\langle x-a\rangle^{-1}$

$= M_0\langle x-a\rangle^{-2}$

moment ⇒ +ve

rd force ⇒ +ve

$-100 \times 5 = 0$

kN

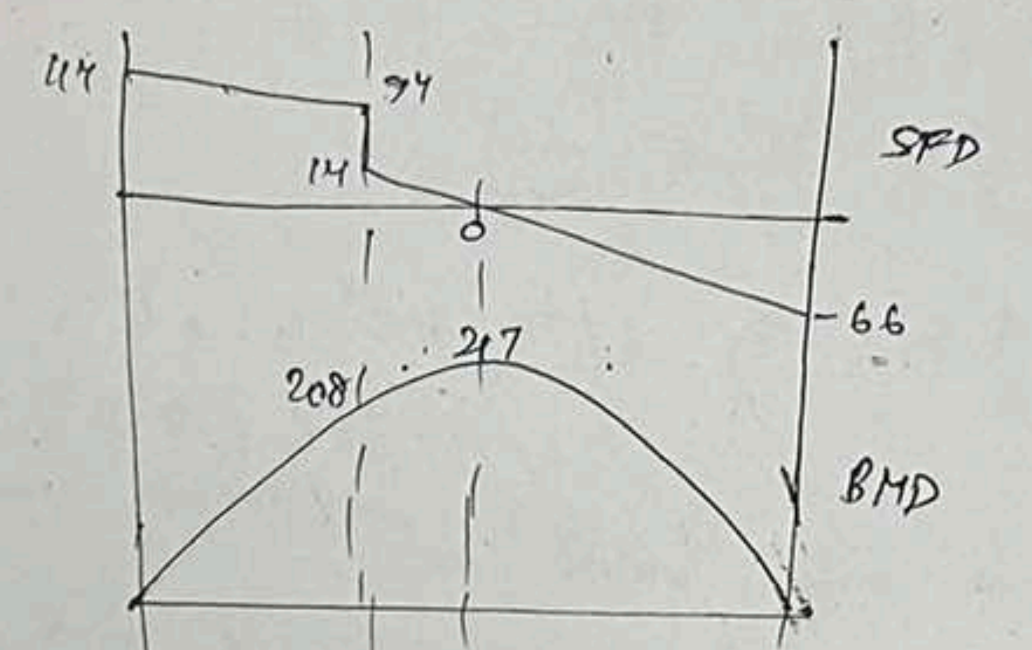
$\langle x-2\rangle^{-1} - 10\langle x-0\rangle^0$

$\langle x-0\rangle^0 - 10\langle x-0\rangle^1 - 80\langle x-2\rangle^0$

x , $V_B = 34$

$x - 80 = 34 - 10x$

$V_B = V|_{x=2} = 14$ $V_C = -66$



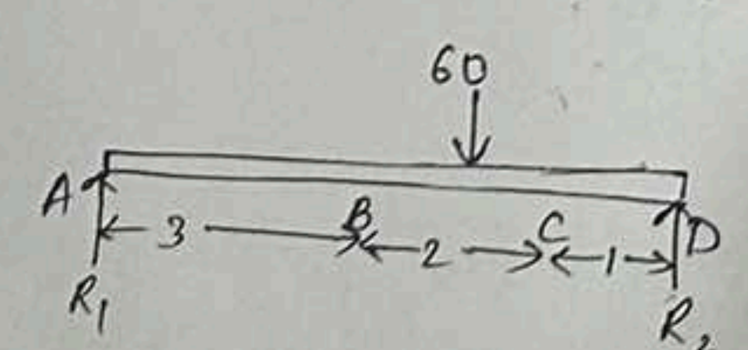
$\frac{dM}{dx} = V$

$\Rightarrow M = \int V\,dx$

$= 114\langle x-0\rangle^1 - \frac{10}{2}\langle x-0\rangle^2 - 80\langle x-2\rangle^1$

$0 < x < 2 \Rightarrow M = 114x - 5x^2 - 80(x-2)$

407)



$\Sigma M_D = 0 \Rightarrow 6R_1 - 60 \times 2 = 0$

$R_1 = 20$ kN

$\omega = 20\langle x-0\rangle^{-1} - \cancel{60\langle x-4\rangle^{-1}}\ 30\langle x-3\rangle^0 + 30\langle x-5\rangle^0$

$V = 20\langle x-0\rangle^0 - 30\langle x-3\rangle^1 + 30\langle x-5\rangle^1$

$M = 20\langle x-0\rangle^1 - \frac{30}{2}\langle 2-3\rangle^2 + \frac{30}{2}\langle x-5\rangle^2$

408) $\Sigma M_D = 0 \Rightarrow 6R_1 - 100 \times 5 - 40 \times 1 = 0$

$R_1 = \frac{540}{6} = 90$ kN

$\omega = 90\langle x-0\rangle^{-1} - 50\langle x-0\rangle^0 + 50\langle x-2\rangle^0 - 20\langle x-4\rangle^0 + \cancel{20\langle x-6\rangle^0}$

411) $\omega = -\frac{\omega_0}{L}\langle x-0\rangle^1 + \omega_0\langle x-0\rangle^0$

$\omega = a\langle x-0\rangle^1 + b\langle x-0\rangle^0$ (For Power ≥ 1)

At $x = 0 \Rightarrow \omega_0 = a \times 0 + b \Rightarrow b = \omega_0$

$x = L,\ \omega = 0 \Rightarrow 0 = aL + b \Rightarrow a = -\frac{\omega_0}{L}$

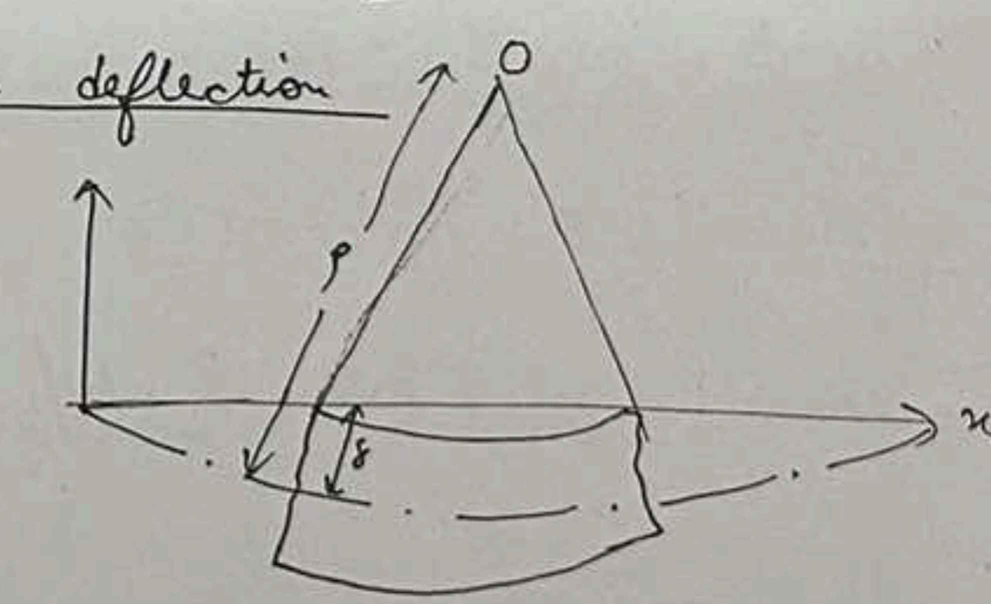
$-30\langle x-3\rangle^0 + 30\langle x-5\rangle^0$

410) At $x = 0 \Rightarrow 0 = a \times 0 + b \Rightarrow b = 0$

$x = L \Rightarrow \omega_0 = aL + b \Rightarrow a = \frac{\omega_0}{L}$

$\therefore \omega = \frac{\omega_0}{L}\langle x-0\rangle^1$

Beam deflection



Edge view of neutral surface is called elastic curve. Deviation of elastic curve from original unloaded position is called deflection of beam.

$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ $\left(\frac{dy}{dx} \ll 1\right)$

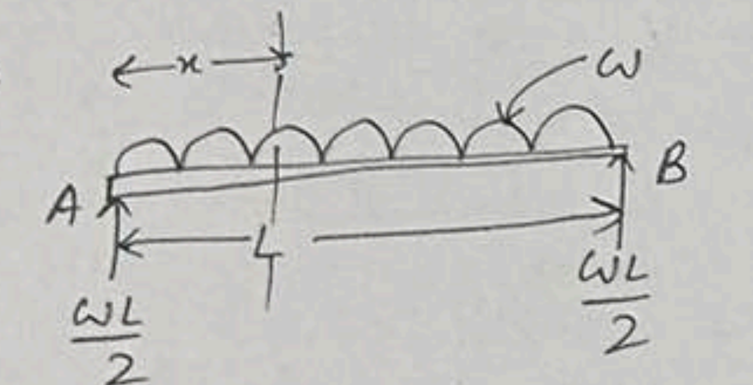
$\Rightarrow \frac{1}{\rho} = \frac{d^2y}{dx^2}$

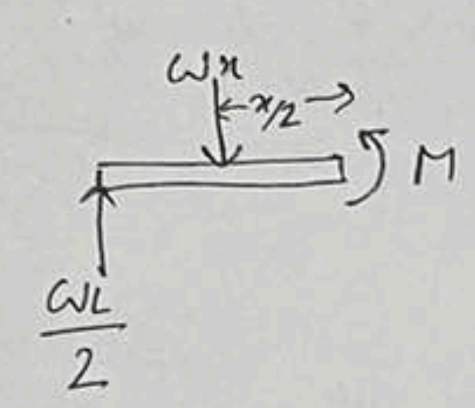
$E\frac{d^2y}{dx^2} = \frac{M}{I}$ $\left(\because \frac{E}{\rho} = \frac{\sigma}{y} = \frac{M}{I}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{EI}\int M\,dx + C_1$

$\Rightarrow y = \frac{1}{EI}\int\left[\int M\,dx + C_1\right]dx + C_2$

Ex:





$M + w\frac{x^2}{2} - \frac{wLx}{2} = 0$

$EI\frac{d^2y}{dx^2} = \frac{wLx}{2} - \frac{wx^2}{2}$

$\Rightarrow EI\frac{dy}{dx} = \frac{wLx^2}{4} - \frac{wx^3}{6} + C_1$

$\Rightarrow EIy = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1x + C_2$

~~At~~ Boundary conditions: $y|_{x=0} = 0$ & $y|_{x=L} = 0$

$C_2 = 0$

$0 = \frac{wL^4}{12} - \frac{wL^4}{24} + C_1L + 0$

$\Rightarrow C_1 = \frac{-wL^3}{24}$

$\therefore EIy = \frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{w_0L^3x}{24}$

To find extrema of $y \Rightarrow \frac{dy}{dx} = 0$

$\frac{wLx^2}{4} - \frac{wx^3}{6} - \frac{wL^3}{24} = 0$

$\Rightarrow wL^3\left(\frac{x^2}{4L^2} - \frac{x^3}{6L^3} - \frac{1}{24}\right) = 0$

Let $\frac{x}{L} = t$ $\Rightarrow 6t^2 - 4t^3 - 1 = 0$

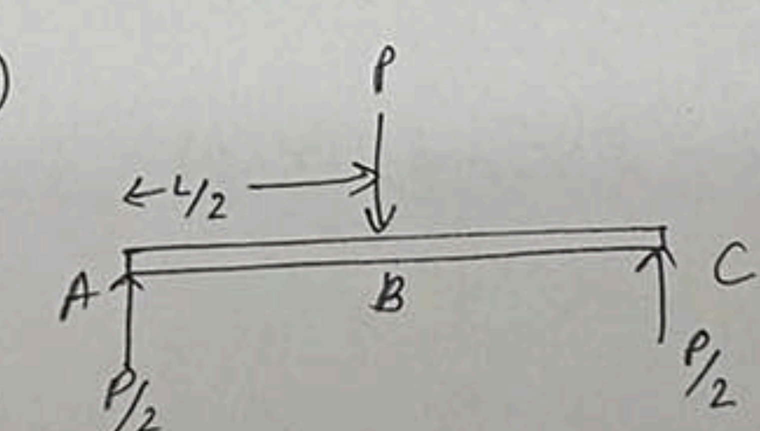
$t = 1.366,\ 0.5,\ -0.366$

$t = 0.5$ is only valid solⁿ

$\delta|_{extrema} = y|_{x=0.5L} = \frac{1}{EI}\left(\frac{-5}{384}\right)wL^4$

$\therefore \delta_{max} = \frac{1}{EI}\cdot\frac{5wL^4}{384}$

605)



$w = \frac{P}{2}\langle x-0\rangle^{-1} - P\langle x-\frac{L}{2}\rangle^{-1}$

$V = \frac{P}{2}\langle x-0\rangle^{0} - P\langle x-\frac{L}{2}\rangle^{0}$

$M = \frac{P}{2}\langle x-0\rangle^{1} - P\langle x-L/2\rangle^{1}$

$EI\frac{d^2y}{dx^2} = \cancel{P}$,,

$\Rightarrow EI\frac{dy}{dx} = \frac{P}{2}\frac{\langle x-0\rangle^2}{2} - \frac{P}{2}\langle x-\frac{L}{2}\rangle^2 + C_1$

$\Rightarrow EIy = \frac{P}{2}\frac{\langle x-0\rangle^3}{6} - \frac{P}{6}\langle x-\frac{L}{2}\rangle^3 + C_1x + C_2$

BC's are $y|_{x=0} = 0$ — (i) & $y|_{x=L} = 0$ — (ii)

Applying (i) $\Rightarrow C_2 = 0$

(ii) $\Rightarrow 0 = \frac{PL^3}{12} -$

$C_1 = -\frac{PL^2}{16}$

$EIy = \frac{P}{12}\langle x-0\rangle^3 - \frac{P}{6}\langle x-\frac{L}{2}$

For max deflection, $\frac{dy}{dx}$

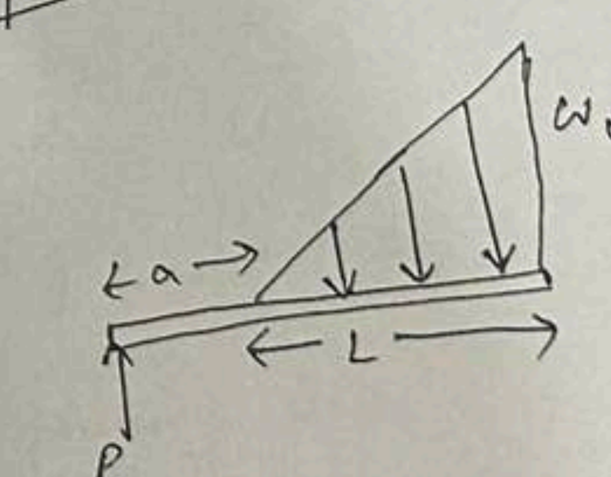
$\frac{P}{4}\langle x-0\rangle^2 - \frac{P}{2}\langle x-\frac{L}{2}\rangle^2$

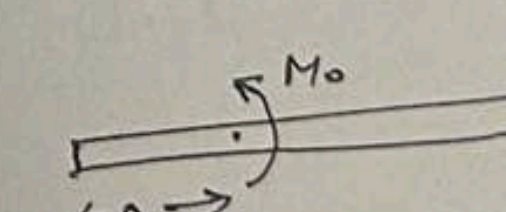
$\Rightarrow x^2 = \frac{PL^2}{16}\times 4$

$EIy|_{x=L/2} = -\frac{PL^3}{48}$

$\therefore \delta_{max} = \frac{PL^3}{48EI}$

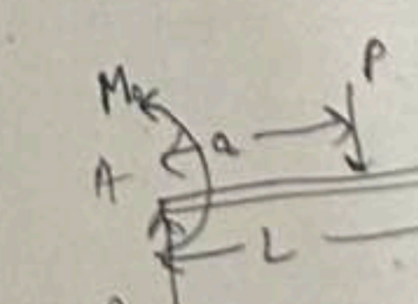
29/10/25





$w = -M_0\langle x-a\rangle^{-2}$

607)



Applying (i) $\Rightarrow$ $c_2 = 0$

(ii) $\Rightarrow$ $0 = \frac{PL^3}{12} - \frac{PL^3}{48} + c_1 L$

$c_1 = -\frac{PL^2}{16}$

$EIy = \frac{P}{12}\langle x-0\rangle^3 - \frac{P}{6}\left\langle x-\frac{L}{2}\right\rangle^3 - \frac{PL^2}{16}x$.

