

Birla Institute of Technology Mesra, Ranchi

Course Materials

Subject Code: ME24201

Subject: Basic of Mechanical Engineering

Prepared by:

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COURSE INFORMATION SHEET

Course Code: ME24101
Course Title: Basics of Mechanical Engineering
Pre-requisite(s): NIL
Co- requisite(s): NIL
Credits: 3 (L: 2 T:1 P: 0)
Class schedule per week: 3
Class: B. Tech
Semester / Level: SECOND
Branch: Mechanical Engineering
Name of Teacher:

COURSE OBJECTIVES

This course envisions to impart to students to:

1.	Introduce system of forces, and write equation of equilibrium.
2.	Analyse motion of particle and rigid body subjected to force.
3.	Grasp the importance of internal and external combustion engines.
4.	Apprehend the fundamentals of friction.
5.	Understand the different sources of energy.

COURSE OUTCOMES (COs)

After the completion of this course, students will be able to:

CO1	Explain the basics of Mechanical Engineering.
CO2	Apply various laws of mechanics on static and dynamic elements and bodies.
CO3	Analyse various problems of mechanics related to static and dynamic bodies.
CO4	Evaluate the real life problem related to mechanics and energy for its probable solution.

SYLLABUS

MODULE	(NO. OF LECTURE HOURS)
Module – I System of Forces and Structure Mechanics; Addition of Forces, Moment of a Force, Couple, Varignon's theorem, Free Body Diagram, Equilibrium in Two and Three Dimensions, Equivalent Forces and Moment. Types of Plane Trusses, Analysis of Plane Trusses by: Method of Joints and Method of Sections. Hooke's Law of elasticity, Stress and Strain, Relation between elastic constants.	8
Module – II Kinematics & Kinetics of rigid bodies: Types of rigid body motion– translation, rotation about fixed axis, equations defining the rotation of a rigid body about a fixed axis, plane motion, absolute and relative velocity in plane motion, instantaneous center of rotation. Equation of motion and D'Alembert's principle.	8
Module – III Friction : Interfacial Friction (a) Laws of dry friction, static & kinetic co-efficient of friction, Analysis of static, kinetic and rolling friction. (b) Analysis of frictional forces in inclined planes, wedges, screw jacks and belt drives.	8
Module – IV Boilers and Internal Combustion Engine; Classification of Boilers, Fire tube and Water Tube boilers. Boiler Mountings and Accessories. Boiler efficiency. Classification of I C Engines. Basic components and terminology of IC engines, working principle of four stroke and two stroke - petrol and diesel engine.	6
Module – V Non-Conventional Energy Sources Renewable and Non-renewable Energy Resources, Advantages and Disadvantages of Renewable Resources, Renewable Energy Forms and Conversion- Solar Energy, Wind Energy, Hydro Energy.	5

TEXTBOOKS:

1. Engineering Mechanics, Irving H. Shames, P H I. ltd, 2011.
2. Boiler operator, Wayne Smith, LSA Publishers, 2013.
3. Internal Combustion Engines, M. L. Sharma and R. P. Mathur, Dhanpat Rai Publications, 2014.
Fundamentals of Renewable Energy Processes, Aldo Vieira Da Rosa, Elsevier publication, 2012.

REFERENCE BOOKS:

1. Engineering Mechanics : statics, James L. Meriam, L. G. Kraige, Wiley, 7th Edition, 2011.
2. Engineering Mechanics, S. Rajasekaran & G. Sankarasubramaniam, Vikash publishing house, 2018.
3. An Introduction to Steam Boilers, David Allan Low, Copper Press Publisher, 2012.
4. Internal Combustion Engines – V Ganesan, McGraw hill, 2017.
5. Non Conventional Energy Resources, B. H. Khan, McGraw Hill Education Publisher, 2017.
6. Principles of Mechanical Engineering, R. P. Sharma & Chilkesh Ranjan, Global Academic Publishers, 2016.

GAPS IN THE SYLLABUS (TO MEET INDUSTRY/PROFESSION REQUIREMENTS) : NIL

POS MET THROUGH GAPS IN THE SYLLABUS: NA

TOPICS BEYOND SYLLABUS/ADVANCED TOPICS/DESIGN: NIL

POS MET THROUGH TOPICS BEYOND SYLLABUS/ADVANCED TOPICS/DESIGN: NA

COURSE OUTCOME (CO) ATTAINMENT ASSESSMENT TOOLS & EVALUATION PROCEDURE

DIRECT ASSESSMENT

Assessment Tool	% Contribution during CO Assessment
Progressive Evaluation	50
End Semester Examination	50

Continuous Internal Assessment	% Distribution
Mid Semester Examination	25
Quiz, Assignment	10 + 10
Teacher's Assessment	5

Assessment Components	CO1	CO2	CO3	CO4
Continuous Internal Assessment	√	√	√	√
Semester End Examination	√	√	√	√

INDIRECT ASSESSMENT

4. Student Feedback on Course Outcome

COURSE DELIVERY METHODS

CD1	Lecture by use of boards/LCD projectors/OHP projectors	√
CD2	Assignments/Seminars	√
CD3	Laboratory experiments/teaching aids	
CD4	Industrial/guest lectures	
CD5	Industrial visits/in-plant training	
CD6	Self- learning such as use of NPTEL materials and internets	√
CD7	Simulation	

MAPPING BETWEEN COURSE OUTCOMES AND POs and PSOs

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO1 0	PO1 1	PO1 2	PSO 1	PSO 2	PSO 3
CO1	3	2	1	1	1		1	2	1	1		2	NA	NA	NA
CO2	3	3	2	2	2	1	1	1	2	1	1	2	NA	NA	NA
CO3	3	3	3	3	2	1	1	1	2	2	2	2	NA	NA	NA
CO4	2	3	3	3	3	2	2	2	2	2	2	3	NA	NA	NA

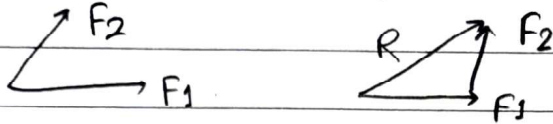
Grading: No correlation – 0, Low correlation - 1, Moderate correlation – 2, High Correlation - 3

MAPPING BETWEEN COURSE OUTCOMES AND COURSE DELIVERY METHOD

Course Outcomes	Course Delivery Method
CO1	CD1, CD2, CD 6
CO2	CD1, CD2, CD 6
CO3	CD1, CD2, CD 6
CO4	CD1, CD2, CD 6



TRIANGLE LAW OF FORCES → if two forces F_1 & F_2 acting simultaneous on a particle, can be represented by two sides of triangle in (magnitude & direction) taken in order. Then the third side (closing side) represent the resultant force in opposite direction.



Q. a simple sling shot about to be 'fired'. If the entire rubberband requires 8 lb/inch elongation. what force does the band exert on hand? Total unstretched length of rubber band is 5.



$$l_e = 2\sqrt{8^2 + 15^2} = 16.28$$

$$l_0 = 5\sqrt{8^2 + 15^2}$$

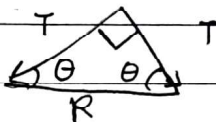
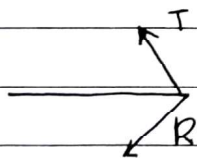
$$\Delta l = 16.28 - 5$$

$$l_0 = 5$$

$$= 11.28$$

$$\Delta l = 16.28 - 5 = 11.28$$

$$\text{Tension} = \Delta l \times 8 = 33.416$$



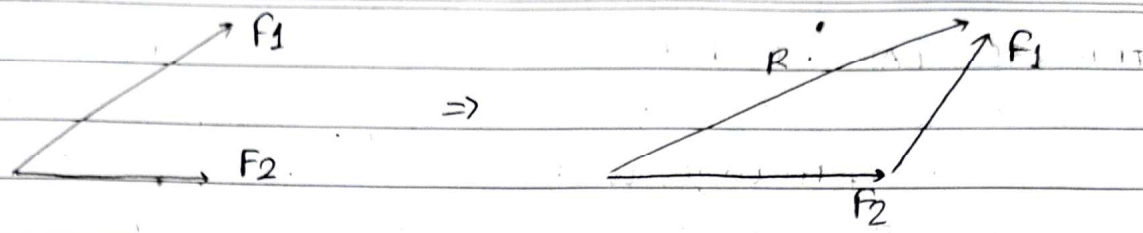
$$\tan \theta = \frac{1.5}{8}$$

$$\theta = 10.62^\circ$$

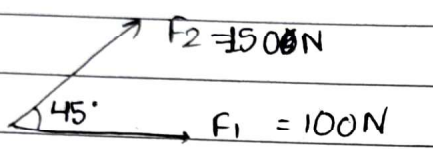
$$\phi = 180 - 2 \times 10.62 = 158.76^\circ$$

$$\frac{T}{\sin \theta} = \frac{R}{\sin (158.76^\circ)}$$

$$R = 66.5 \text{ lb}$$



A particle is subjected to forces F_1 and F_2 , find the resultant.



$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_1 = F_1 \cos \theta_1 \hat{i} + F_1 \sin \theta_1 \hat{j}$$

$$\vec{F}_2 = F_2 \cos \theta_2 \hat{i} + F_2 \sin \theta_2 \hat{j}$$

$$\vec{F}_1 = \frac{100}{\sqrt{2}} \hat{i} + \frac{100}{\sqrt{2}} \hat{j}$$

$$= 70.71 (\hat{i} + \hat{j})$$

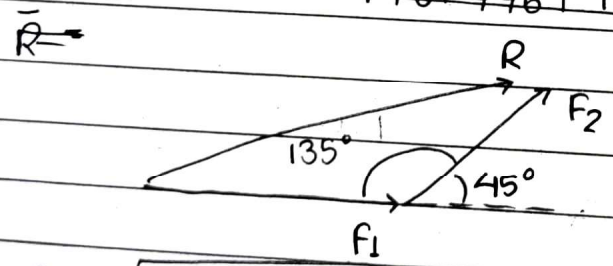
$$\vec{F}_2 = \frac{1500}{\sqrt{2}} \hat{i} + \frac{1500}{\sqrt{2}} \hat{j}$$

$$= 1060.66 (\hat{i} + \hat{j})$$

$$= 106.066 \hat{i} + 106.066 \hat{j}$$

$$\vec{R} = 364.26 \hat{i} + 364.26 \hat{j}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = 176.776 \hat{i} + 176.776 \hat{j}$$



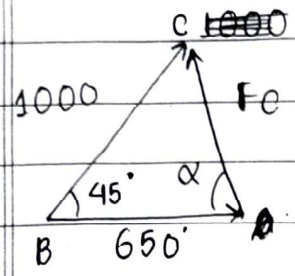
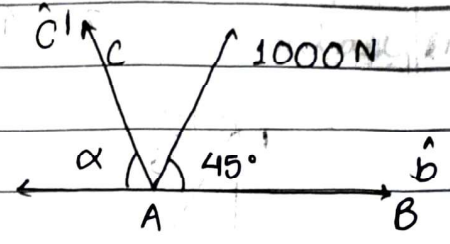
$$R = \sqrt{F_1^2 + F_2^2 - 2 F_1 F_2 \cos 135^\circ}$$

$$R = \sqrt{53713.203} = 231.76 \text{ N}$$

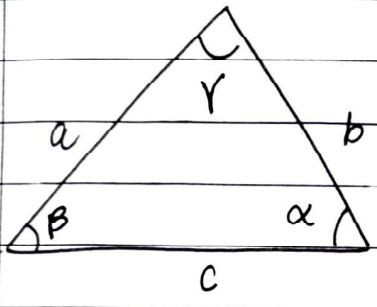
What if the vectors are not orthogonal? In other words, what if a force needs to be resolved on non-orthogonal vectors?