For max deflection, $\frac{dy}{dx} = 0$

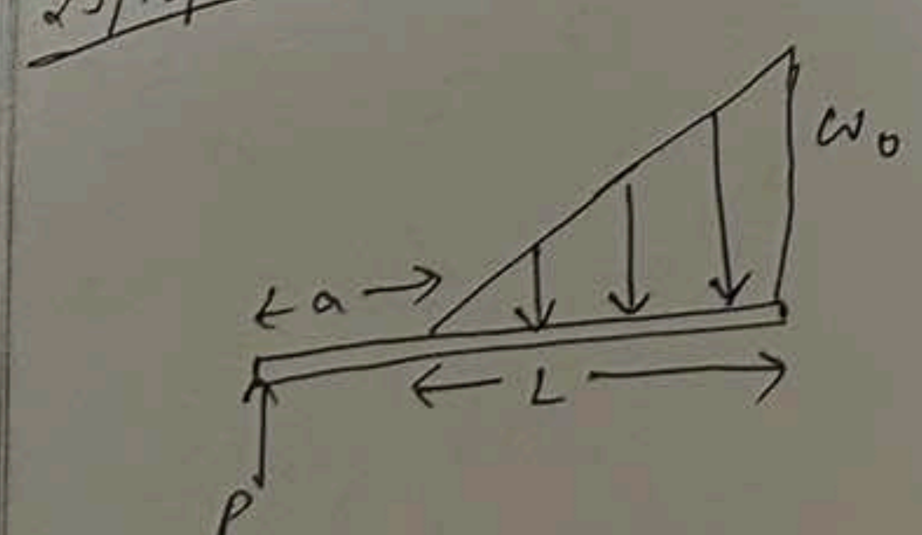
$\frac{P}{4}\langle x-0\rangle^2 - \frac{P}{2}\left\langle x-\frac{L}{2}\right\rangle^2 - \frac{PL^2}{16} = 0$

$\Rightarrow x^2 = \frac{PL^2}{16} \times 4$ $\qquad x = \frac{L}{2}$

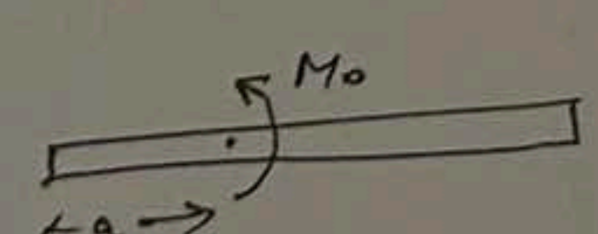
$EIy\big|_{x=L/2} = -\frac{PL^3}{48}$

$\therefore \delta_{max} = \frac{PL^3}{48EI}$ —— Ans

29/10/25

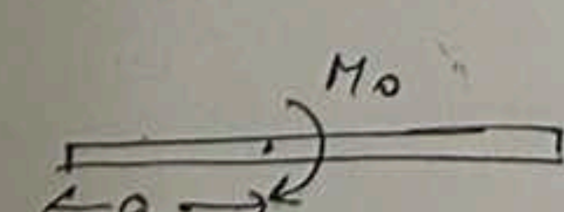


$w = P\langle x-0\rangle^{-1} - \frac{w_0}{L}\langle x-a\rangle^1$

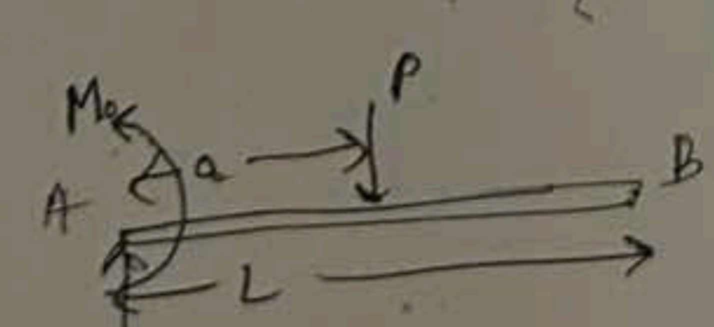


$w = -M_0\langle x-a\rangle^{-2}$

$w = M_0\langle x-a\rangle^{-2}$



607)



Cantilever beam $\Rightarrow$ Initial value problem so apply initial conditions : $y\big|_{x=0} = 0$ & $\frac{dy}{dx}\Big|_{x=0} = 0$

$\Sigma F_y = 0 \Rightarrow A_y = P$

$\Sigma M_A = 0 \Rightarrow M_0 = Pa$

$w = A_y\langle x-0\rangle^{-1} - M_0\langle x-0\rangle^{-2} - P\langle x-a\rangle^{-1}$

$V = P\langle x-0\rangle^0 - M_0\langle x-0\rangle^{-1} - P\langle x-a\rangle^0$

$M = P\langle x-0\rangle^1 - Pa\langle x-0\rangle^0 - P\langle x-a\rangle$

$EI\frac{dy}{dx} = \frac{P}{2}\langle x-0\rangle^2 - Pa\langle x-0\rangle^1 - \frac{P}{2}\langle x-a\rangle^2 + c_1$ —— (i)

$EIy = \frac{P}{6}\langle x-0\rangle^3 - \frac{Pa}{2}\langle x-0\rangle^2 - \frac{P}{6}\langle x-a\rangle^3 + c_1x + c_2$ —— (ii)

Applying IC's $\Rightarrow$

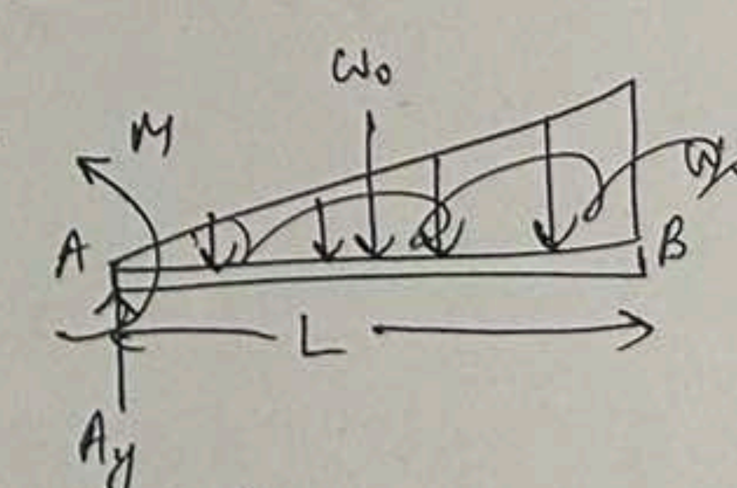
$y\big|_{x=0}$ in (ii) $\Rightarrow$ $c_2 = 0$

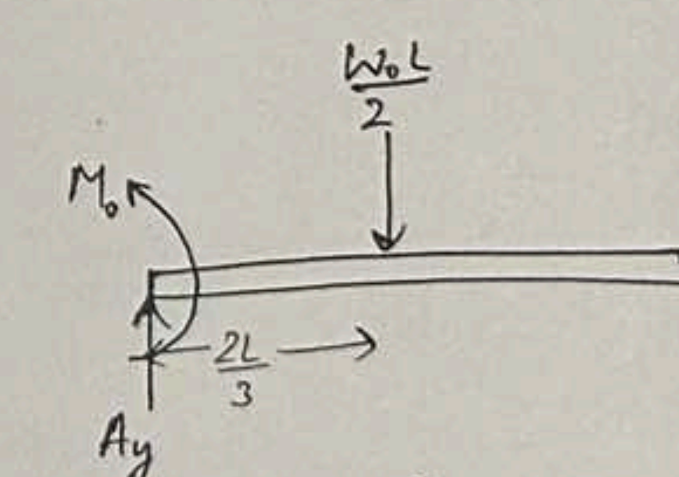
$\frac{dy}{dx}\Big|_{x=0}$ in (i) $\Rightarrow c_1 = 0$

$\therefore EIy = \frac{P}{6}\langle x-0\rangle^3 - \frac{Pa}{2}\langle x-0\rangle^2 - \frac{P}{6}\langle x-a\rangle^3$

For $(EIy)_{max}$, $x = L$

608)





$w = A_y\langle x-0\rangle^{-1} - \frac{w_0}{L}\langle x-0\rangle^1 - M_0\langle x-0\rangle^{-2}$

$\Sigma F_y = 0 \Rightarrow A_y = \frac{w_0 L}{2}$

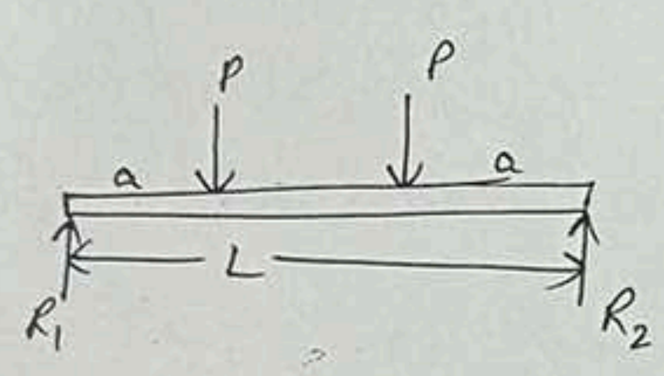
$\Sigma M_A = 0 \Rightarrow M_0 = \frac{w_0 L}{2} \times \frac{2L}{3} = \frac{w_0 L^2}{3}$

$V = A_y \langle x-0\rangle^0 - M_0 \langle x-0\rangle^{-1} - \frac{\omega_0}{2L} \langle x-0\rangle^2$

$M = \frac{\omega_0 L}{2} \langle x-0\rangle^1 - \frac{\omega_0 L^2}{3} \langle x-0\rangle^0 - \frac{\omega_0}{6L} \langle x-0\rangle^3$

$EI\frac{dy}{dx} = \frac{\omega_0 L}{4}\langle x-0\rangle^2 - \frac{\omega_0 L^2}{3}\langle x-0\rangle^1 - \frac{\omega_0}{24L}\langle x-0\rangle^4$

609)



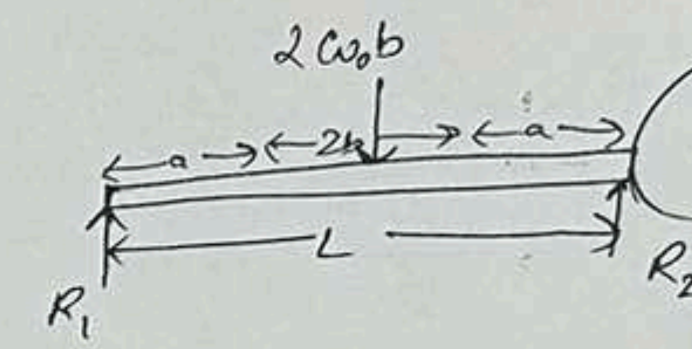
$R_1 + R_2 = 2P$

$\Sigma M_{R_1} = 0 \Rightarrow Pa + P(L-a) = R_2 L$

$R_2 = P \qquad \Rightarrow R_1 = P$

$\omega = R_1\langle x-0\rangle^{-1} - P\langle x-a\rangle^{-1} - P\langle x-(L-a)\rangle^{-1}$ ~~+R2~~

610)



09/10

BTECH

$\Sigma F_y = 0 \Rightarrow R_1 + R_2 = 2b\omega_0$

$\Sigma M_{R_1} = 0 \Rightarrow R_2 L = 2\omega_0 b(L-a-b)$

$\omega = R_1\langle x-0\rangle^{-1} - \omega_0\langle x-a\rangle^{-1} + \omega_0\langle x-(a+2b$

$R_1 = 2b\omega_0\left(1 - \frac{(L-a-b)}{L}\right)$

$= 2b\omega_0 \frac{(a+b)}{L}$

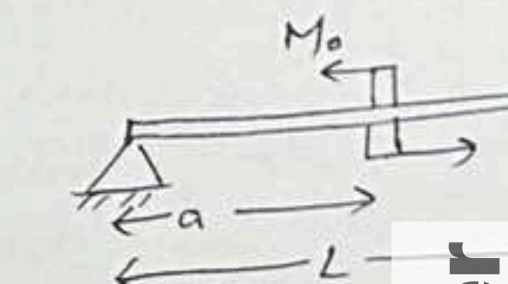
$2a + 2b = L$

$\Rightarrow a + b =$

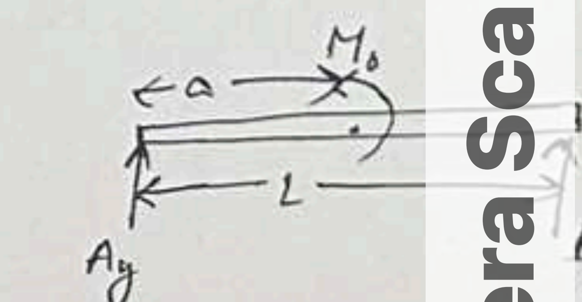
$= b\omega_0$

HW - 609, 611, 612, 612, 618, 613, 620, 605 - 608

618)