Q: Resolve 1000N force about AB and AC. The component about AB is 650N. also find α .

$$\vec{F} = F_B \hat{b} + F_C \hat{c}$$



$$\frac{1000}{\sin \alpha} = \frac{F_C}{\sin 45^\circ}$$



sine law $\rightarrow \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$

cosine law

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$F_C^2 = 1000^2 + 650^2 - 2 \times 1000 \times 650 \cos 45^\circ$$

$$= 503261.1845$$

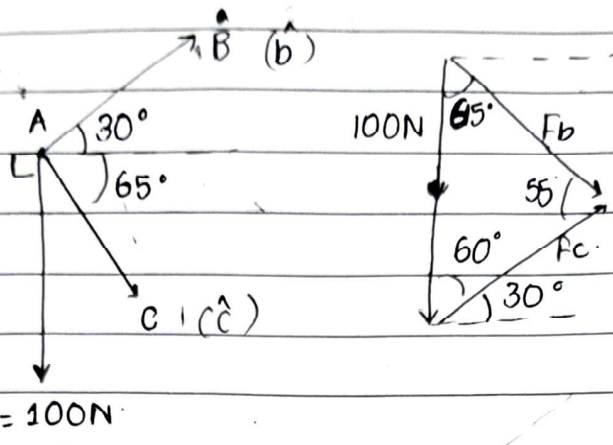
$$F_C = 709.4$$

$$\frac{1000}{\sin \alpha} = \frac{709.4}{\sin 45}$$

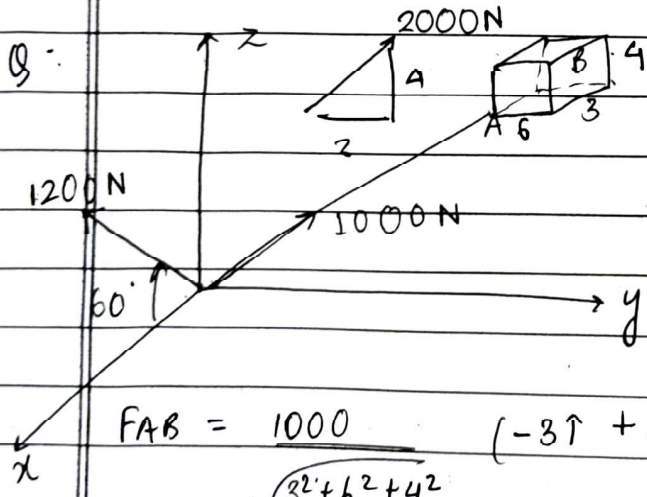
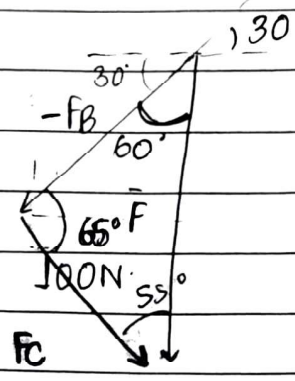
$$= \frac{1000}{709.4} \times \frac{1}{\sqrt{2}} = 0.99676$$

$$\alpha = 85.386^\circ$$

Q: find resultant of force F along AB & AC



$$\frac{F_b}{\sin 60^\circ} = \frac{100}{\sin 55^\circ}$$



$$AB = 6\hat{j} - 3\hat{i} + 4\hat{k}$$

$$\hat{AB} = \frac{-3\hat{i} + 6\hat{j} + 4\hat{k}}{\sqrt{3^2 + 6^2 + 4^2}}$$

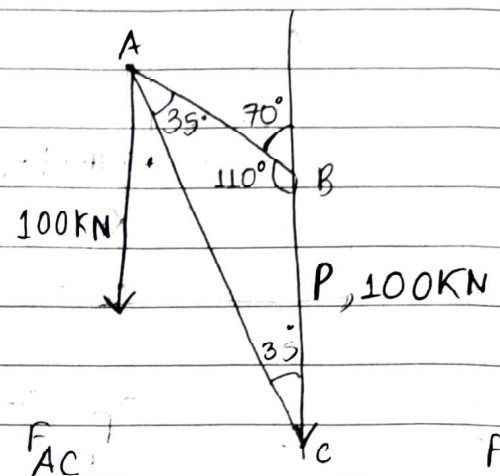
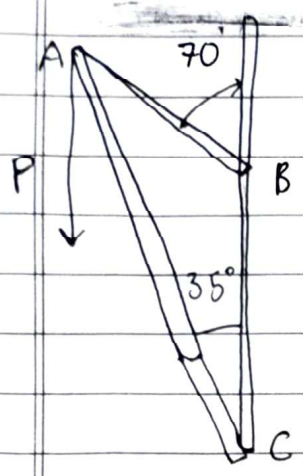
$$F_{AB} = \frac{1000}{\sqrt{3^2 + 6^2 + 4^2}} (-3\hat{i} + 6\hat{j} + 4\hat{k})$$

$$F_{yz} = \frac{(2000)(8\hat{j} + 4\hat{k})}{\sqrt{3^2 + 4^2}}$$

100kN
163.8 kN

Teacher's Signature.....

1.1. The vertical force P of magnitude 100 kN is applied to the frame shown in the figure. Resolve P into components that are parallel to the members AB and AC.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

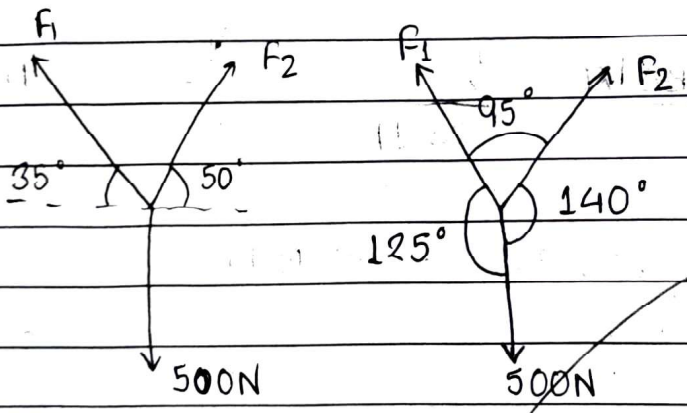
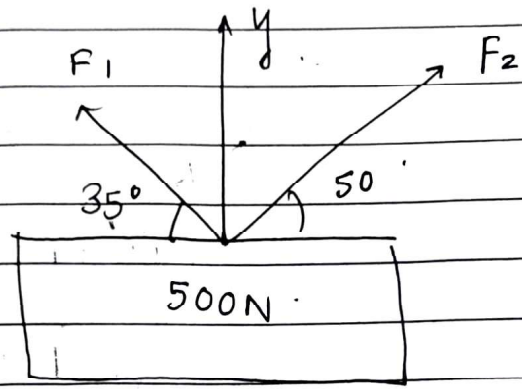
$$\frac{100}{\sin 35^\circ} = \frac{F_{BC}}{\sin 35^\circ} = \frac{F_{AC}}{\sin 110^\circ}$$

$F_{BC} = 100 \text{ kN}$

$$F_{AB} = \frac{100}{\sin 35^\circ} \times \sin 35^\circ = 100 \text{ kN}$$

~~$$F_{AC} = \frac{100 \sin 110^\circ}{\sin 35^\circ} = \frac{100 \times 0.9396}{0.573} = 163.979 \text{ kN}$$~~

1.3 The 500 N weight is supported by two cables, the cable forces being F_1 and F_2 . Knowing that the resultant of F_1 and F_2 is a force of magnitude 500 N acting in y direction determine F_1 and F_2



Using Lami's theorem

$$\frac{F_1}{\sin 140^\circ} = \frac{F_2}{\sin 125^\circ} = \frac{500}{\sin 95^\circ}$$

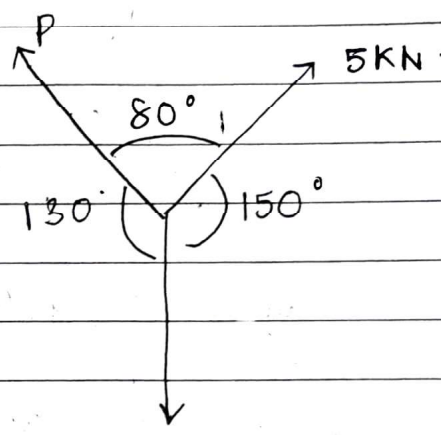
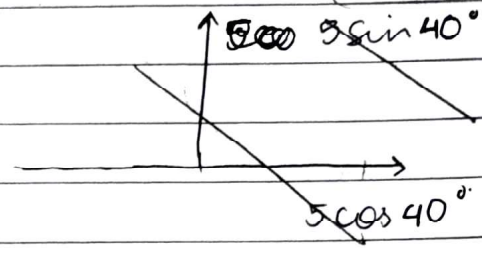
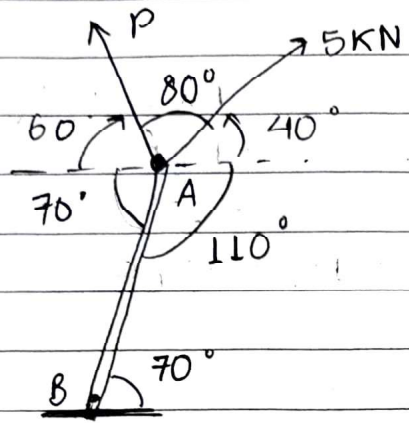
$$F_1 = \frac{500 \times \sin 140^\circ}{\sin 95^\circ} = \frac{500 \times 0.643}{0.996} = 322.79 \text{ N}$$

$$F_2 = \frac{500 \times \sin 125^\circ}{\sin 95^\circ} = \frac{500 \times 0.819}{0.996} = 411.22 \text{ N}$$

Teacher's Signature.....

1.5 Two forces shown act on AB. Determine the magnitude of \bar{P} such that the resultant of these forces is directed along AB

(3.26 KN)



$$\frac{P}{\sin 150^\circ} = \frac{5}{\sin 30^\circ}$$

$$P = 5 \frac{\sin 150^\circ}{\sin 30^\circ}$$

$$P = 5 \times 0.5$$

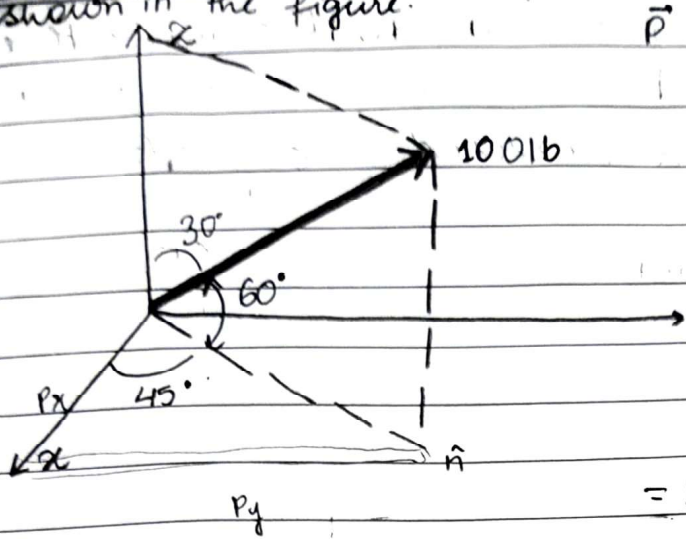
$$0.766$$

$$= 3.263 \text{ KN}$$

19/11

Find cartesian components of 100 lb force shown in the figure.

8:



$$\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$$

$$P_z = P \sin 60^\circ = P \cos 30^\circ$$

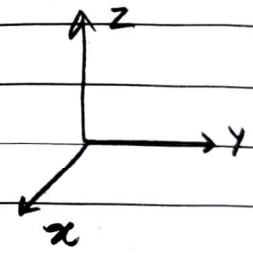
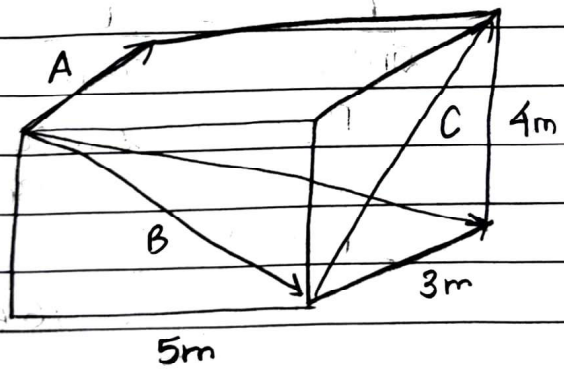
$$P_x \hat{i} + P_y \hat{j} = P_n \hat{n} = 100 \cos 60^\circ \hat{n}$$

$$= 100 \cos 60^\circ (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$$

$$P_x = 100 \cos 60^\circ \cos 45^\circ$$

$$P_y = 100 \cos 60^\circ \sin 45^\circ$$

9: Determine the following resultant of position vectors given in the figure, show the results in sketch of the box - a) $\vec{A} + \vec{B}$ b) $\vec{B} + \vec{C}$



$$\vec{A} = -4\hat{i} - 3\hat{j} - 3\hat{k} \quad \vec{A} + \vec{B} = 5\hat{j} - 8\hat{k}$$

$$B = 5\hat{j} - 4\hat{k} \quad = \sqrt{25+64}$$

$$\vec{C} = -3\hat{i} + 4\hat{k} \quad = \sqrt{89}$$

$$\vec{A} + 8\hat{j} - 4\hat{k} = \sqrt{64+16} = \sqrt{80}$$

$$\vec{A} + \vec{B} = -3\hat{i} + 5\hat{j} - 4\hat{k} \quad \rho = \sqrt{9+25+16} = \sqrt{50} \text{ m}$$

$$\vec{B} + \vec{C} = -3\hat{i} + 5\hat{j} \Rightarrow \sqrt{9+25} = \sqrt{34} \text{ m}$$

Teacher's Signature.....

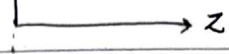
Direction cosines

10109

$$P \cos \alpha = P_x$$

$$P \cos \beta = P_y$$

$$P \cos \gamma = P_z$$



$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

α, β, γ are direction cosines of vector \vec{P}

§1.6 to 1.12 - Vector Representation

Pg. 25 to 27 → Dot & Cross product - Vector Multiplication
1.13 to 1.19

TREASURE HUNT

where thou wilt find a deserted island. There lieth a large meadow; not pint on the north shore of the island where standeth a lonely oak and a lonely pine. There thou wilt see also an old gallows. They once were ~~unto~~ ~~ha~~ want to hang traitors.

Start thou from the gallows and walk to the oak counting thy steps. At the oak, thou must turn right by a right angle and take a same no of steps. Put here a spike on the ground. Now must thou return to the gallows and walk to the pine counting thy steps. At the pine, thou must turn left by a right angle and see that thou takes same no. of steps and put another spike into the ground. Dig halfway between the spikes. The treasure is there.

FORCE

Force is a physical quantity that causes deformation on the body it is applied on

There are 2 types of forces:

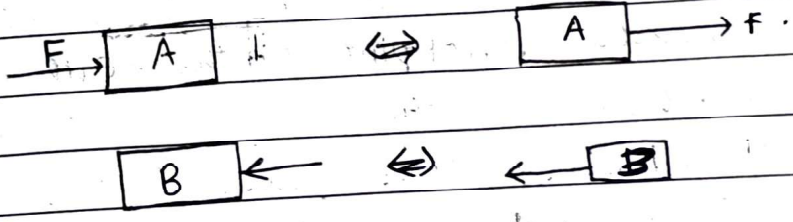
↳ contact force : appears due to direct contact of body. (Friction force, Normal force)

↳ Body force : distributed throughout the volume of the body

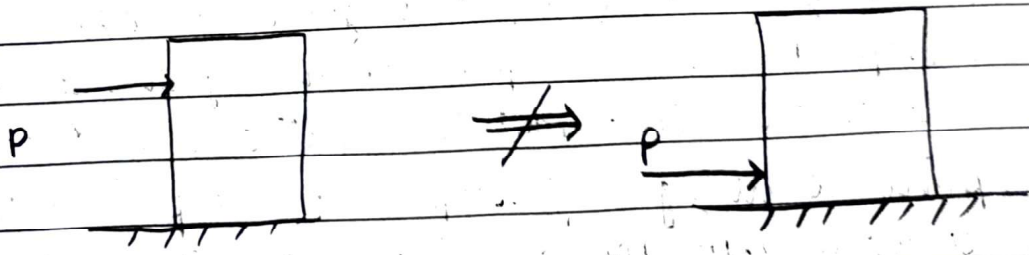
(gravitational force, buoyant force, Electrostatic force)

★ Force as sliding vector →

If A is a rigid body,



∴ A and B are rigid bodies, the system of force is causing same effect. Therefore the vector of force can be slid along the line of force.

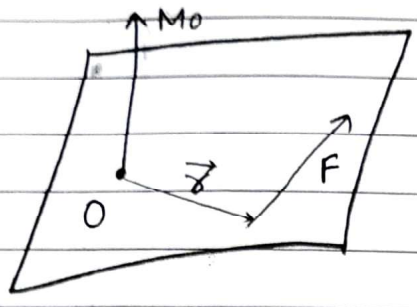


Not equivalent moment about a point due to force P will be different

Teacher's Signature.....

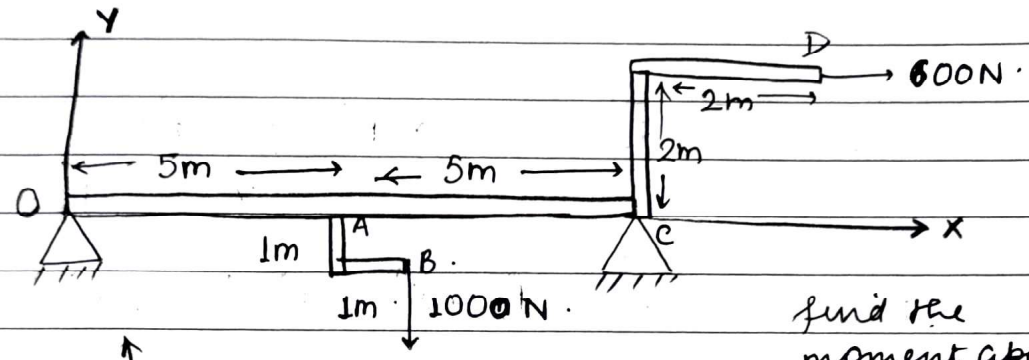
MOMENT

$\vec{M}_O = \vec{r} \times \vec{F}$

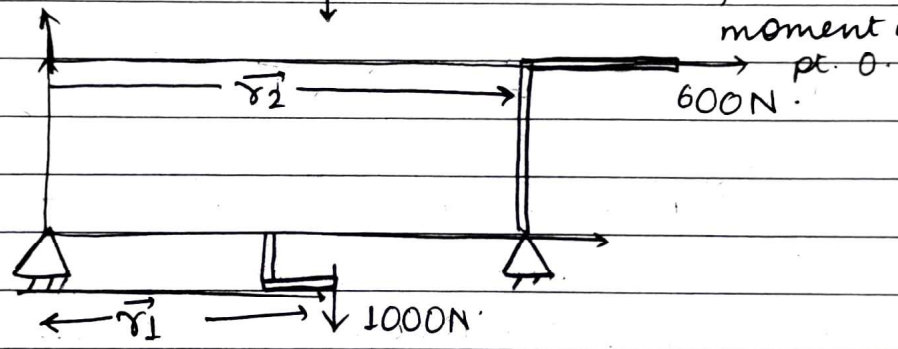


\vec{M}_O is moment of force \vec{F} about point O .
Here \vec{r} stems at O and acts as a point on the line of force

Q:



find the moment about pt. O.



One can write moment about O as

$$\vec{M}_O = \vec{OB} \times (1000) (-\hat{j}) + \vec{OD} \times (600) \hat{i}$$

$$\vec{r}_1 = 6\hat{i}, \quad \vec{r}_2 = 2\hat{j}$$

$$\vec{M}_O = 6\hat{i} \times 1000(-\hat{j}) + 2\hat{j} \times 600\hat{i}$$

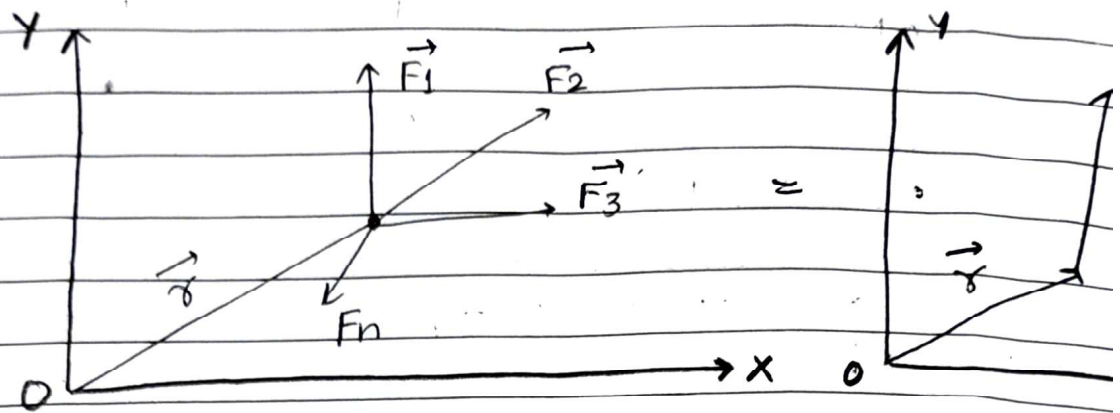
Since force is a sliding vector, we can simplify the arm vector ~~(\vec{r})~~ as (\vec{r}) as :-

$$\vec{r}_1 = 6\hat{i}, \quad \vec{r}_2 = 2\hat{j}$$

$$\vec{M}_O = 6\hat{i} \times 1000(-\hat{j}) + 2\hat{j} \times 600\hat{i}$$

$$\vec{M}_O = -6000\hat{k} - 1200\hat{k} = -7200\hat{k} \text{ Nm}$$

SYSTEM OF CONCURRENT FORCES



$$\vec{M}_0 = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \vec{r} \times \vec{F}_3 + \dots + \vec{r} \times \vec{F}_n$$

$$\vec{M}_0 = \vec{r} \times (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n)$$

$$\vec{M}_0 = \vec{r} \times \vec{R}$$

where $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n$

This is Varignon's Theorem.