FBD:



$\Sigma M_B = 0 \Rightarrow -LA_y$

$\Rightarrow A_y$

$\omega = A_y\langle x-0\rangle^{-1}$

$V = \frac{M_0}{L}\langle x-0\rangle^0$

$M = \frac{M_0}{L}\langle x-0\rangle^1$

$EI\frac{dy}{dx} = \frac{M_0}{2L}\langle x-0$

$EI\,y = \frac{M_0}{6L}\langle x-0\rangle^3$

Applying BC's,

$y|_{x=L} = 0 \Rightarrow \frac{M_0 L^3}{6L} -$

$\frac{M_0 L^2}{6} -$

$-\frac{M_0 L^2}{36}$

$\frac{09}{10}$

R_2 BTECH/10348/24

$a-b)$

$\rangle^{-1} + \omega_0 \langle x-(a+2b)\rangle$

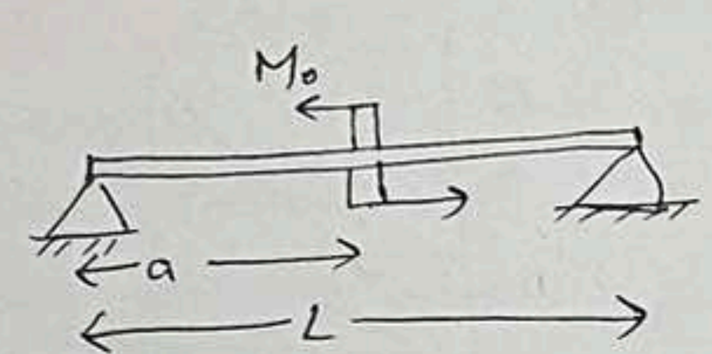
$)$

$2a+2b = L$

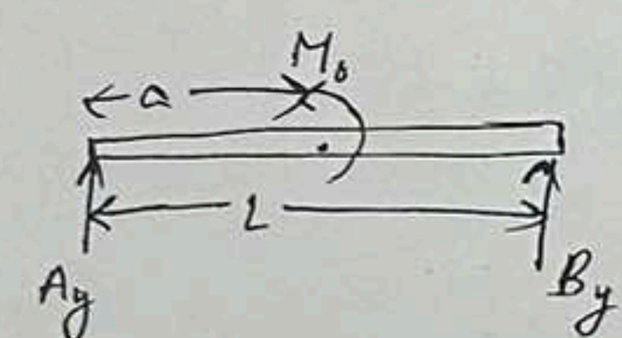
$\Rightarrow a+b = \frac{L}{2}$

0, 605 - 608

618)



FBD:



$\Sigma M_B = 0 \Rightarrow -LA_y + M_0 = 0$

$\Rightarrow A_y = \frac{M_0}{L}$

$\omega = A_y \langle x-0\rangle^{-1} - M_0 \langle x-a\rangle^{-2}$

$V = \frac{M_0}{L}\langle x-0\rangle^0 - M_0\langle x-a\rangle^{-1}$

$M = \frac{M_0}{L}\langle x-0\rangle^1 - M_0\langle x-a\rangle^0$

$EI\frac{dy}{dx} = \frac{M_0}{2L}\langle x-0\rangle^2 - M_0\langle x-a\rangle^1 + c_1$

$EIy = \frac{M_0}{6L}\langle x-0\rangle^3 - \frac{M_0}{2}\langle x-a\rangle^2 + c_1x + c_2$

Applying BC's,

$y|_{x=L} = 0 \Rightarrow \frac{M_0L^3}{6L} - \frac{M_0}{2}(L-a)^2 + c_1L + \cancel{c_2} = 0$

$\frac{M_0L^2}{6} - \frac{M_0}{2}(L^2+a^2-2La) + c_1L + \cancel{c_2} = 0$

$-\frac{M_0L^2}{3} - \cancel{\frac{M_0a^2}{2}} + M_0La + c_1L + \cancel{\frac{M_0a^2}{2}} = 0$

$c_1 = \frac{M_0L}{3} - M_0a + \frac{M_0a^2}{2L}$

$y|_{x=0} = 0 \Rightarrow \cancel{\frac{M_0}{2}}0 = c_2$

Putting $x = a$,

$EIy = \frac{M_0a^3}{6L} - \cancel{\frac{M_0}{2}} + a\left(\frac{M_0L}{3} - M_0a + \frac{M_0a^2}{2L}\right)$

$= \frac{M_0a^3}{6L} + \frac{M_0a^3}{2L} - M_0a^2 + \frac{M_0aL}{3}$

$= \frac{2}{3L}M_0a^3 - M_0a^2 + \frac{M_0aL}{3}$

$= \frac{M_0a}{3L}\left(2a^2 - 3La + L^2\right)$ — Ans

629) $y = a\langle x-0\rangle^1 + b\langle x-\frac{L}{2}\rangle^1 + \underbrace{c\langle x-0\rangle^0}_{\text{constant}}$

$y|_{x=0} = -\omega_0 \Rightarrow c = -\omega_0$

$y|_{x=\frac{L}{2}} = 0 \Rightarrow \frac{aL}{2} + c = 0 \Rightarrow a = \frac{2\omega_0}{L}$

$y|_{x=L} = -\omega_0 \Rightarrow aL + \frac{bL}{2} + c = -\omega_0$

$b = \frac{2}{L}\left(\omega_0 - \frac{2\omega_0 L}{L}\right) = -\frac{4\omega_0}{L}$

$\therefore \omega = R_1\langle x-0\rangle^{-1} + \frac{2\omega_0}{L}\langle x-0\rangle^1 - \frac{4\omega_0}{L}\langle x-\frac{L}{2}\rangle^1 - \omega_0\langle x-0\rangle^0$

$R_1 + R_2 = \frac{1}{2}\omega_0\frac{L}{2} + \frac{1}{2}\omega_0\frac{L}{2} = \frac{\omega_0L}{2}$

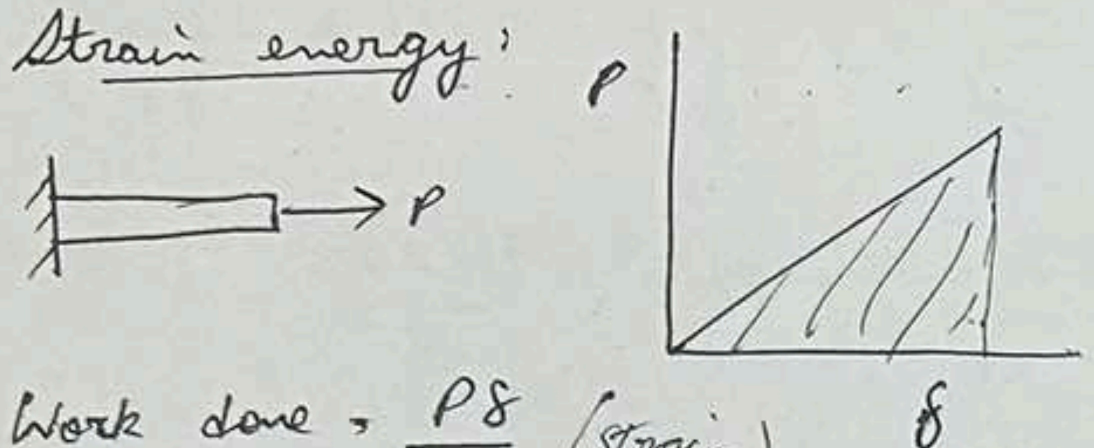
$\Sigma M_{R_1} = 0 \Rightarrow R_2L = \frac{\omega_0L}{4}\times\frac{L}{4} + \frac{\omega_0L}{4}\times\frac{3L}{4} = \frac{\omega_0L^2}{4}$

$R_2 = \frac{\omega_0L}{4}$ so $R_1 = \frac{\omega_0L}{4}$

Energy method for deflection in structures

Strain energy:

i)



Work done = $\frac{P\delta}{2}$ (strain energy)

(we apply load gradually)

$U = W = \frac{1}{2}P\delta = \frac{1}{2}\left(\frac{P}{A}\right)\left(\frac{\delta}{L}\right)AL$

$\Rightarrow \frac{U}{AL} = \frac{\sigma\epsilon}{2}$

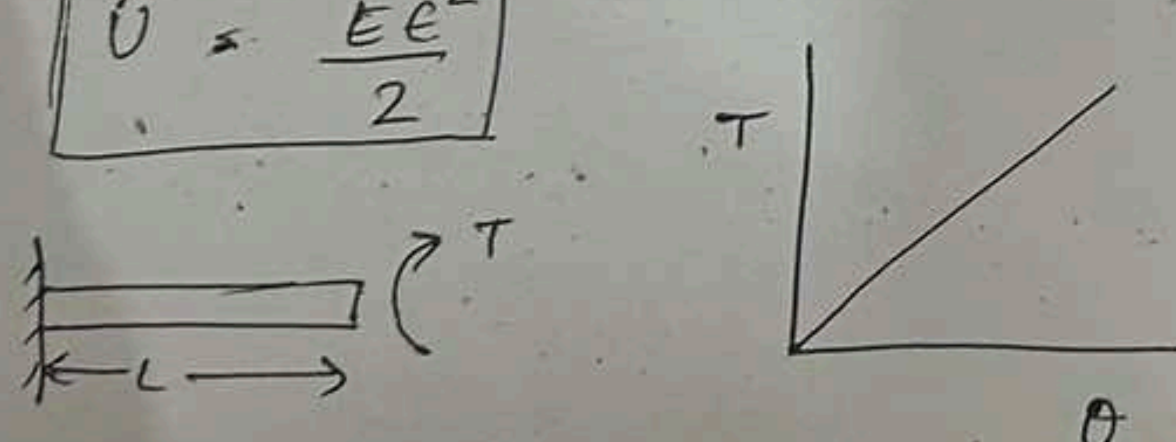
Deformation is small enough in body so that Hooke's law is applicable.

Energy density (Energy/vol): $\hat{U} = \frac{U}{AL}$

$\hat{U} = \frac{1}{2}\sigma\left(\frac{\sigma}{E}\right)$

$$\boxed{\hat{U} = \frac{E\epsilon^2}{2}}$$

ii)



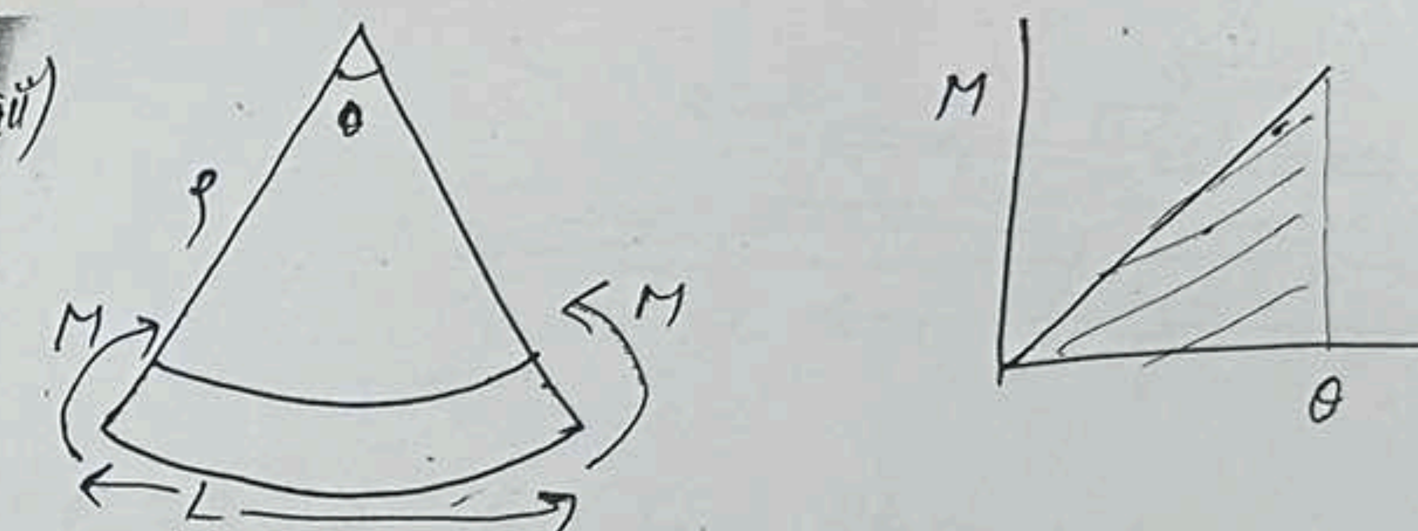
$U = \frac{T\theta}{2}$ Using, $\frac{T}{J} = \frac{G\theta}{L}$

$\Rightarrow U = \frac{T}{2} \times \frac{TL}{GJ} = \frac{1}{2}\frac{T^2L}{GJ}$

Strain energy density (Energy/length) $\Rightarrow \hat{U} = \frac{U}{L}$

$$\boxed{\hat{U} = \frac{1}{2}\frac{T^2}{GJ}}$$

iii)

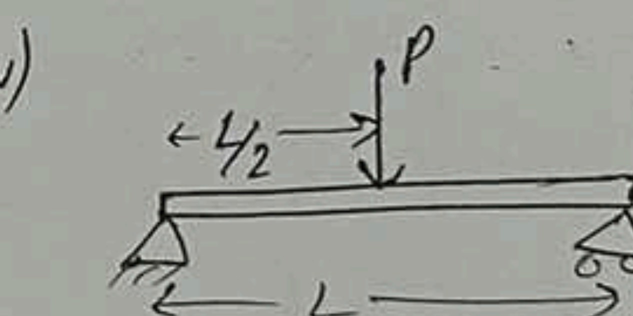


Using $\frac{M}{I} = \frac{E}{\rho}$ & $\rho\theta = L$

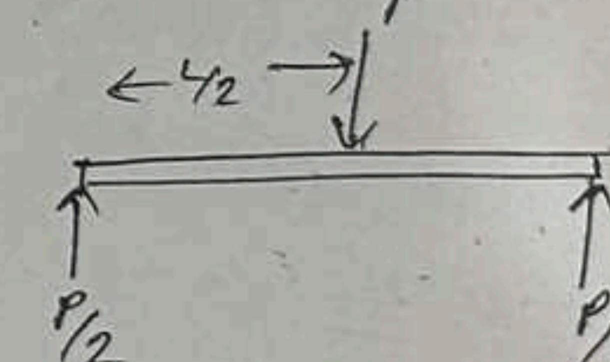
$\Rightarrow U = \frac{M}{2}\left(\frac{ML}{IE}\right)$