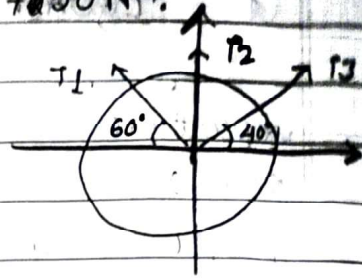
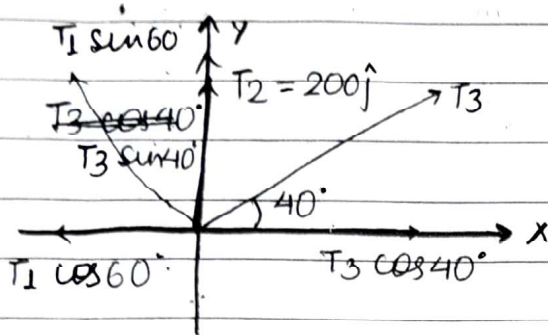
With help of this theorem, we can ~~simplify~~ the system simplify / equivalent / reduce to a single force system.

CONDITION APPLIED: -

The ~~result~~ line of resultant force R must pass through point of concurrency.

Q: 1.20 $T_1 = 550\text{ N}$, $T_2 = 200\text{ N}$, $T_3 = 750\text{ N}$.

Draw equivalent force \vec{R} .



$$\sum F_x = (T_3 \cos 40^\circ - T_1 \cos 60^\circ)$$

$$= 299.53 \hat{i}$$

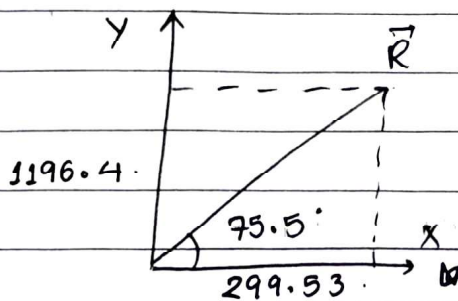
$$\sum F_y = 200 + T_1 \sin 60^\circ + T_3 \sin 40^\circ$$

$$\sum F_y = 1158.4 \hat{j}$$

$$R = \sqrt{1158.4^2 + 299.53^2}$$

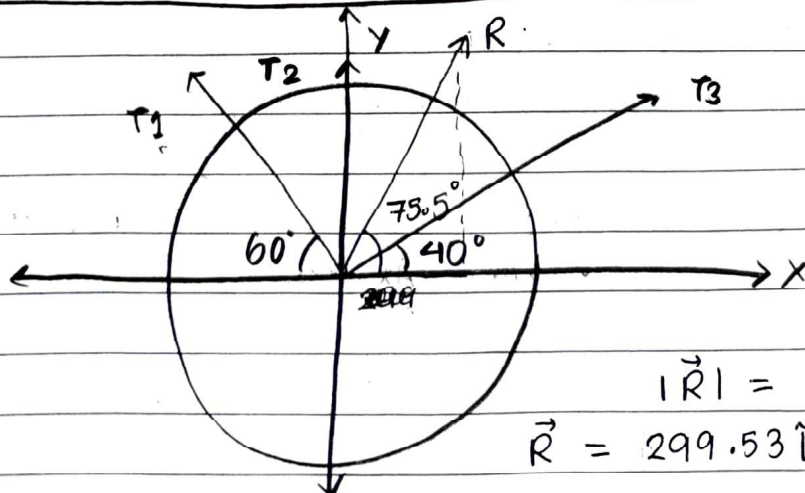
$$= \sqrt{1431608.78}$$

$$= 1196.4$$



$$\alpha = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right) = 75.50^\circ$$

$$\alpha = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right) = \tan^{-1} \left(\frac{1158.4}{299.53} \right) = 75.5^\circ$$



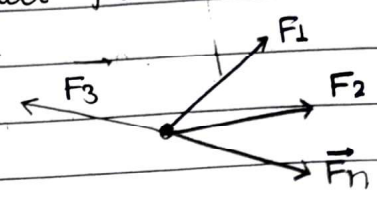
$$|\vec{R}| = 1196.4\text{ N}$$

$$\vec{R} = 299.53 \hat{i} + 1158.4 \hat{j}$$

EQUATIONS OF EQUILIBRIUM

A particle in equilibrium, is one, that is stationary or that moves uniformly relative to an inertial plane. In other words, total force on the particle is zero. That is

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = 0$$



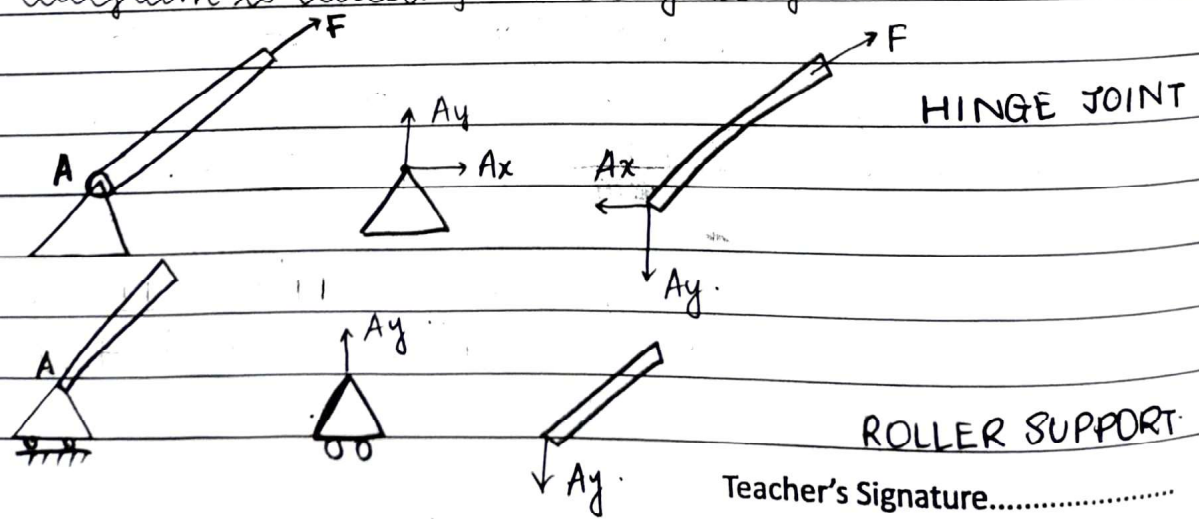
A body (rigid body) is in equilibrium if all the particles that may be considered to comprise the body are in equilibrium, that is, no particle can even rotate about any axis

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = 0$$

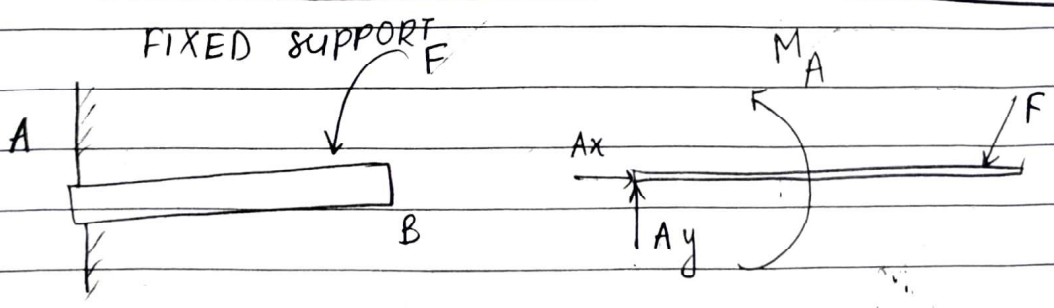
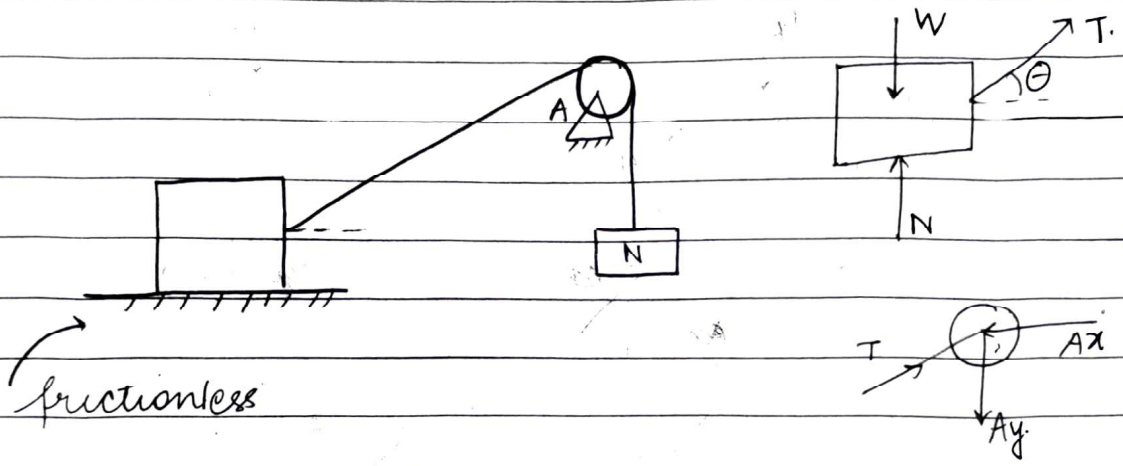
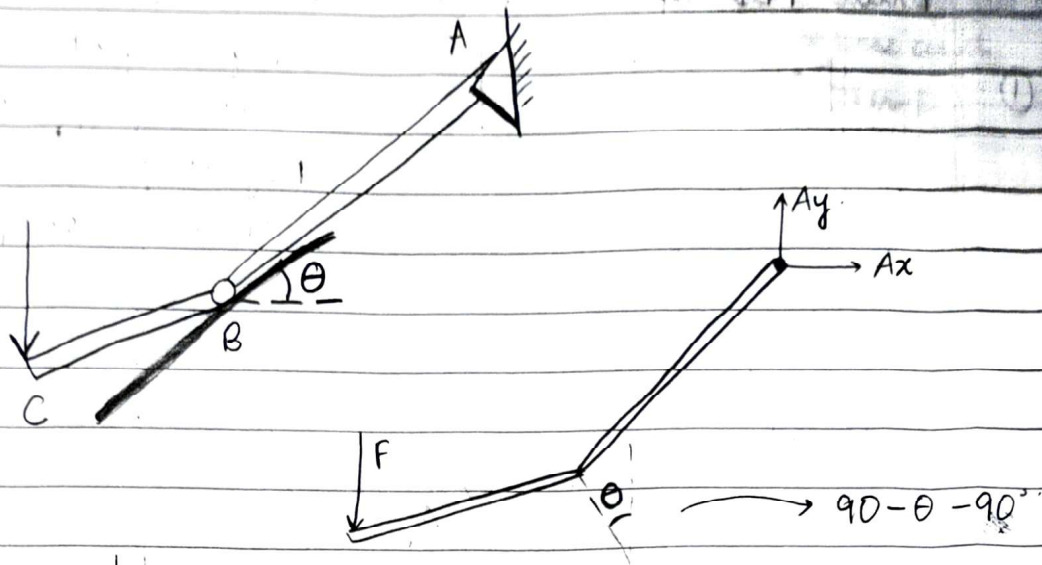
$$\vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \dots + \vec{r}_n \times \vec{F}_n = \vec{M} = 0$$

FREE BODY DIAGRAM

To help identify all the forces, we isolate the body in a simple diagram and show all the forces from the surroundings that act on the body, such diagram is called free body diagram.