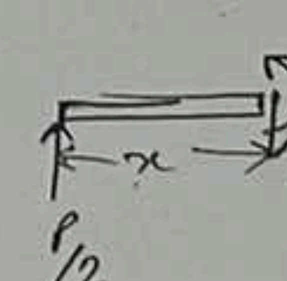
$$\boxed{\hat{U} = \frac{U}{L} = \frac{M^2}{2EI}}$$

iv)



FBD:





$M = \frac{Px}{2}$

$dU = \frac{M^2}{2EI}dx$

$U = 2\int_0^{L/2}\frac{M^2}{2EI}dx$

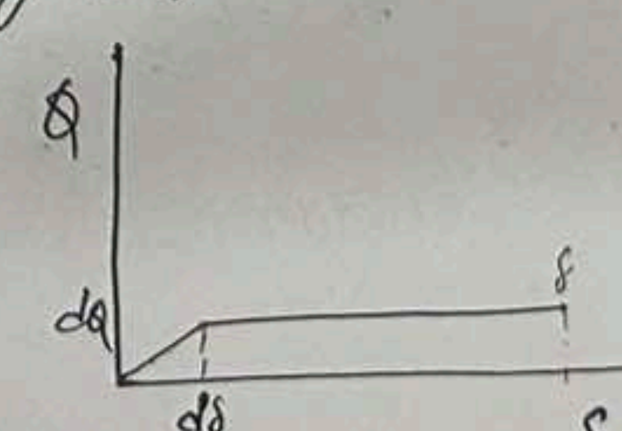
$= 2\int_0^{L/2}\frac{P^2x^2}{4EI}dx$

$= \frac{P^2L^3}{96EI}$

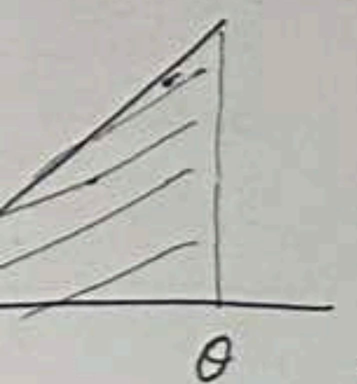
v) $dU = dQ\,\delta$ (Pg 452)

$dU = \frac{\partial U}{\partial Q}dQ$

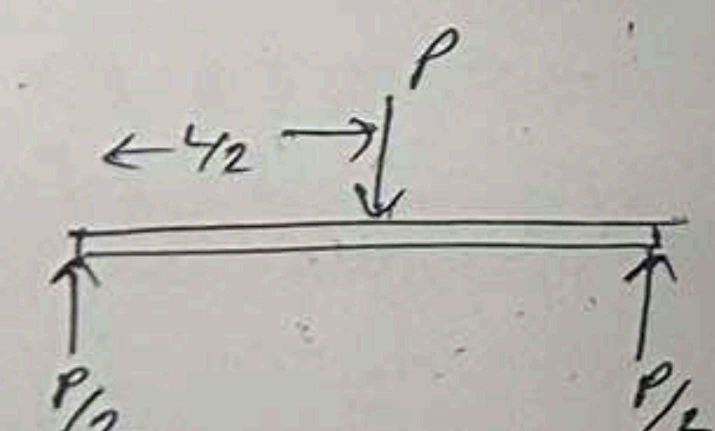
$$\boxed{\delta = \frac{dU}{dQ}}$$



$W = \frac{M\theta}{2}$



$W = \frac{M\theta}{2}$



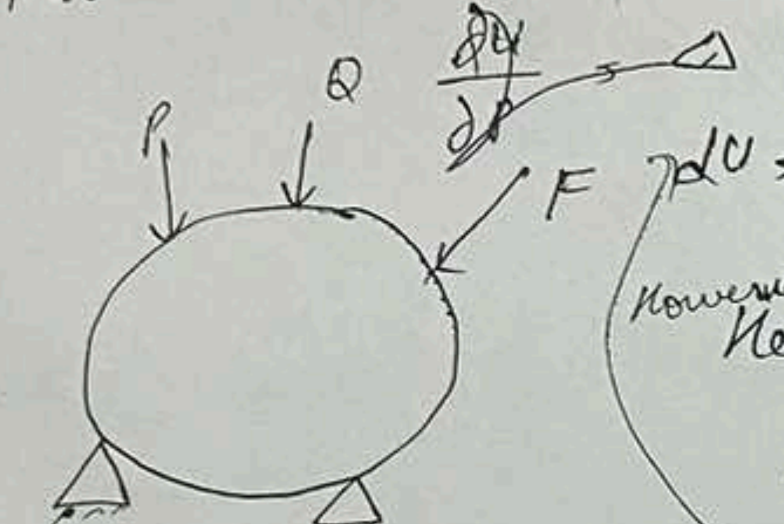
$U = \int \frac{M^2}{2EI}\,dx$

$= 2\int_0^{L/2} \frac{M^2}{2EI}\,dx$

$= 2\int_0^{L/2} \frac{P^2x^2}{4EI}\,dx$

$= \frac{P^2L^3}{96EI}$

• <u>Castigliano's Theorem</u>: Partial derivative of strain energy wrt one of the external loads equals displacement of the pt. of application of that load in direction of load.



$dU = \frac{\partial U}{\partial P}dP + \frac{\partial U}{\partial Q}dQ + \frac{\partial U}{\partial F}dF$

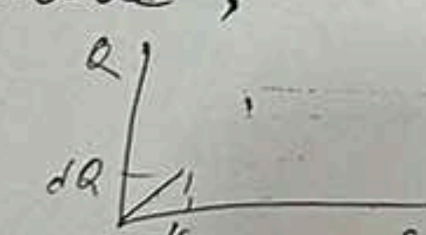
However, Here, $dP, dF = 0$

$\therefore dU = \frac{\partial U}{\partial Q}dQ$ ——— (i)

Initially, we applied P, Q, F, which results in strain energy as $U = U(P,Q,F)$. They, only Q is ↑ by dQ which may results:

Since superposition is applicable, this time we 1st apply dQ

This results $dU = \frac{dQ\,d\delta}{2}$

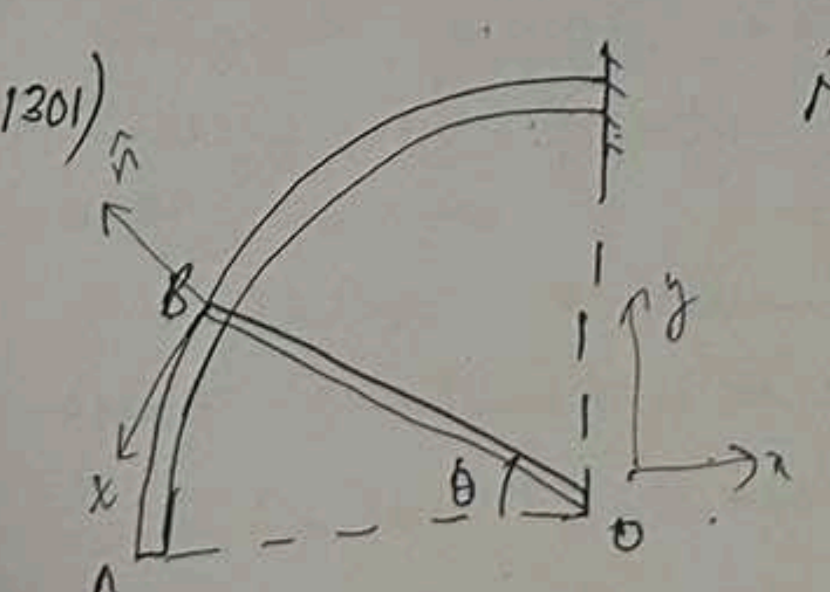


Later, we apply Q, P, F as before then at pt. of application of Q, deformation is δ as before

$\therefore dU = \frac{dQ}{2}d\delta + dQ.\delta \approx dQ.\delta$ ——— (ii)

Comparing (i) & (ii) $\rightarrow$ $\boxed{\frac{dU}{dQ} = \delta}$

1301)



$\vec{M_B} = \vec{BA} \times \vec{P}$

$\vec{BA} = \vec{OA} - \vec{OB}$

$= -R\hat{i} - (-R\cos\theta\hat{i} + R\sin\theta\hat{j})$

$= (R\cos\theta - R)\hat{i} - R\sin\theta\hat{j}$

$\vec{P} = -P\hat{k}$

$\vec{M_B} = -PR(1-\cos\theta)\hat{j} + PR\sin\theta\hat{i}$

$\vec{M} = \vec{M_B}.\hat{n}$

$\hat{n} = \frac{\vec{OB}}{OB} = \frac{-R\cos\theta\hat{i} + R\sin\theta\hat{j}}{R}$

$\hat{t} = -\sin\theta\hat{i} - \cos\theta\hat{j}$

$\vec{M} = -PR\sin\theta\cos\theta - PRs_\theta + PRs_\theta c_\theta$

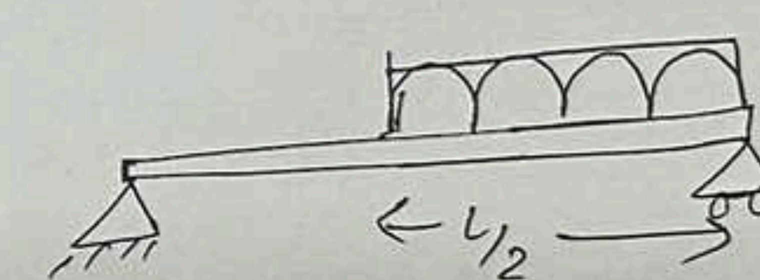
$= -PR\sin\theta$

$T = \vec{M_B}.\hat{t}$

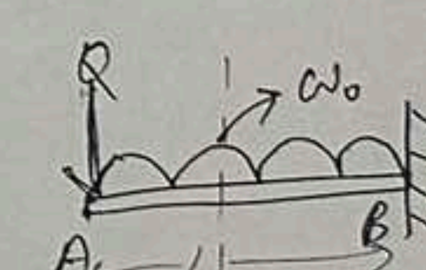
$= -PRs_\theta^2 + PRc_\theta - PRc_\theta^2$

$= -PR(1-\cos\theta)$

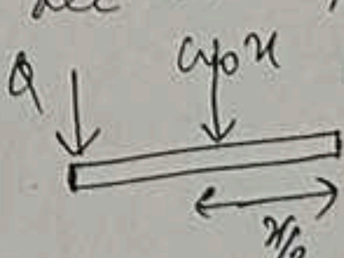
1303)



Ex- Determine δ_A



$\rightarrow$ Let us apply dummy load Q & taking section



$M = -Q.x - \frac{w_0x^2}{2}$

$U = \int_0^L \frac{M^2}{2EI}dx = \int_0^L \frac{\left(Qx + \frac{wx^2}{2}\right)^2}{2EI}dx$

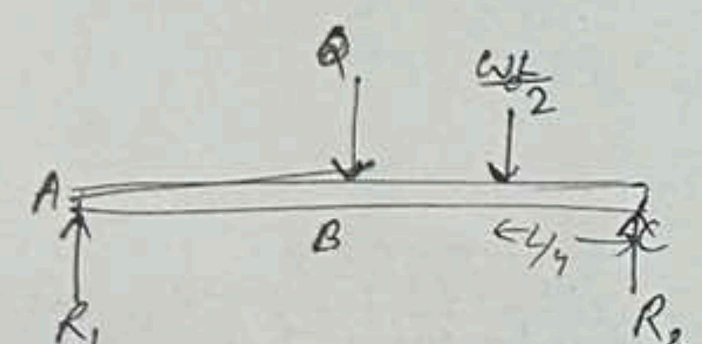
$\Rightarrow \frac{\partial U}{\partial Q} = \int_0^L \frac{2\left(Qx + \frac{wx^2}{2}\right)x}{2EI}dx$

Now, put $Q = 0$

$$\delta = \frac{dU}{dQ} = \int_0^L \frac{\frac{wx^2}{2} \cdot x \, dx}{EI} = \frac{wL^4}{8EI}$$

• It is imp° to ~~stress~~ note that U is total strain energy, expressed in terms of loads (not including statically determinate rexn) & partial derivative wrt each load in turn (considering others as const) gives deflection at load pt in direction of load.
(1309)

1303)



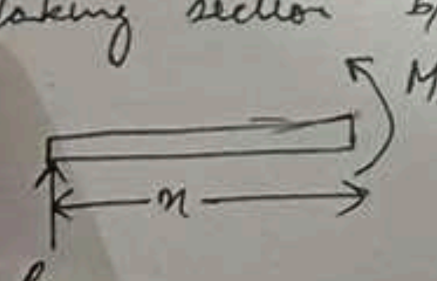
$$R_1 + R_2 = Q + \frac{\omega_0 L}{2}$$

$$\Sigma M_A = 0 \Rightarrow LR_2 = \frac{QL}{2} + \frac{\omega_0 L}{2} \times \frac{3L}{4}$$

$$R_2 = \frac{Q}{2} + \frac{3\omega_0 L}{8}$$

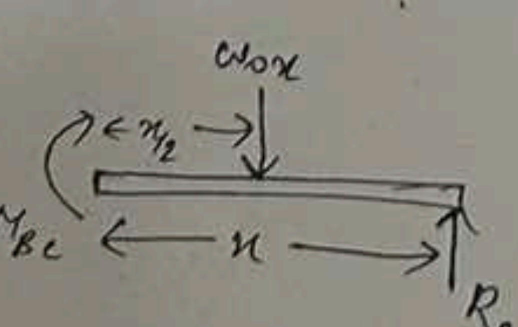
$$R_1 = Q + \frac{\omega_0 L}{2} - \frac{Q}{2} - \frac{3\omega_0 L}{8} = \frac{Q}{2} + \frac{\omega_0 L}{8}$$