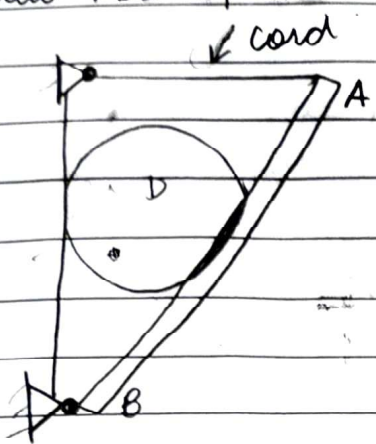
Teacher's Signature.....



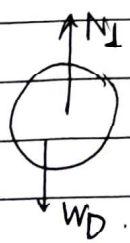
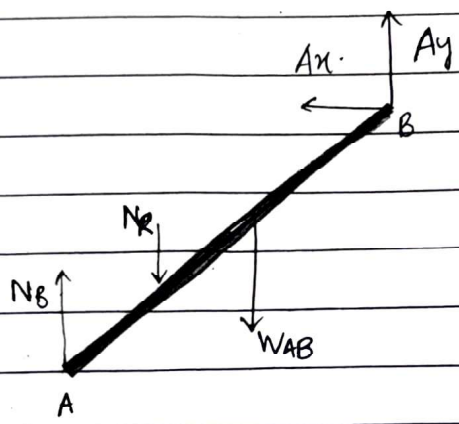
Teacher's Signature.....

Q: Draw FBD of member AB and cylinder D.

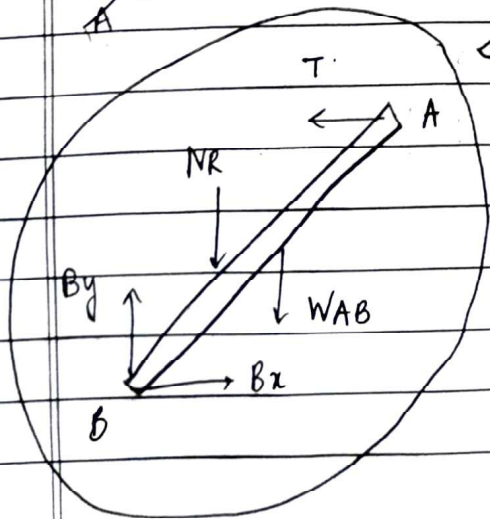
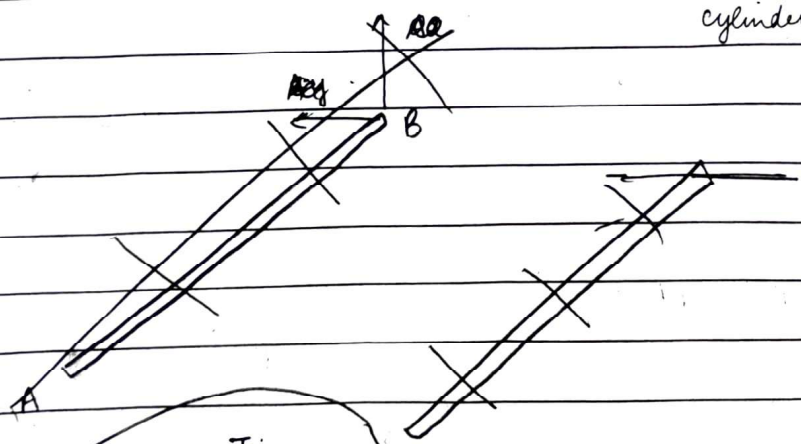
①



Weight of AB = W_{AB}
 Weight of cylinder = W_D
 Neglect friction on cylinder surface

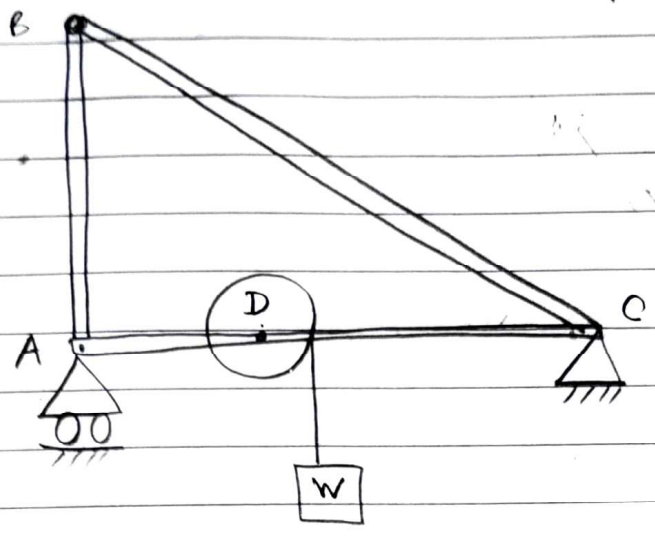


Tension force on A
 Hinge on B.
 cylinder \rightarrow normal force

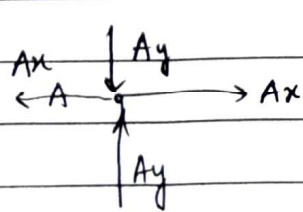
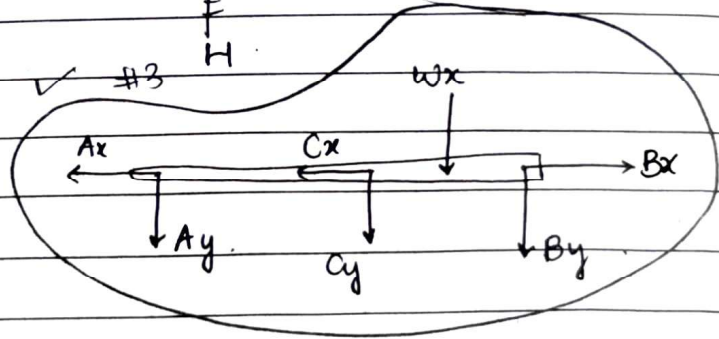
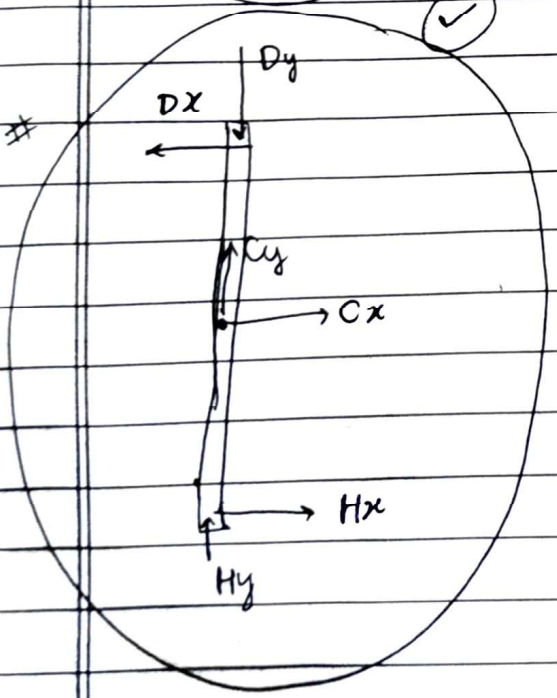
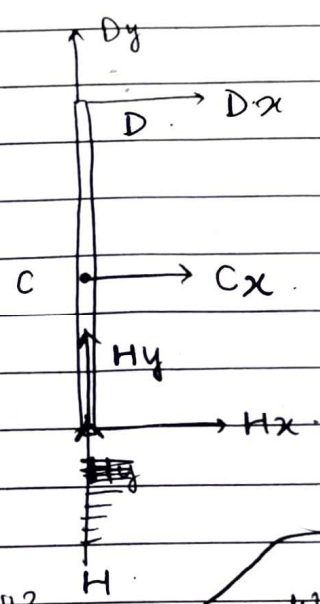
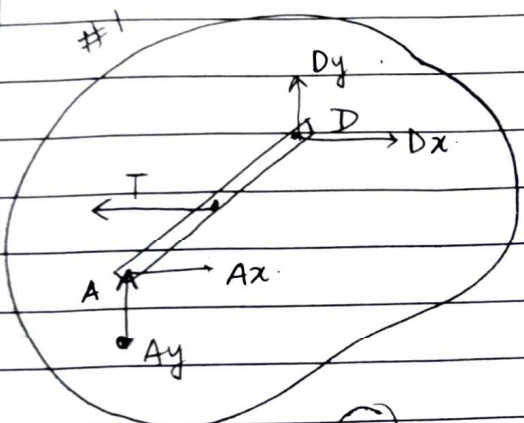
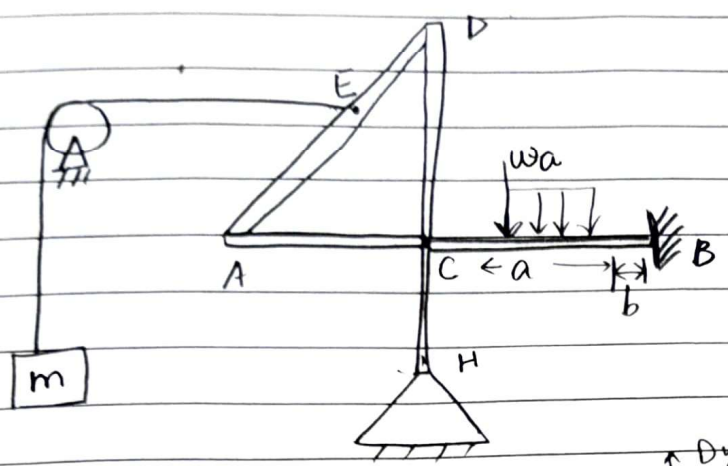


Teacher's Signature.....

3



Include weight of
members and
cylinder D

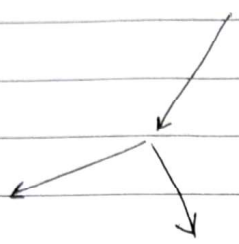


from #1 and #2
 all forces at
 @ point A will
 sum upto 0.

Teacher's Signature.....