Taking section b/w A & B,



$$M_{AB} = \left(\frac{Q}{2} + \frac{\omega_0 L}{8}\right)x$$

Taking section b/w B & C



$$M_{BC} = R_2 x - \frac{\omega_0 x^2}{2}$$

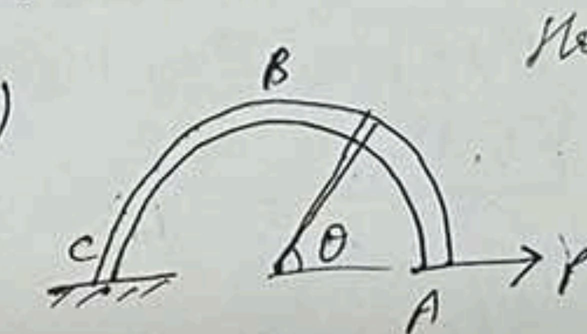
$$U = U_{AB} + U_{BC} = \int_0^{L/2} \frac{M_{AB}^2}{2EI} dx + \int_0^{L/2} \frac{M_{BC}^2}{2EI} dx$$

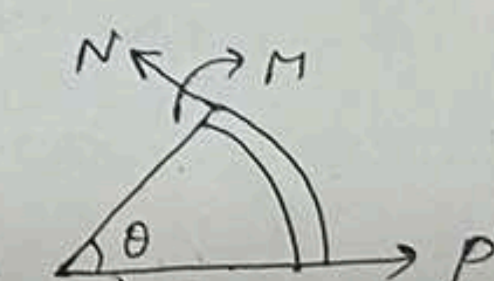
$$\delta = \frac{\partial U}{\partial Q}\bigg|_{Q=0} = \int_0^{L/2} \frac{\frac{\partial M_{AB}^2}{\partial Q}}{2EI} dx + \int_0^{L/2} \frac{\frac{\partial M_{BC}^2}{\partial Q}}{2EI} dx$$

$$= \int_0^{L/2} \frac{2M_{AB}\frac{x}{2} \, dx}{2EI} + \int_0^{L/2} \frac{2M_{BC}\frac{x}{2} \, dx}{2EI}$$

$$= \frac{1}{2EI}\int_0^{L/2} \frac{\omega_0}{8} L x^2 dx + \int_0^{L/2} \left(\frac{3\omega_0}{8} L x - \frac{\omega_0 x^2}{2}\right) x \, dx$$

1307) Horiz deflection of A?





$$M = PR \sin\theta$$

$$U = \int_0^{\pi} \frac{M^2}{2EI} R \, d\theta = \int_0^{\pi} \frac{P^2 R^3 \sin^2\theta}{2EI} d\theta = \frac{\pi P^2 R^3}{4EI}$$

$$(\delta_H)_A = \frac{\partial U}{\partial P} = \frac{\pi P R^3}{2EI}$$

$\boxed{1303 - 1310}$

Buckling of column

A column is a member undergoes compression as shown in Fig 1. The linear elastic theory suggests, there will be no bending moment or deflection in transverse direction (of axis of column). However, sometimes due to unavoidable eccentricity or perturbation in transverse direction column buckle as shown in Fig. 2.

Fig 1

If P isn't large enough, member regains its original sh[ape]. On other hand [if] of P, column [...] shown in Fig [...] phenomenon is [...]

★ Derivation of Euler buckling [...]

Consider a column with [...] in Fig-3, assuming it [...] compressive load P.

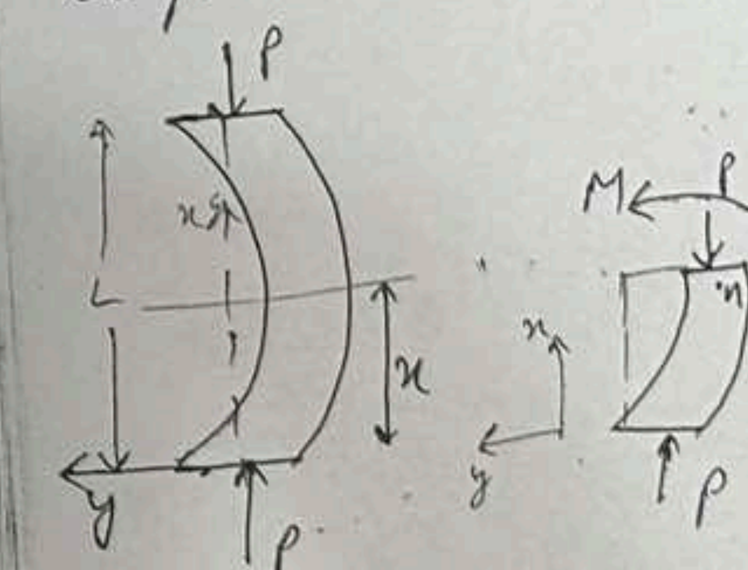
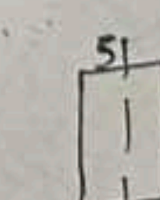


Fig-3



$$\frac{d^2y}{dx^2} [...]$$

$\rightarrow y = A \sin [...]$

BC's are $y|_{x=0} = 0$ & $y|_{x=L} [...]$

Applying I in (ii) ⇒ B [...]
" II in (ii) ⇒ A [...]

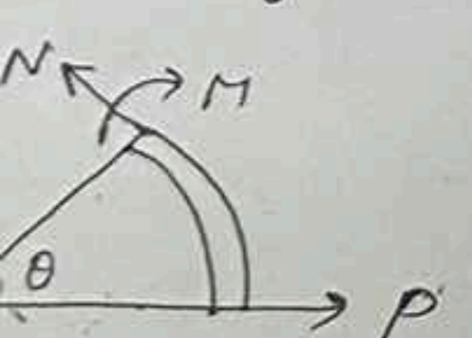
If A = 0 then y = 0 & [...] Fig 1. However, we are lo[...] represents coupling as sh[...]

$\int_0^{l/2} \frac{\partial^2 M_{BC}^2}{\partial Q \, 2EI} \, dx$

$\frac{2 M_{BC} \frac{x}{2}}{2EI} \, dx$

$x - \frac{w_0 x^2}{2} \Big) x \, dx$

...lection of A?

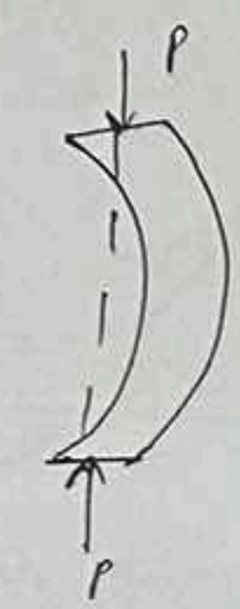


$\sin^2\theta \, d\theta = \frac{P^2 R^3}{4EI}$

$\boxed{1303 - 1310}$

...ergoes compression ... theory suggests, there ... or deflection in ... column). However, ... eccentricity or ... direction column ...

...member regains



its original shape as in Fig 1; On other hand, for larger values of P, column remains deflected as shown in Fig 2. The later phenomenon is known as buckling.

★ Derivation of Euler buckling formula/critical load

Consider a column with pinned ends as shown in Fig-3, assuming it buckled due to compressive load P.

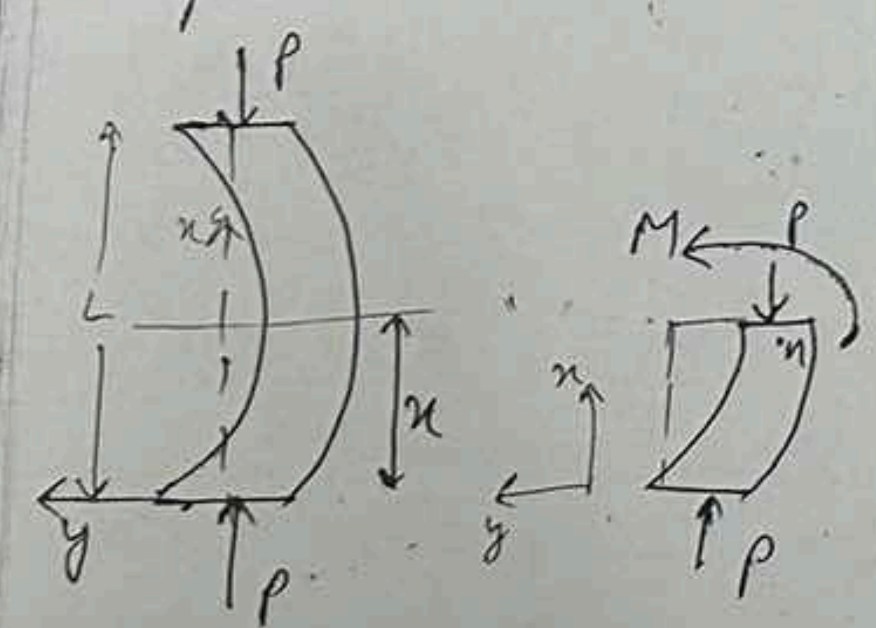
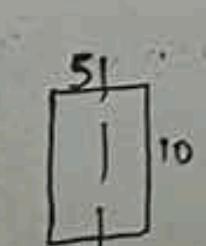


Fig-3



$\Sigma M_n = 0$

$\Rightarrow M - P(-y) = 0$

$M + Py = 0$ —— (i)

Applying flexural formula

$\frac{M}{I} = \frac{E}{\rho} = E\frac{d^2y}{dx^2}$

$\Rightarrow EI_z \frac{d^2y}{dx^2} + Py = 0$

$\frac{d^2y}{dx^2} + \left(\frac{P}{EI_z}\right) y = 0$

$\Rightarrow \frac{d^2y}{dx^2} + k^2 y = 0$

$\Rightarrow y = A \sin kx + B \cos kx$ —— (ii)

BC's are $y|_{x=0} = 0$ (I) & $y|_{x=L} = 0$ (II)

Applying I in (ii) $\Rightarrow B = 0$

" II in (ii) $\Rightarrow A \sin kL = 0$

If $A = 0$ then $y = 0$ & we go to situation in Fig 1. However, we are looking for solution which represents coupling as shown in Fig 2 & 3.

Thus $A \neq 0$ then $kL = n\pi$ $\therefore (n \neq 0)$

i) For $n = 1$, $kL = \pi$

$\sqrt{\frac{P}{EI_z}} \, L = \pi$

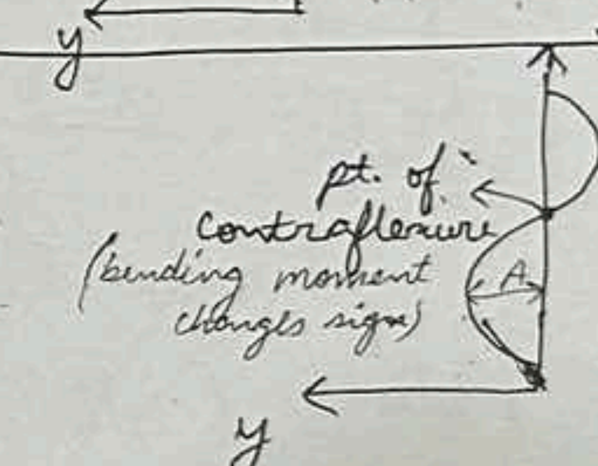
$\boxed{P_{cr} = \frac{\pi^2 EI_z}{L^2}}$ Critical load of buckling

$\therefore y = A \sin\left(\frac{\pi x}{L}\right)$

ii) $n = 2$, $kL = 2\pi$

$P_{cr} = \frac{4\pi^2}{L^2} EI_z$

$y = A \sin\left(\frac{2\pi x}{L}\right)$



$x = 0 \Rightarrow y = 0$ $\quad x = \frac{L}{4} \Rightarrow y = A$

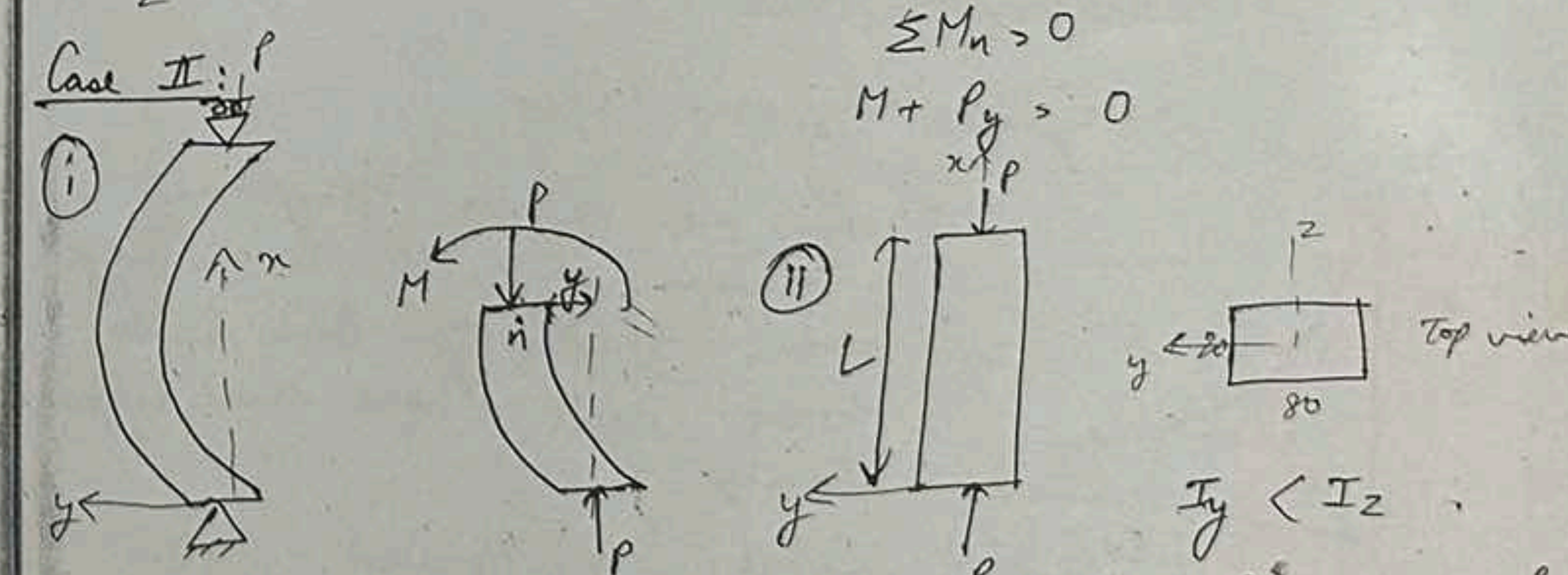
$x = L \Rightarrow y = 0$ $\quad x = \frac{3L}{4} \Rightarrow y = -A$

$x = \frac{L}{2} \Rightarrow y = 0$

Case II:

$\Sigma M_n = 0$

$M + Py = 0$



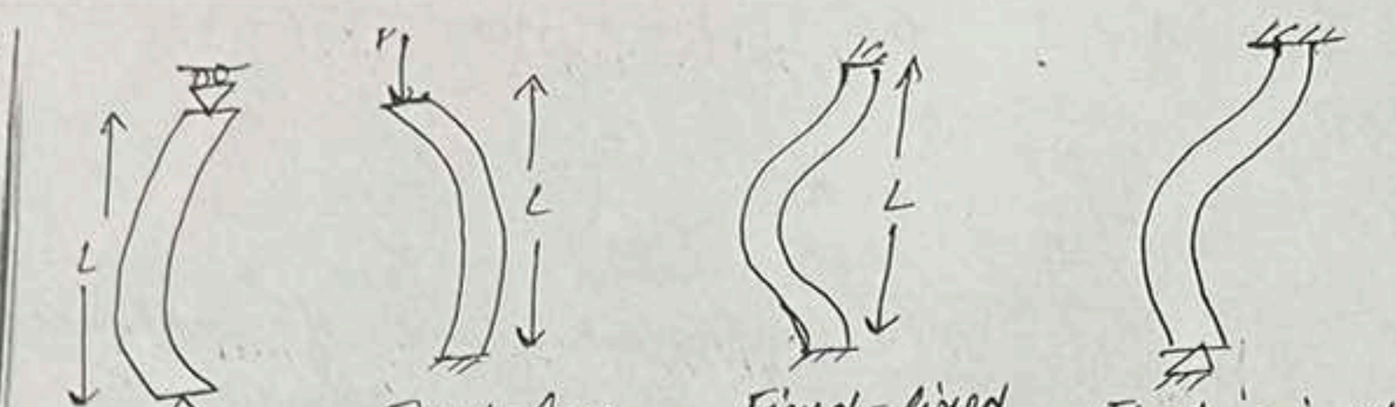
$I_y < I_z$

$\therefore P_{cr} = EI_y \frac{\pi^2}{L^2}$

(Take smaller I)

• General formula for critical load

$P_{cr} = \frac{\pi^2}{L_e^2} EI$ $\quad$ (L_e: equiv length)



Pinned-pinned $L_e = L$ | Fixed-free $L_e = 2L$ | Fixed-fixed $L_e = 0.5L$ | Fixed-pinned $L_e = 0.7L$

• **Limit of applicability of Euler formula**

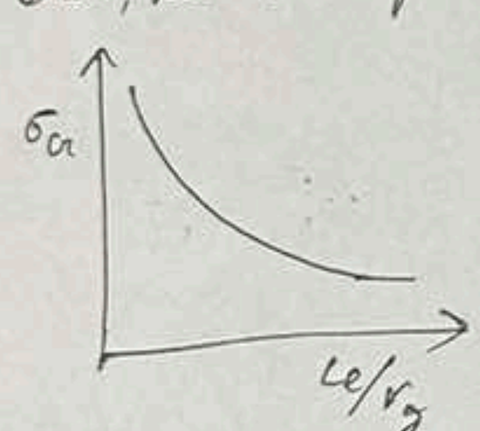
$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E A r_g^2}{L_e^2}$$

A = area of cross-section of column
r_g = radius of gyration

$$\frac{P_{cr}}{A} = \sigma_{cr} = \frac{\pi^2 E}{(L_e/r_g)^2}$$

$\frac{L_e}{r_g}$ = Slenderness ratio

Then, we can plot σ_{cr} vs L_e/r_g



Euler Curve

If we reduce length of column, there'll be a length when $\sigma_{cr} \geq \sigma_y$ then Euler curve will have an upper bound.

Since, we derived Euler formula based on linear elastic theory (Hooke's Law), thus it has upper bound.

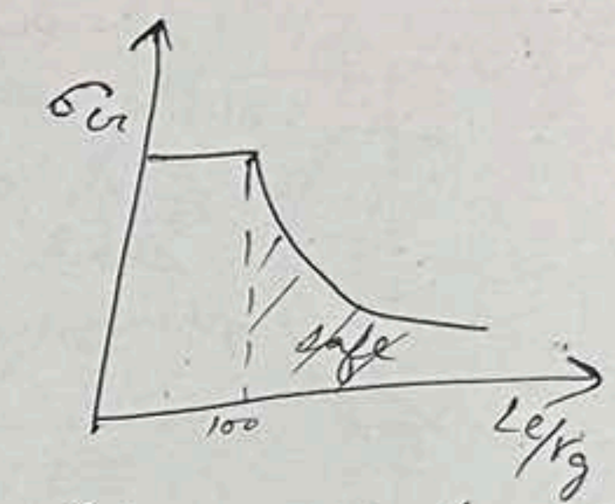
For ex, consider steel with $\sigma_y = 200$ MPa, $E = 200$ GPa

$$\sigma_{cr} = \frac{200 \times 10^3 \pi^2}{(L_e/r_g)^2}$$

If $\sigma_{cr} = \sigma_y$

Then, $\left(\frac{L_e}{r_g}\right)^2 = \frac{200 \times 10^3 \pi^2}{200}$

$\Rightarrow \frac{L_e}{r_g} \approx 100$



Euler curve for steel

• Factor of safety = $\frac{\text{Ultimate load}}{\text{Service load}} = \frac{(\sigma_{cr})_{ultimate/theory}}{(\sigma_{cr})_{service}}$ (1104)

(1102) $A = 50 \times 100 = 5000$ mm²
$E = 10000$ MPa
$\sigma_y = 10$ MPa

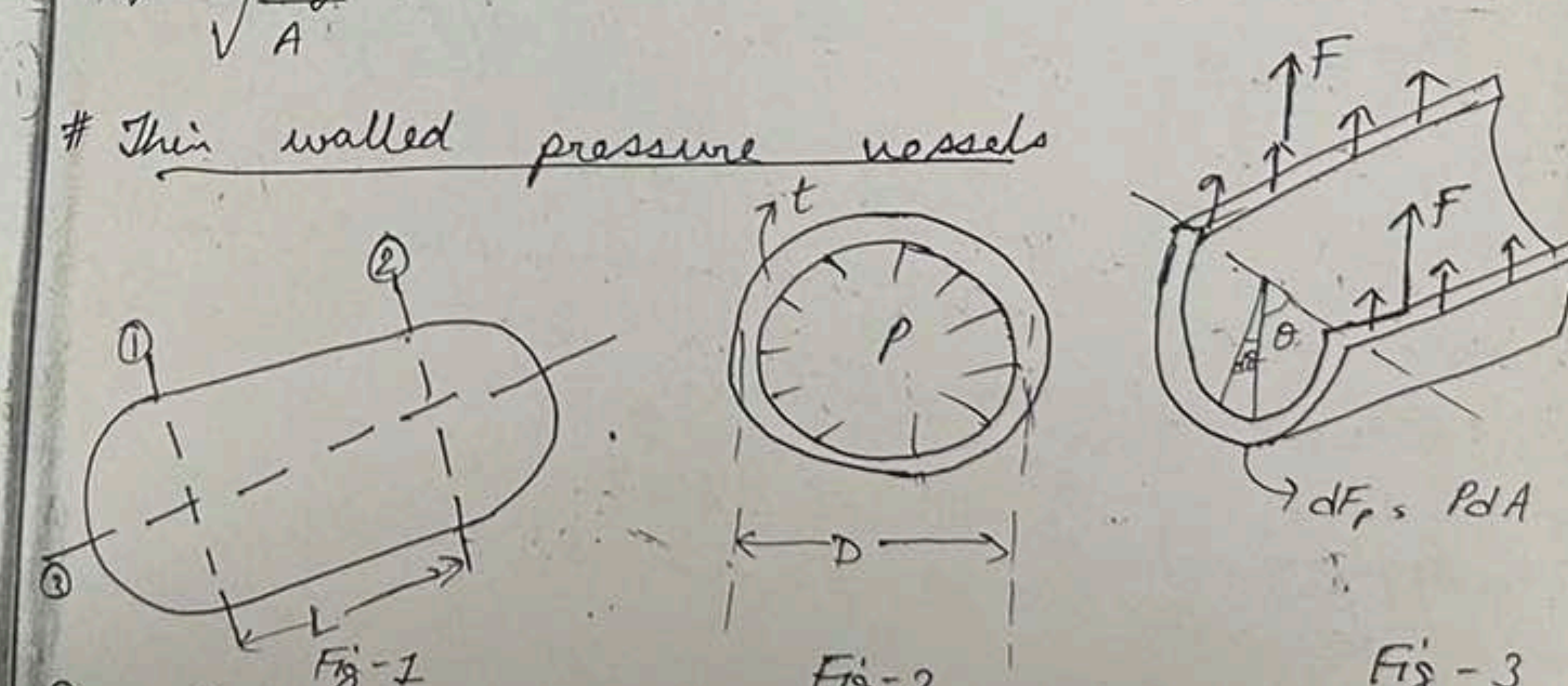
$\frac{L_e}{r_g} = \pi\sqrt{\frac{E}{\sigma_y}}$

$I_y = \frac{100 \times 50^3}{12}$ $\quad I_x = \frac{50 \times 100^3}{12}$

Using I_y $\because I_y < I_x$

$r_g = \sqrt{\frac{I_y}{A}}$

Thin walled pressure vessels



Due to press inside cylinder,
$dF_r = PdA$ at angular position θ
$= P L \frac{D}{2} d\theta$

This dF_r has 2 com[...]
Horiz component wil[...]
Vertical "
cylinder material as
$t < \frac{D}{20}$

$F_{PV} =$

F_{P}[...]

$\Sigma F_y = 0$
$\Rightarrow 2F = F_{PV}$
$F = \frac{PDL}{2}$

Normal stress [...]

$\sigma_t = \frac{F}{Lt}$

$\boxed{\sigma_t = \frac{PD}{2t}}$ Tange[...]

σ_t is known as [...]
girth stress.

From eq (i), we lear[...]
projected area wi[...]
due to press ⊥ to [...]

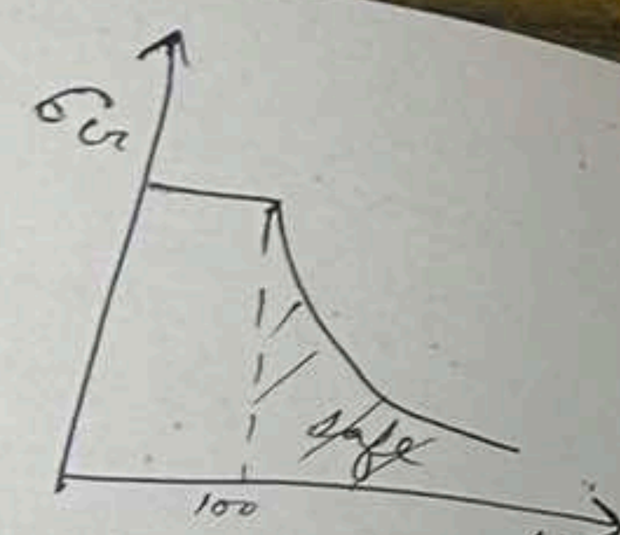
$F_{PL} = P\frac{\pi D^2}{4}$

$\sigma_L = \frac{F_{PL}}{\pi D t}$

$\boxed{\sigma_L = \frac{PD}{4t}}$ Longitudi[...] stre[...]

σ_y

$= \frac{200 \times 10^3 \pi^2}{200}$



$= 100$

ty = Ultimate ~~Load~~ / Service load = $\frac{(\sigma_{cr})_{ultimate/theory}}{(\sigma_{cr})_{service}}$

Euler curve for steel

00 = 5000 mm^2

MPa

1104

$I_x = \frac{50 \times 100^3}{12}$

$I_y < I_x$

ssure vessels

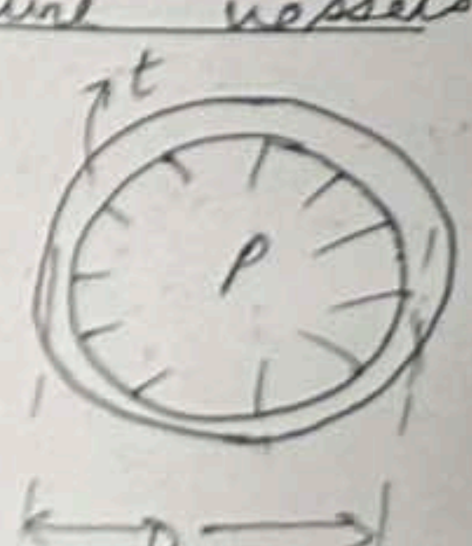


Fig-2

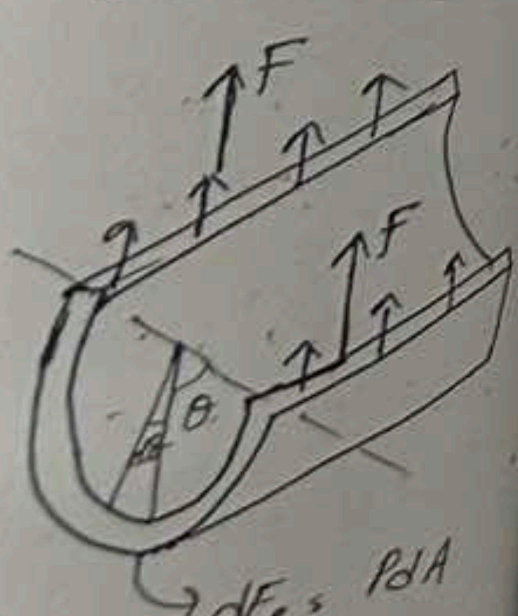


Fig-3

cylinder,

at angular position θ

dθ

This dF_p has 2 components (i) Horiz (ii) Vertical

Horiz component will vanish by opp signs.