EQUILIBRIUM EQUATIONS

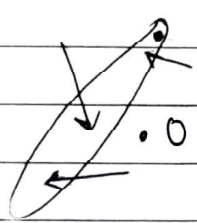
★ Forces are conservative



Equilibrium eqⁿs

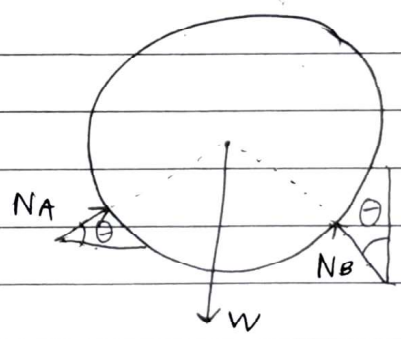
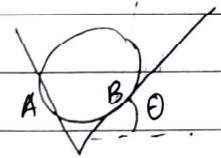
2D	3D
$\Sigma F_x = 0$	$\Sigma F_x = 0$
$\Sigma F_y = 0$	$\Sigma F_y = 0$
	$\Sigma F_z = 0$

★ Forces are not conservative



$\Sigma F_x = 0$	$\Sigma F_x = 0$
$\Sigma F_y = 0$	$\Sigma F_y = 0$
$\Sigma M_o = 0$	$\Sigma F_z = 0$
	$\Sigma M_o = 0$

Q1.49 The homogeneous cylinder of weight W rests in a frictionless right angled corner. Determine the contact forces N_A and N_B in terms of W and θ . find θ when $N_B = 1.5 N_A$.



eqⁿ of eq^m $\Sigma F_x = 0, \Sigma F_y = 0$

$\Sigma F_x = 0$

$N_A \cos \theta - N_B \sin \theta = 0$

$\tan \theta = \frac{N_A}{N_B} = \frac{1}{1.5} = \frac{2}{3}$

$\theta = \tan^{-1} (2/3) = 33.7^\circ$

$\Sigma F_y = 0$

$N_A \sin \theta - W + N_B \cos \theta = 0$

$W = N_B \sin \theta + N_A \sin \theta + N_B \cos \theta$

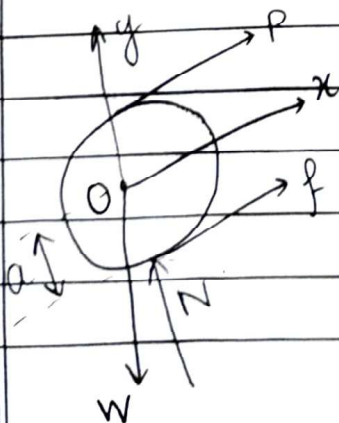
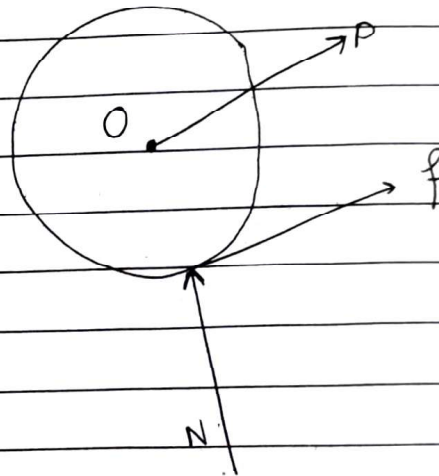
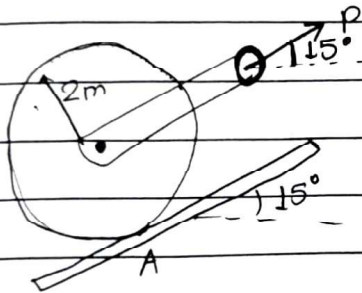
Teacher's Signature.....

$$\begin{aligned}
 W &= N_A \sin \theta + N_B \sin \theta \cos \theta \\
 &= N_A \sin (33.7^\circ) + 1.5 N_A (\cos (33.7^\circ)) \\
 &= 0.555 N_A + 1.248 N_A \\
 W &= 1.8 N_A
 \end{aligned}$$

$$N_A = \frac{1}{1.8} W = 0.555 W$$

$$N_B = 1.5 N_A = 1.5 \times 0.555 W = 0.833 W$$

Q: calculate the force P that is required to hold the 600N roller at rest on the rough incline.



$$\begin{aligned}
 \sum M_O &= 0 \\
 f a - P a &= 0 \\
 f &= P
 \end{aligned}$$

$$\begin{aligned}
 \sum F_x &= 0 \\
 P + f - W \sin \theta &= 0 \\
 \text{or} \\
 2P &= W \sin \theta \\
 P &= \frac{W}{2} \sin \theta
 \end{aligned}$$

Teacher's Signature.....

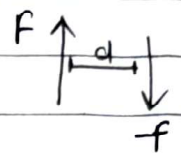
MOMENT OF FORCE

$[\vec{r} \times \vec{F}]$

Bending Moment

Turning Moment (Torque)

couple moment — effect of couple moment is pure turning



The couple moment is formed by any two equal parallel forces that are opposite in sense.

$\vec{C} = F d \hat{n}$

\hat{n} is \perp to the plane formed by F and d

→ Moment about an axis :

WHY?

Sometimes we need to know the rotation tendency about an axis due to load applied.

How TO DETERMINE IT?

- 1) Select a suitable point on the axis (O)
- 2) Find the moment about that point M_0 .
- 3) Take dot product and moment M_0 with the axis $M_0 \hat{n}$.

$\vec{M}_0 = \vec{r} \times \vec{F}$
 $\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$

$M_n = (\vec{r} \times \vec{F}) \cdot \hat{n}$

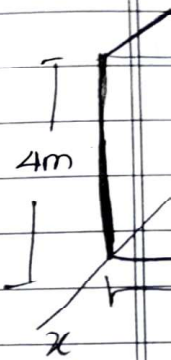
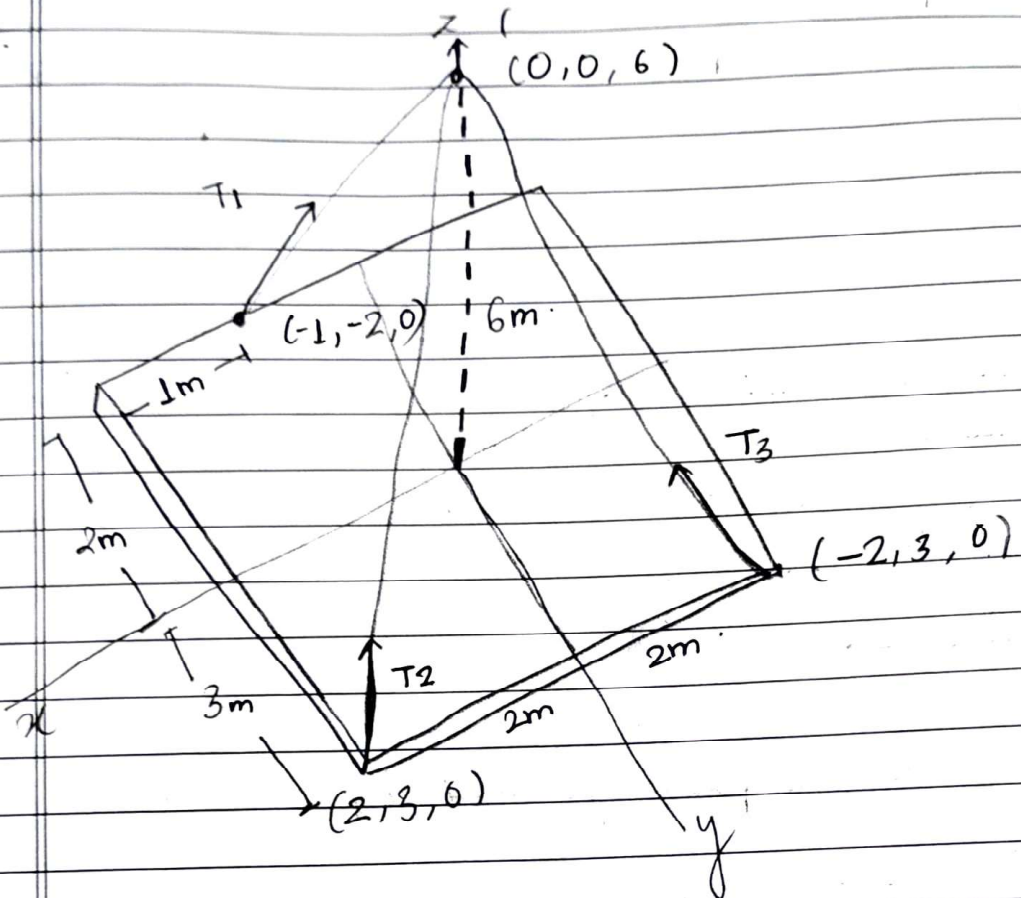
$M_n =$	r_x	r_y	r_z
	F_x	F_y	F_z
	n_x	n_y	n_z

Teacher's Signature

1.23

1.23

1.34

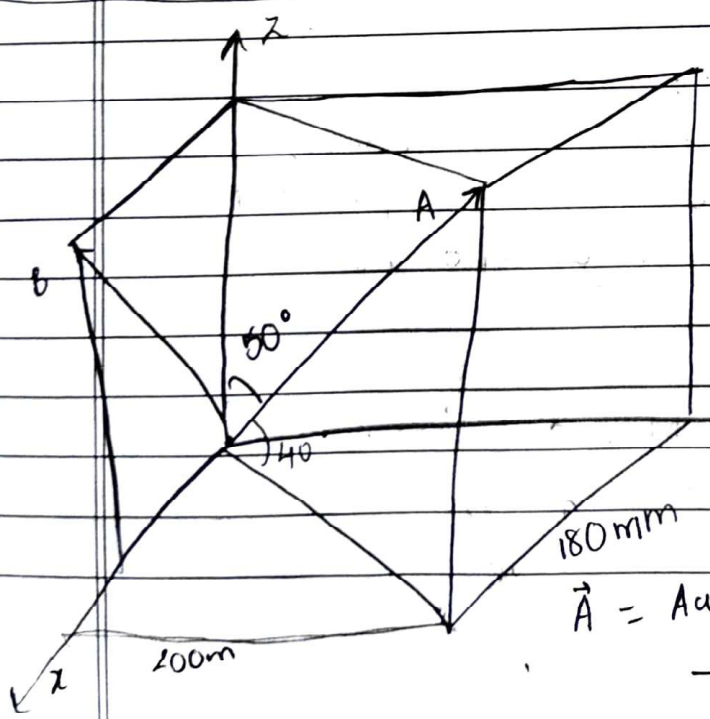


$$\vec{T}_1 = T_1 \hat{\gamma}_1$$

$$\vec{T}_2 = T_2 \hat{\gamma}_2$$

$$\vec{T}_3 = T_3 \hat{\gamma}_3$$

$$\vec{\gamma} = \gamma (\cos\theta \hat{i} + \sin\theta \hat{j})$$



$$\hat{n} = \frac{180\hat{i} + 200\hat{j}}{\sqrt{180^2 + 200^2}}$$

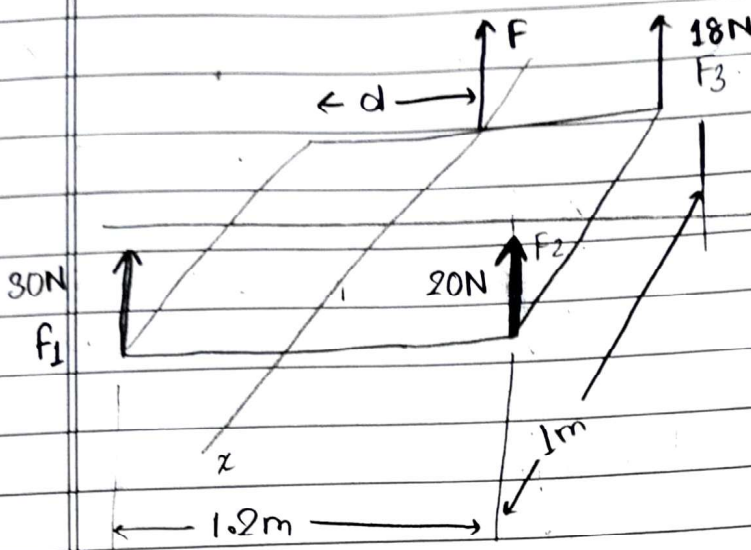
$$\vec{A} = A \cos 50^\circ \hat{k} + A \sin 50^\circ \hat{n}$$

$$B_x = A \cos 50^\circ$$

$$\vec{A} = A \cos 50^\circ \hat{k} + \frac{A \sin 50^\circ}{\sqrt{180^2 + 200^2}} (180\hat{i} + 200\hat{j})$$

Teacher's Signature.....