Vertical " " cause normal stress in cylinder material as shown in fig-3

$t < \frac{D}{20}$

$F_{PV} = \int dF_p \sin\theta$

$= \int_0^{\pi} \frac{PDL}{2} \sin\theta \, d\theta$

$\boxed{F_{PV} = PDL}$ —(i)

$\Sigma F_y = 0$

$\Rightarrow 2F = F_{PV}$

$F = \frac{PDL}{2}$

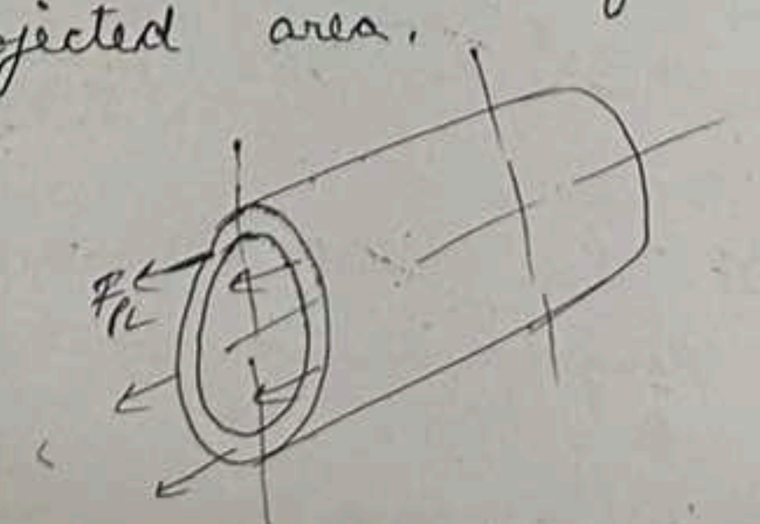
Normal stress in cylinder material

$\sigma_t = \frac{F}{Lt}$

$\boxed{\sigma_t = \frac{PD}{2t}}$ Tangential stress (ii)

σ_t is known as tangential or circumferential or girth stress.

From eq (i), we learn that if we multiply projected area with pressure, we get force due to press ⊥ to projected area.



$F_{PL} = \frac{P\pi D^2}{4}$

$\sigma_L = \frac{F_{PL}}{\pi D t}$

$\boxed{\sigma_L = \frac{PD}{4t}}$ Longitudinal stress

$\therefore \boxed{\sigma_L = \frac{\sigma_t}{2}}$

~~135, 137, 141, 140~~ $\boxed{133-142}$

141) $P = 125$ psi $\quad L = 2$ ft $\quad t = \frac{1}{8}$ in

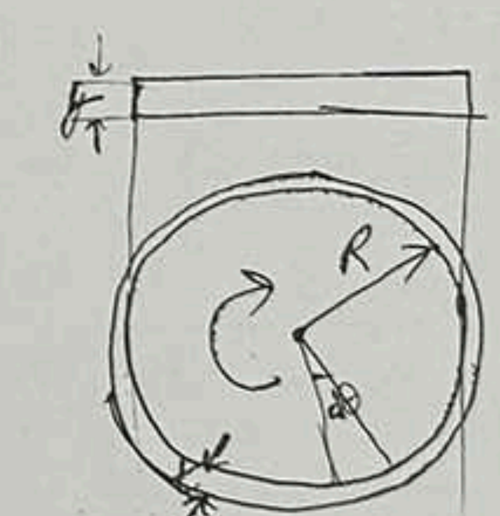
$D = 1.5$ ft

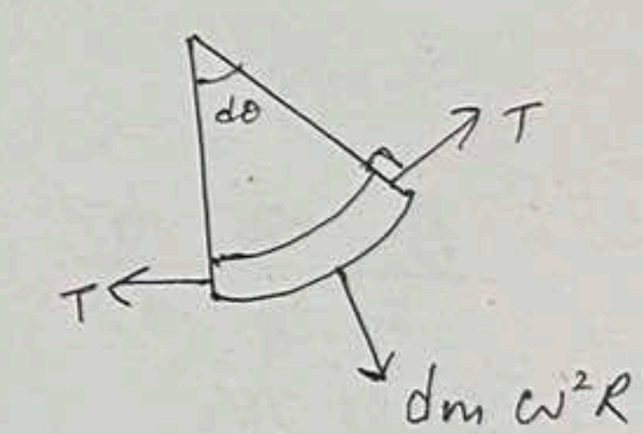
$\sigma_t = \frac{PD}{2t}$

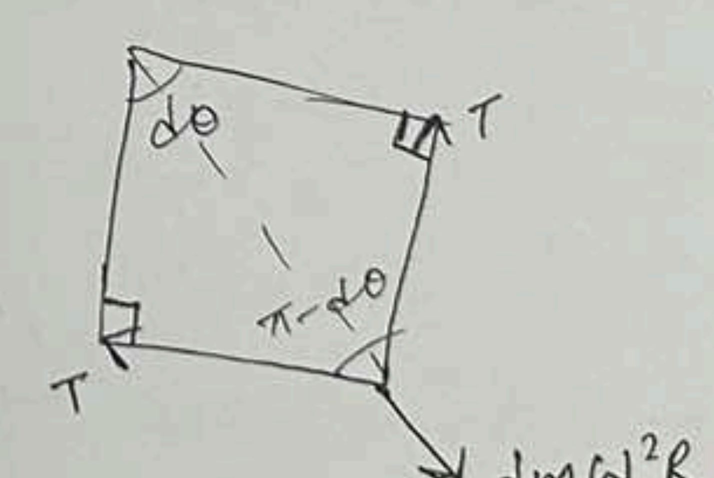
$= \frac{125 \times 1.5 \times 12 \times 8}{2 \times 1} = 9000$ psi $= 9$ ksi

$\sigma_L = \frac{PD}{4t} = \frac{125 \times 1.5 \times 12}{4 \times \frac{1}{8}} = 4500$ psi

139)







$2T \sin\frac{d\theta}{2} = dm\,\omega^2 R$

$\because \theta$ is very small

$\Rightarrow \boxed{2T \frac{d\theta}{2} = dm\,\omega^2 R}$

$dm = \rho t L R d\theta$

$T = \rho L t \omega^2 R^2$

$\boxed{\sigma_t = \rho \omega^2 R^2}$

134) $\sigma_L = 8000$ psi $\quad D = 48$ in

$t = \frac{5}{16}$ in

$8000 = \frac{P_{max} \times 48 \times 16}{(2 \times 2) \times 5}$

$\Rightarrow P_{max} = \frac{5000 \times 2}{48} = 208$ psi

135) $P = 1400$ psi
$D = 24$ in
$\sigma_L = 12000$ psi

$12000 = \frac{1400 \times 24}{4t}$ $\Rightarrow t = 0.7$ in

136) $t = 20 \times 10^{-3}$ $\sigma_L = 140 \times 10^6$
$D = 450 \times 10^{-3}$ $\sigma_t = 60 \times 10^6$
$L = 2m$

$140 \times 10^6 = \frac{P_1 \times 450 \times 10^{-3}}{4 \times 20 \times 10^{-3}}$

$\Rightarrow P = 533$ MPa

& $60 \times 10^6 = \frac{P_2 \times 450 \times 10^{-3}}{2 \times 20 \times 10^{-3}}$

$\Rightarrow P_2 = 5.33$ MPa

137) $D = 22 \times 12$ $t = 0.5$ in $\sigma_t = 6000$
$\rho g = \frac{62.4}{12^3}$ lb/in³

$6000 = \frac{P \times 254}{2 \times 0.5}$

$\Rightarrow P = \frac{6000}{254} = \rho g h$

$\Rightarrow h = \frac{6000 \times 12^3}{254 \times 62.4} = 52.4$ ft

138) $\sigma_{Lt} = \frac{33000}{12}$ $\sigma_{tt} = \frac{16000}{12}$ $P = 150$

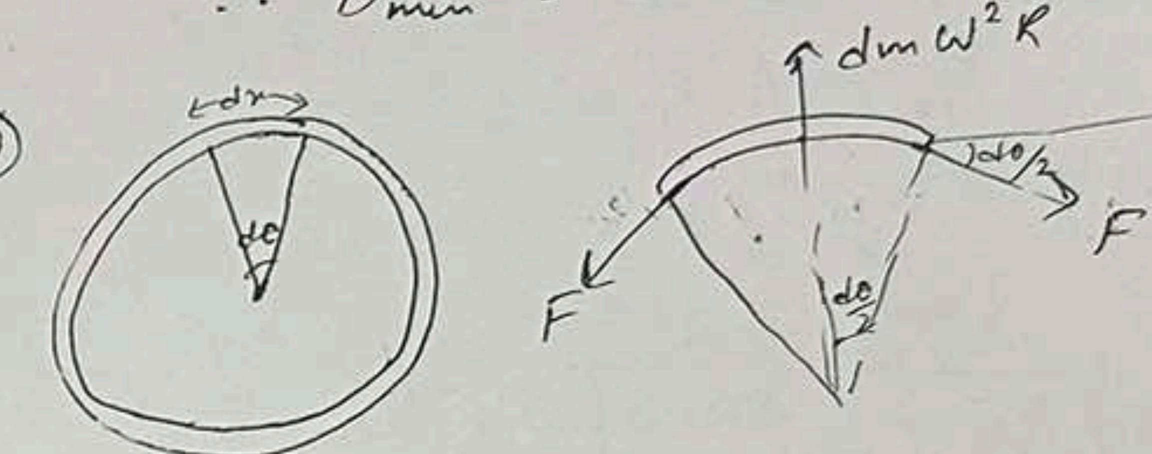
Strength $= \sigma t$

Longitudinal jt will fail by tangential stress & vice versa

$\frac{33000}{12t} = \frac{150 D_1}{12t}$ $\Rightarrow D_1 = 36.67$ in

$\frac{4000}{3t} = \frac{150 D_2}{4t}$ $\Rightarrow D_2 = 35.56$ in

$\therefore D_{min} = 35.56$ in

139) 

$2F \frac{d\theta}{2} = dm\,\omega^2 R$

$\therefore dm = \rho (R\,d\theta)\,Lt$

$\Rightarrow F\,d\theta = \rho R\,d\theta\,Lt\,\omega^2 R$

$\Rightarrow \frac{F}{Lt} = \sigma_t = \rho \omega^2 R^2$

$\sigma_t = 20000$ $\rho = \frac{490}{12^3}$ lb/in³
$R = 10$ in

$\Rightarrow 20000 = \frac{490}{12 \times 12 \times 12} \times \omega^2 \times 100$

$\Rightarrow \omega = \frac{12}{7}\sqrt{240} = 26.56$ rad/s

140) $150 \times 10^6 = 7.85 \times 10^3\,\omega^2 \times 22^2 \times 100 \times 10^{-6}$

$\Rightarrow \omega^2 = \frac{10^4}{22}\sqrt{\frac{15}{7.85}} = 628.33$ rad/s

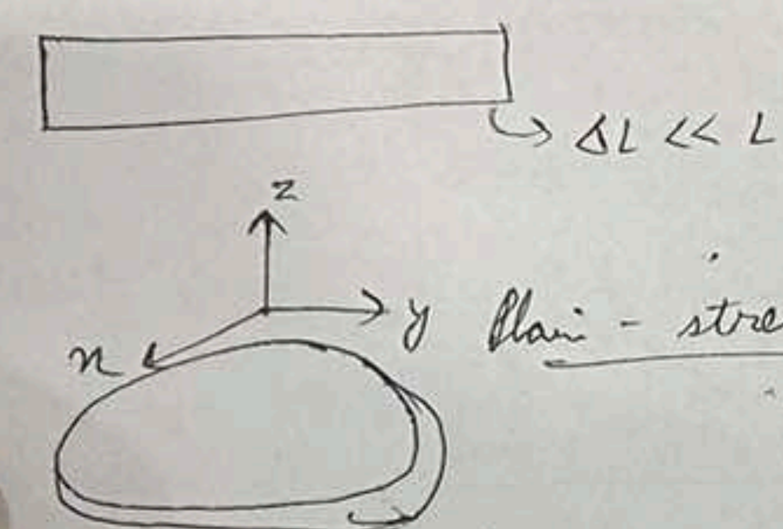
1104) $P = 20000$
Hinged-hinged $\Rightarrow L_e = L = 10$ ft $= 120$ in
$E = 29 \times 10^6$

$P = \frac{\pi^2}{L_e^2} EI$

$20000 = \frac{\pi^2}{120 \times 120} \times 29 \times 10^6 \times \frac{x^4}{12}$ $\Rightarrow x = 1.86$ in

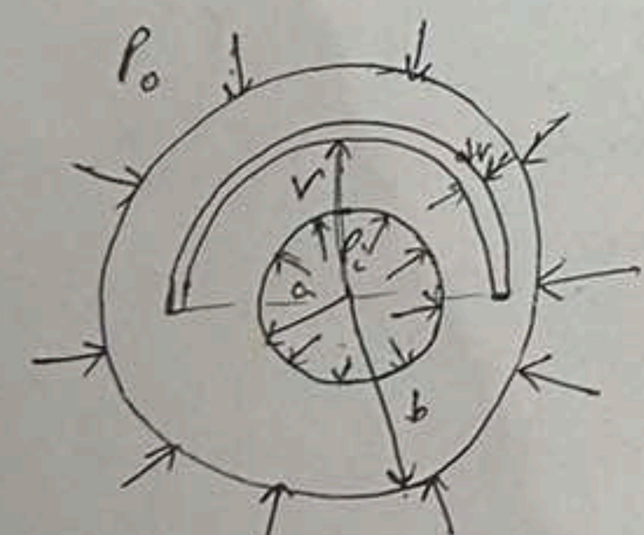
Plane-strain problem

$\epsilon_x = \frac{\Delta L}{L} \approx 0$



Plain-stress problem

Thick walled cylinder (Pg 490)



Consider a long thick cylinder of internal radius 'a' & outer radius 'b', subjected to internal press 'P_i' & out press 'P_o'.

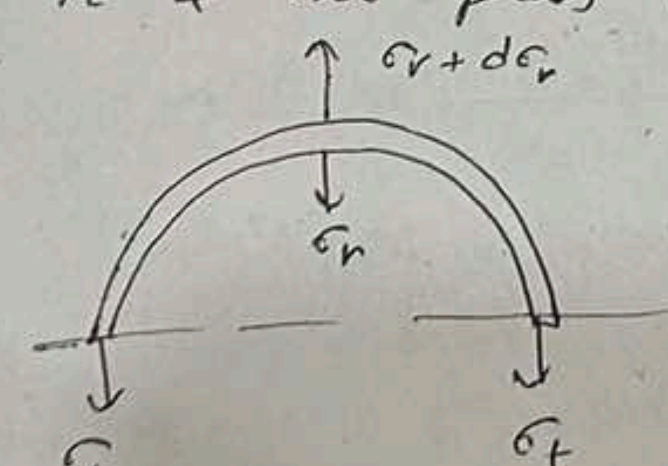


Fig 13-40

Q/ Why V is not here?
Ans/ Because of symmetry

$$(\sigma_r + d\sigma_r)(2(r+dr)) - \sigma_r \cdot 2r - \sigma_t \cdot 2dr = 0 \quad (\cancel{d\sigma_r\,dr} \approx 0)$$

$$r\,d\sigma_r + (\sigma_r - \sigma_t)\,dr = 0$$

$$\Rightarrow r\frac{d\sigma_r}{dr} + (\sigma_r - \sigma_t) = 0 \quad \text{——— (i)}$$

From generalised Hooke's law, strain in axial direction: $\epsilon_a = \frac{1}{E}[\sigma_a - \nu(\sigma_r + \sigma_t)]$

Since, we assumed plane strain problem i.e.: $\epsilon_a = 0$.