1.35



$\Sigma M_x = 0$
 $\Sigma M_y = 0$

$\vec{M}_1 = \vec{r}_1 \times \vec{F}_1$
 $= -0.6 \hat{j} \times 30 \hat{k}$
 $= -18 \hat{i}$

$M_x = 0, M_y = 0$

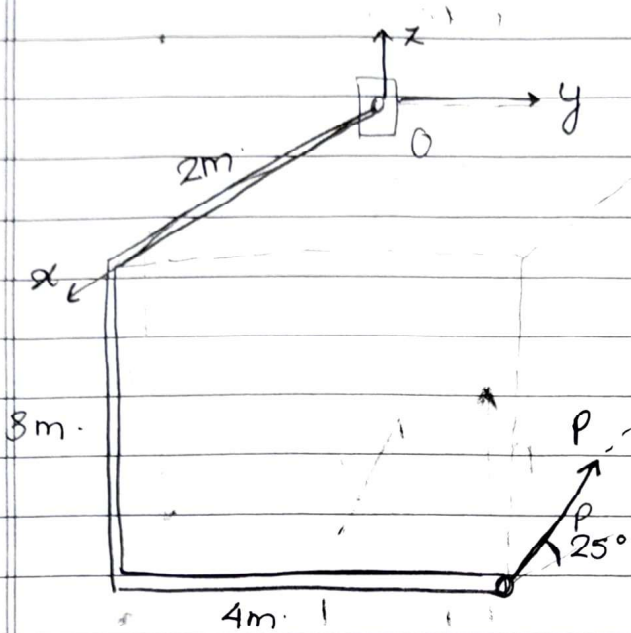
$M_{1x} = -18$
 $M_{2x} = 12 \times 0.6$
 $M_{3x} = 18 \times 0.6$
 $M_{4x} = -F \times (0.6 - d)$

$\Sigma M_x = M_{1x} + M_{2x} + M_{3x} + M_{4x} = 0$

$\Sigma M_y = M_{1y} + M_{2y} + M_{3y} + M_{4y} = 0$

Teacher's Signature.....

1.30. The magnitude of moment of force P about point O is 200 kNm . Determine the magnitude of P .

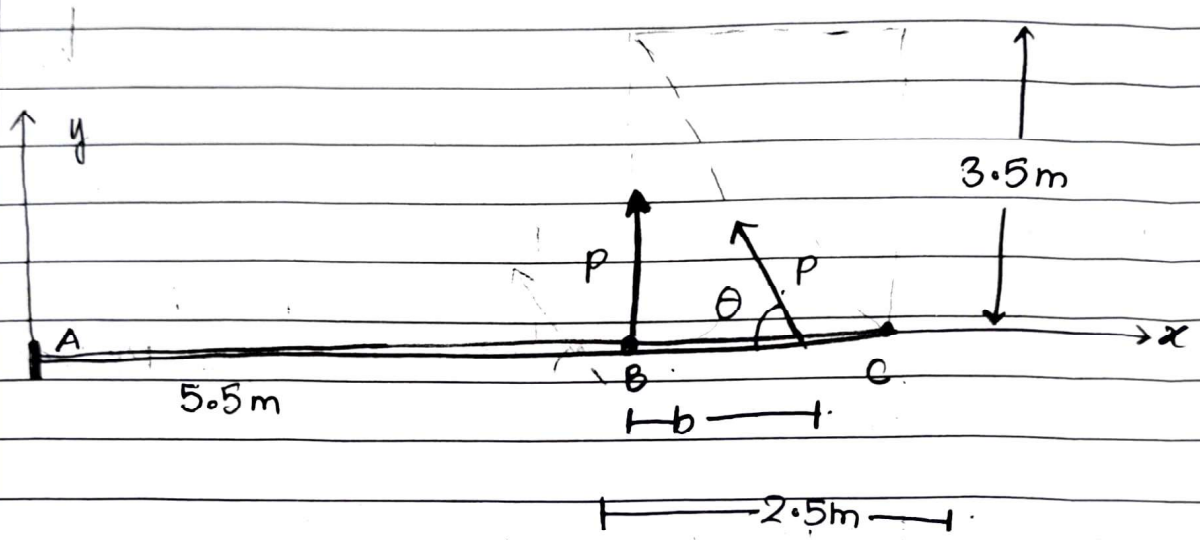


$$\begin{aligned} \vec{M}_O &= \vec{r} \times \vec{P} \\ \vec{r} \times \vec{P} & \\ \vec{r} &= \vec{OC} \\ &= \vec{OA} + \vec{AB} + \vec{BC} \\ &= 2\hat{i} + 3(-\hat{k}) + 4\hat{j} \\ \vec{r} &= 2\hat{i} + 4\hat{j} - 3\hat{k} \\ \vec{P} &= P \cos 25^\circ (-\hat{i}) \\ &\quad + P \sin 25^\circ \hat{k} \end{aligned}$$

$$\begin{aligned} \vec{M}_O &= (2\hat{i} + 4\hat{j} - 3\hat{k}) \times P(-\cos 25^\circ \hat{i} + \sin 25^\circ \hat{k}) \\ &= -2P \sin 25^\circ \hat{j} + 4P \cos 25^\circ \hat{k} + 4P \sin 25^\circ \hat{i} \\ &\quad + 3P \cos 25^\circ \hat{j} \\ &= 4P \sin 25^\circ \hat{i} + (3P \cos 25^\circ - 2P \sin 25^\circ) \hat{j} \\ &\quad + 4P \cos 25^\circ \hat{k} \end{aligned}$$

$$200 \times 10^3 = P \sqrt{(\sin 25^\circ)^2 + (3 \cos 25^\circ - 2 \sin 25^\circ)^2 + (4 \cos 25^\circ)^2}$$

1.25 Two forces can be replaced by an equivalent force acting at point B on the beam. Determine the distance b that locates B.



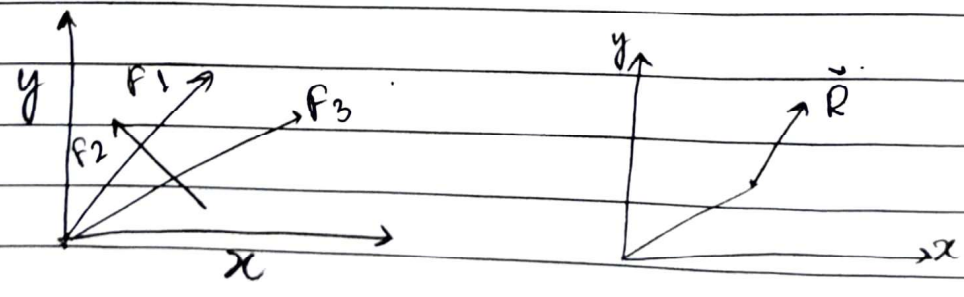
$$\vec{R} = P\hat{j} + P(-\cos\theta\hat{i} + \sin\theta\hat{j})$$

$$R_{\text{net}} = -P\cos\theta + P(1 + \sin\theta)\hat{j}$$

Here $\tan\theta = 3.5/2.5 \Rightarrow \theta = \tan^{-1} 54.46^\circ$

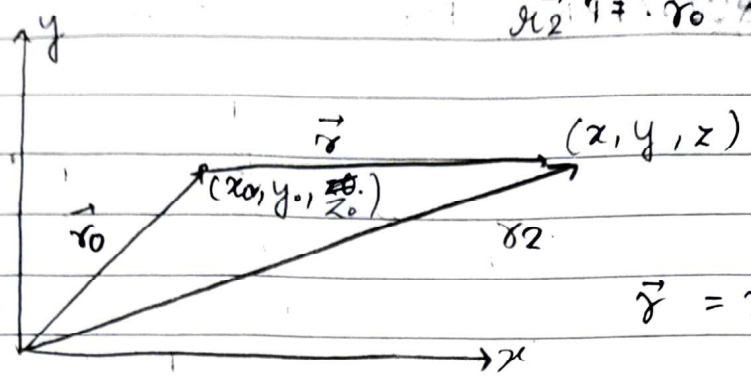
$$\vec{R} = -0.58P\hat{i} + 1.814P\hat{j}$$

Resultant of a concurrent force system must pass through the point of concurrency.



Teacher's Signature.....

$$r_2 = r_0 + ar$$



$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + a \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$x = x_0 + ar_x, \quad y = y_0 + ar_y, \quad z = z_0 + ar_z$$

$$x = x_0 + \frac{y_0 (0.58)}{1.814}, \quad y = 5$$

$$= 5.5 + \frac{3.5 \times 0.58}{1.814} = 6.62$$

$$b = 8 - 6.62$$

$$\vec{R} = -0.58P \hat{i} + 1.814P \hat{j}$$

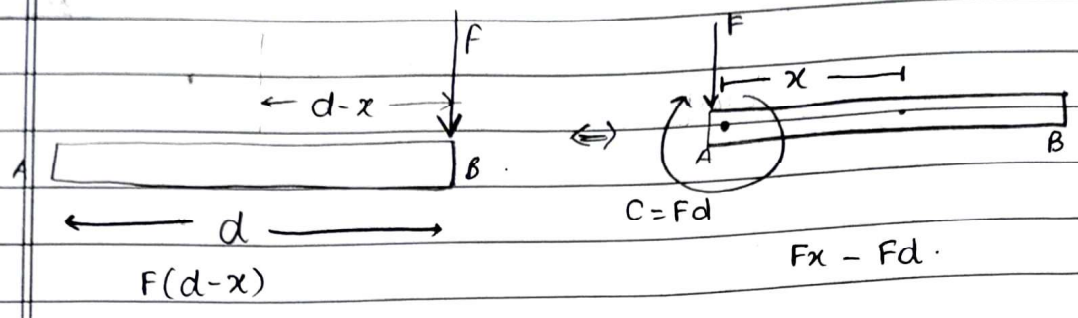
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + a \begin{pmatrix} -0.58P \\ 1.814P \end{pmatrix}$$

$$x = x_0 + aP \times (-0.58)$$

$$y = y_0 + 1.814 aP$$

In this case, $y = 0 \Rightarrow aP = \frac{-y_0}{1.814}$

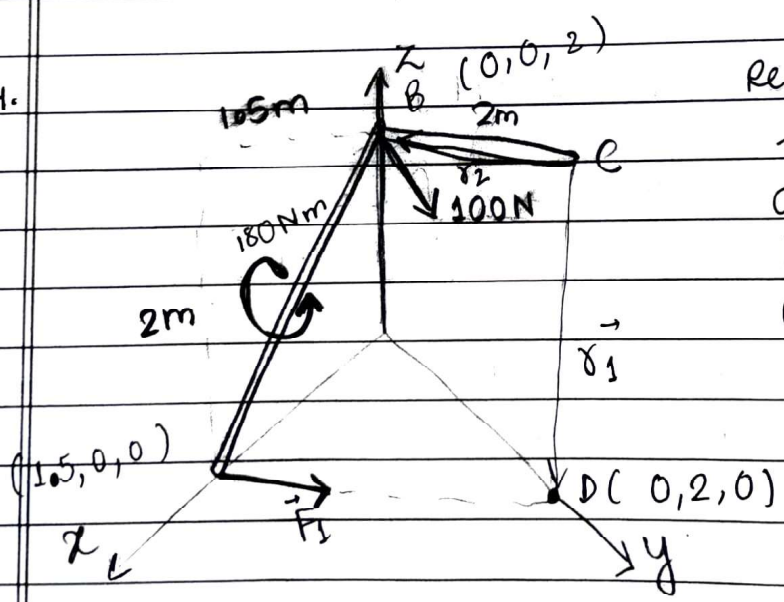
EQUIVALENT FORCE SYSTEM



Two force systems are equivalent if

- Resultant force is same in both systems
- moment about any point is same.