$\Rightarrow \sigma_a = \nu(\sigma_r + \sigma_t)$

$\sigma_a \neq f(r) \quad \therefore \sigma_a$ is const

Say, $\sigma_r + \sigma_t = 2A$

Substituting in (i) $\Rightarrow$

$$r\frac{d\sigma_r}{dr} + \sigma_r - 2A + \sigma_r = 0$$

$$\Rightarrow \int \frac{d\sigma_r}{A - \sigma_r} = \int 2\frac{dr}{r}$$

$\Rightarrow -\ln(A - \sigma_r) = 2\ln r + C$

$\Rightarrow \ln[r^2(A - \sigma_r)] = -C$

$\Rightarrow r^2(A - \sigma_r) = e^{-C} = B$

$\Rightarrow \boxed{\sigma_r = A - \frac{B}{r^2}}$

$\therefore \boxed{\sigma_t = 2A - \sigma_r = A + \frac{B}{r^2}}$

At $r = a$, $\sigma_r = -P_i$; $r = b$, $\sigma_r = -P_o$ Find A & B.

$\rightarrow$ $\cancel{\sigma_r = +}\; -P_i = A - \frac{B}{a^2}$

$-P_o = A - \frac{B}{b^2}$

$P_o - P_i = B\left(\frac{1}{b^2} - \frac{1}{a^2}\right)$

$\therefore A = \frac{B}{a^2} - P_i = \frac{P_o - P_i}{a^2} \times \left(\frac{a^2 b^2}{\cancel{a^2} - b^2}\right) - P_i$

$= \frac{P_o b^2 - \cancel{P_i b^2} - P_i a^2 + \cancel{P_i b^2}}{a^2 - b^2}$

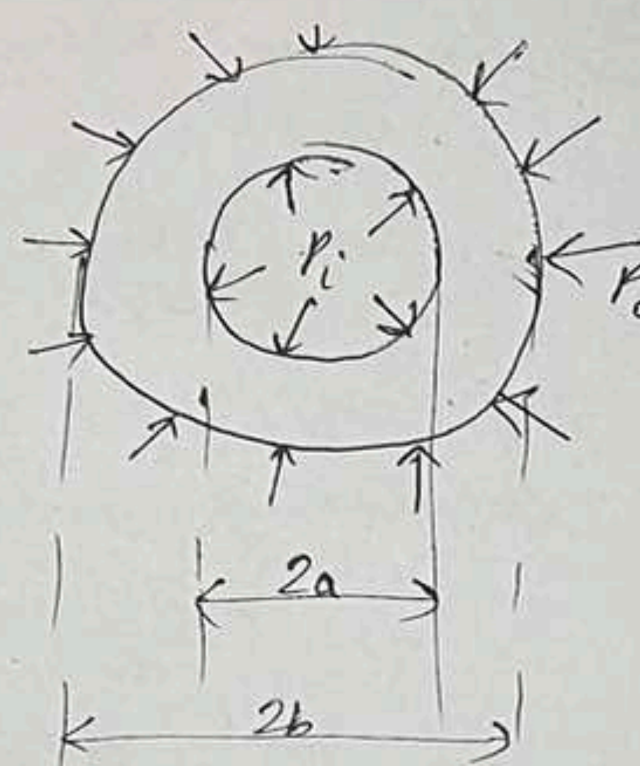
12/11/25

$\sigma_r = A - \frac{B}{r^2}$

$\sigma_t = A + \frac{B}{r^2}$

$A = \frac{P_i a^2 - P_o b^2}{b^2 - a^2}$

$B = \frac{a^2 b^2 (P_i - P_o)}{b^2 - a^2}$



Q) Determine & plot stress-distribution in a cylinder, $d_i = 50$ mm, $d_o = 150$ mm, $P_i = 35$ MPa, $P_o = 0$

$\rightarrow a = \frac{50}{2} = 25$ mm

$b = \frac{150}{2} = 75$ mm

$A = \frac{35 \times 25^2}{75^2 - 25^2} = 4.375$

$B = \frac{35 \times 25^2 \times 25^2}{75^2 - 25^2} = 24609.35$

$\sigma_t|_{r=25} = 4.375 + \frac{24609.35}{25^2} = 43.75$ MPa

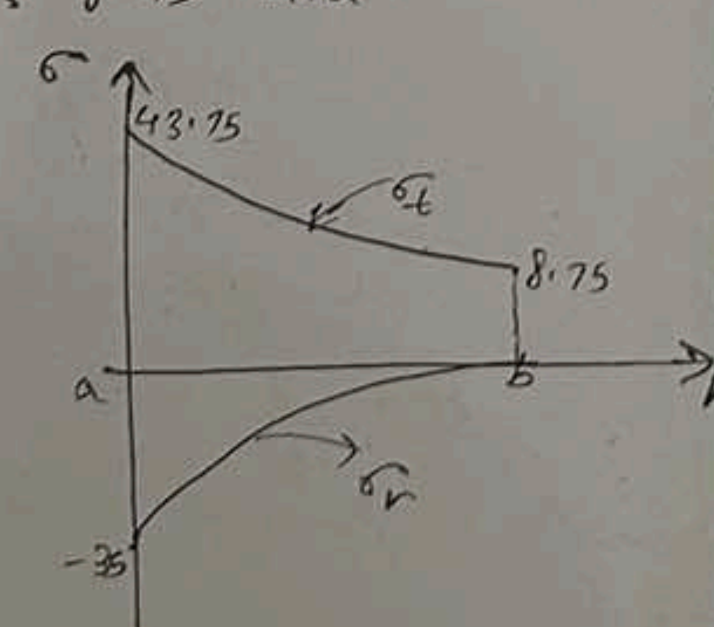
$\sigma_t|_{r=75} = 4.375 + \frac{24609.35}{75^2} = 8.75$ MPa

$\therefore \sigma_t = 4.375 + \frac{24609.35}{r^2}$

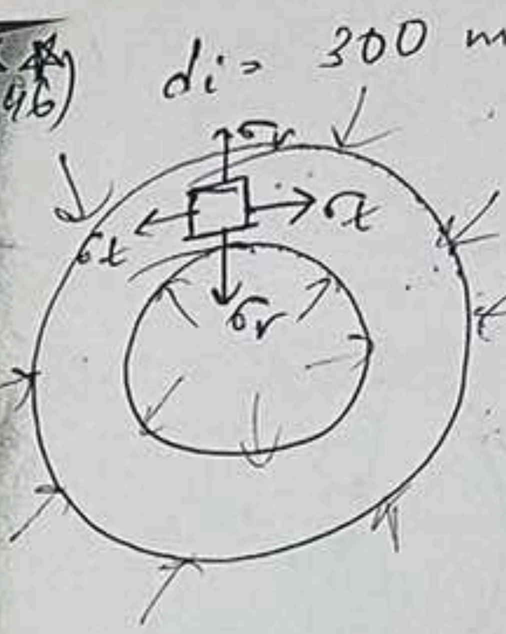
$\sigma_r = 4.375 - \frac{24609.35}{r^2}$

$\sigma_r|_{r=a} = -P_i = -35$

$\sigma_r|_{r=b} = P_o = 0$



(1346) $d_i = 300$ mm, $P_i = 60$ MPa, $P_o = 0$



~~Shear stress~~ $= \frac{\sigma_t - \sigma_r}{2} = \tau_{max}$

Thickness $= b - a$

$\sigma_t = A + \frac{B}{b^2}$

$B = \frac{150^2 \times b^2 \times 60}{b^2 - 150^2}$

$\frac{A + \frac{B}{r^2} - A + \frac{B}{r^2}}{2} = 90$ MPa $= \frac{B}{r^2}$

τ_{max} will have biggest value at $r = a$

$\Rightarrow 90 = \frac{B}{a^2}$

$\Rightarrow \frac{P_i a^2 b^2}{b^2 - a^2} = 90 a^2$

$\Rightarrow 60 b^2 = 90(b^2 - a^2)$

$\Rightarrow b = 259.8$

$\therefore$ thickness $= 109.8$ mm

(1347) $k = \frac{(\sigma_t)_{max} - (\sigma_t)_{avg}}{(\sigma_t)_{avg}} \times 100 = f\left(\frac{t}{a}\right)$

$(\sigma_t)_{max} = A + \frac{B}{a^2}$

$(\sigma_t)_{avg} = \frac{1}{b-a} \int_a^b \left(A + \frac{B}{r^2}\right) dr$

$= A - \frac{B}{b-a}\left(\frac{1}{b} - \frac{1}{a}\right)$

$= A + \frac{B}{ab}$

$\therefore k = \frac{\frac{B}{a^2} - \frac{B}{ab}}{A + B/ab}$

$k = \frac{\left(\frac{b^2}{b^2 - a^2} - \frac{ab}{b^2 -}\right.}{\left(\frac{P_i a^2}{b^2 - a^2} + \frac{ab}{b^2 -}\right.}$

(1102) $L = 6$ ft

$I_A = \frac{bh^3}{12} = \frac{0.75 \times 2^3}{12}$

$I_B = \frac{2 \times (0.75)^3}{12}$

For $k = 1$, $L_e = 1.7$ ft

Fixed end $\Rightarrow k =$

Euler formula $=$

$P_{cr,A} = 9.8696 \times$

$P_{cr,B} = 9.8696 \times$

$P_{cr,min} = P_{cr,B} =$

$P_{allow} = \frac{P_{cr,min}}{2}$

(1104) $\therefore$ ~~for~~ $P = 20$

$L = 10$ ft

$L_e = 120$ in

$I = \frac{a^4}{12}$

$P = \frac{\pi^2 EI}{L_e^2} \Rightarrow a^4$

$\Rightarrow a^4 =$

mm $\quad P_i = 60$ MPa $\quad , P_o = 0$

~~Shear stress~~ $= \dfrac{\sigma_t - \sigma_r}{2} = \tau_{max}$

Thickness $= b - a$

$\sigma_t = A$

$B =$

r^2

$\dfrac{B}{r^2}\Big) dr$

$-a\left(\dfrac{1}{b} - \dfrac{1}{a}\right)$

$\dfrac{B}{ab}$

$$k = \frac{\left(\dfrac{b^2}{b^2-a^2} - \dfrac{ab}{b^2-a^2}\right)P_i}{\left(\dfrac{a^2}{b^2-a^2} + \dfrac{ab}{b^2-a^2}\right)P_i} = \frac{b^2-ab}{a^2+ab} = \frac{(a+t)t}{a(a+a+t)} \quad (\because b - a \ldots)$$

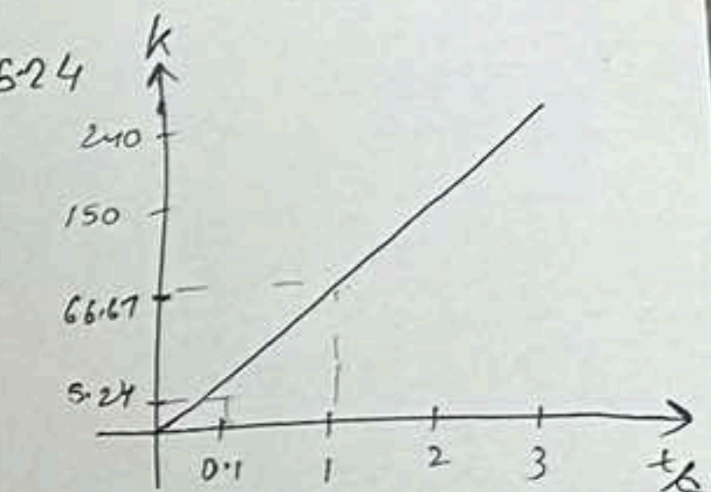
$$= \frac{a\left(1+\frac{t}{a}\right)\frac{t}{a}}{a\left(2+\frac{t}{a}\right)} \times 100$$

At $\dfrac{t}{a} = 0.1 \Rightarrow k = 5.24$

At $t/a = 1 \Rightarrow k = 66.67$

$2 \Rightarrow k = 150$

$3 \Rightarrow 240$



$\therefore (\sigma_t)_{avg} = A + \dfrac{B}{ab}$

$= \dfrac{P_i a^2}{b^2-a^2} + \dfrac{a^2b^2P_i}{(b^2-a^2)ab}$

$= \dfrac{P_i a(a+b)}{(a-b)(a+b)} = \dfrac{P_i a}{a-b} = \dfrac{P_i D_i}{2t} \quad (D_i = 2a)$

1102) $L = 6$ ft $= 72$ in $\qquad h = 2$ in

$I_A = \dfrac{bh^3}{12} = \dfrac{0.75 \times 2^3}{12} = 0.5$ in^4

$I_B = \dfrac{2 \times (0.75)^3}{12} = \dfrac{2 \times 0.421875}{12} = 0.0703125$ in^4

For $k = 1$, $L_e = 1.72$ ft

Fixed end $\Rightarrow k = 0.5$, $L_e = 0.5 \times 72 = 36$ in

Euler formula $= \dfrac{\pi^2 E I}{L_e^2}$

$P_{cr,A} = \dfrac{9.8696 \times (10.3 \times 10^6 \times 1/2)}{72^2} = 9.8 \times 10^3$ lb

$P_{cr,B} = \dfrac{9.8696 \times 10.3 \times 10^6 \times 0.07}{36^2} = 5.52 \times 10^3$ lb

$P_{cr,min} = P_{cr,B} = 5520$ lb $\quad$ for $n = 2$

$P_{allow} = \dfrac{P_{cr,min}}{2} = 2758$ lb $\quad$ ——— Ans

1104) $\therefore$ ~~P_{cr}~~ $P = 20$ kpsi $= 20000$ lb

$L = 10$ ft $\qquad E = 29 \times 10^6$ psi

$L_e = 120$ in

$I = \dfrac{a^4}{12}$

$P = \dfrac{\pi^2 E I}{L_e^2} \Rightarrow a^4 = \dfrac{12 L^2 P}{\pi^2 E}$

$\Rightarrow a^4 = \dfrac{3.456 \times 10^9}{2.86 \times 10^8} = 12.075 \Rightarrow a = 1.86$ in