1.44.



Replace the two forces and a couple acting on the bent rod ABC with an equivalent force couple system with force acting at C

Teacher's Signature.....

we have to put the resultant of \vec{F}_1 and \vec{F}_2 at point C.
also, put the couple at C due to shifting of forces \vec{F}_1 and \vec{F}_2 required

The shifting couples will be \vec{r}_i moment due to \vec{F}_1 and \vec{F}_2

$$\vec{F}_1 = 120 \left[\frac{(0-1.5)\hat{i} + (2-0)\hat{j}}{\sqrt{1.5^2 + 2^2}} \right]$$

$$\vec{F}_1 = \frac{-180\hat{i} + 240\hat{j}}{2.5} = -72\hat{i} + 96\hat{j} = \vec{F}_1$$

$$\vec{F}_2 = 100 \left[\frac{(2-0)\hat{j} + (0-2)\hat{k}}{\sqrt{2^2 + 2^2}} \right] = 70.7\hat{j} - 70.7\hat{k}$$

$$\vec{C}_1 = \vec{r}_1 \times \vec{F}_1 = -2\hat{k} \times (-72\hat{i} + 96\hat{j})$$

$$= 144\hat{j} + 192\hat{i}$$

$$\vec{C}_1 = 192\hat{i} + 144\hat{j} \text{ Nm}$$

$$\vec{C}_2 = \vec{r}_2 \times \vec{F}_2 = -2\hat{j} \times (70.7\hat{j} - 70.7\hat{k})$$

$$= 141.4\hat{i} \text{ Nm}$$

$$\vec{C}_3 = 180 \vec{AB} = 180 \left[\frac{(0-1.5)\hat{i} + (2-0)\hat{k}}{\sqrt{1.5^2 + 2^2}} \right]$$

$$\vec{C}_3 = -108\hat{i} + 144\hat{k} \text{ N-m}$$

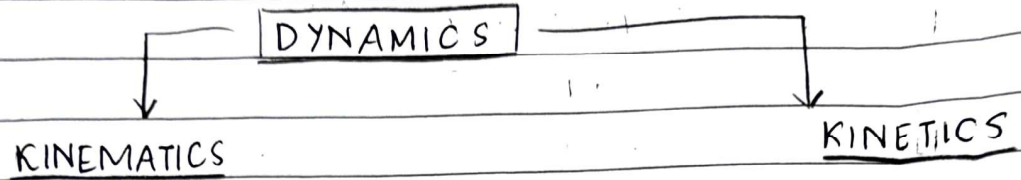
$$\text{couple at } \vec{C} = \vec{C}_1 + \vec{C}_2 + \vec{C}_3$$

$$= 225.4\hat{i} + 144\hat{j} + 144\hat{k} \text{ N-m}$$

Hence \vec{R} is the resultant of point C and \vec{C} is the required couple for equivalent force system.

DYNAMICS

Dynamics is that branch of mechanics which deals with the motion of bodies under the action of forces. We divide this motion analysis in 2 parts



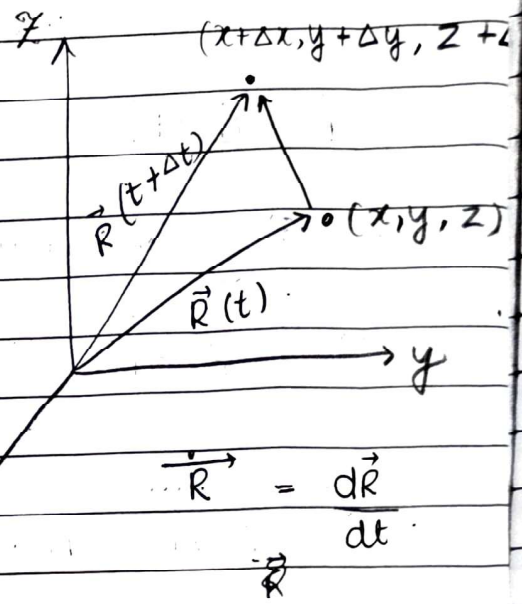
KINEMATICS
 study of motion without reference to the forces cause the motion

KINETICS
 It relates the action of forces on bodies with the resulting motion

★ **RECTILINEAR MOTION :-**

$$\vec{R}(t) = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{R}(t+\Delta t) = (x+\Delta x)\hat{i} + (y+\Delta y)\hat{j} + (z+\Delta z)\hat{k}$$



$$\lim_{\Delta t \rightarrow 0} \frac{\vec{R}(t+\Delta t) - \vec{R}(t)}{\Delta t}$$

$$= \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k} \right)$$

$$\vec{v} = \frac{d\vec{R}}{dt}$$

$$\frac{d\vec{R}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k}$$

$$\vec{v} = \dot{\vec{R}} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

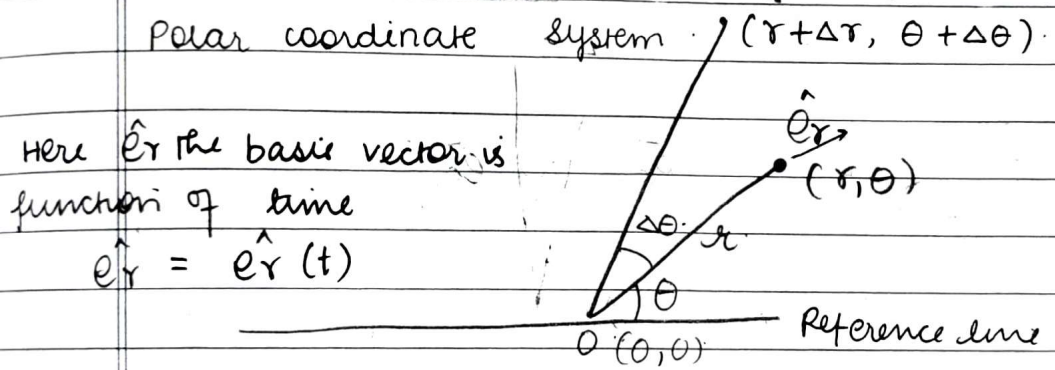
$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k} = \ddot{\vec{R}}$$

Teacher's Signature.....

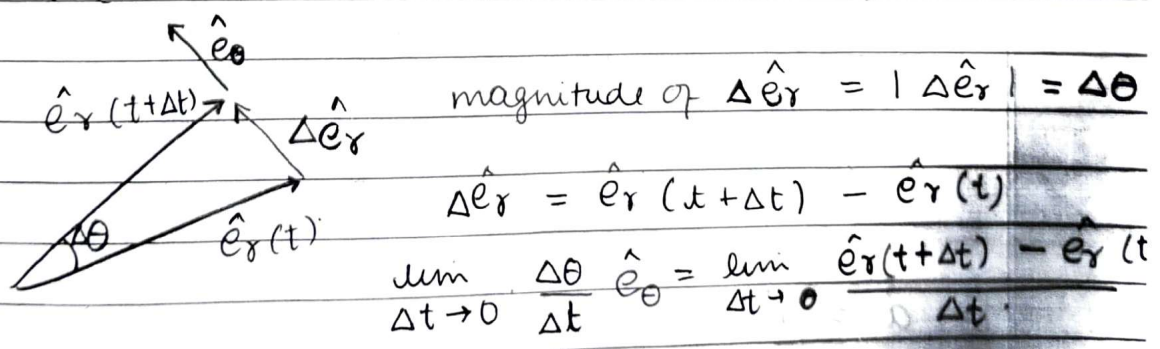
Q: The position of a particle moving in a 3D space is defined as $x = 3t^2 + 2t + 1$, $y = 4t + 7$, $z = 7t^3 + 5t + 1$. Find its position, velocity and acceleration at time $t = 2$ sec.

$$\begin{aligned} \vec{R} &= (3t^2 + 2t + 1)\hat{i} + (4t + 7)\hat{j} + (7t^3 + 5t + 1)\hat{k} \\ &= (12 + 4 + 1)\hat{i} + (18)\hat{j} + (7 \times 8 + 10 + 1)\hat{k} \\ &= 17\hat{i} + 18\hat{j} + 67\hat{k} \\ \vec{V} &= (6t + 2)\hat{i} + (4)\hat{j} + (21t^2 + 5)\hat{k} \\ &= 14\hat{i} + 4\hat{j} + 89\hat{k} \\ \vec{a} &= 6\hat{i} + 42 \times 2\hat{k} \\ &= 6\hat{i} + 84\hat{k} \end{aligned}$$

* CURVILINEAR MOTION :- $\hat{e}_{r1} = \hat{e}_r(t + \Delta t)$.



$$\begin{aligned} \vec{R}(t) &= r \hat{e}_r(t) \\ \vec{R}(t + \Delta t) &= (r + \Delta r) \hat{e}_{r1} = (r + \Delta r) \hat{e}_r(t + \Delta t) \end{aligned}$$



\hat{e}_θ is \perp to \vec{e}_r
 \hookrightarrow unit vector

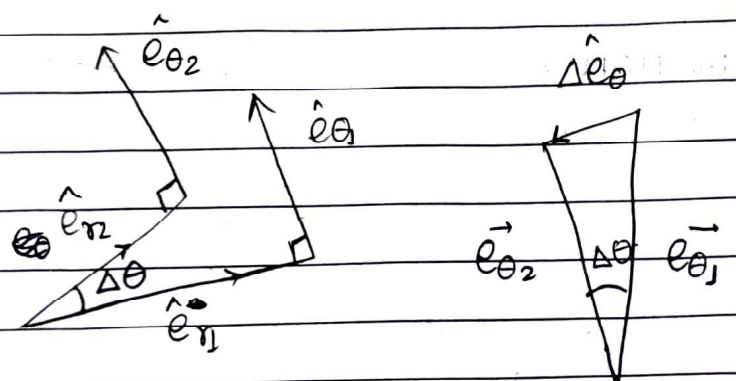
$$\frac{d\theta}{dt} \hat{e}_\theta = \frac{d\hat{e}_r}{dt}$$

$$\dot{\hat{e}}_r = \dot{\theta} \hat{e}_\theta$$

$$\frac{d\vec{R}}{dt} = \frac{d(r\hat{e}_r)}{dt} = \frac{dr}{dt} \hat{e}_r + r \frac{d\hat{e}_r}{dt}$$

$$\vec{v} = \dot{\vec{R}} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta$$

$$\vec{a} = \dot{\vec{v}} = \frac{d\vec{v}}{dt} = \ddot{r} \hat{e}_r + \dot{r} \dot{\hat{e}}_r + \dot{r} \dot{\theta} \hat{e}_\theta + r \ddot{\theta} \hat{e}_\theta + r \dot{\theta} \dot{\hat{e}}_\theta$$



$$\Delta \hat{e}_\theta = \hat{e}_{\theta 2} - \hat{e}_{\theta 1} = \Delta \theta$$

$$\frac{\Delta \hat{e}_\theta}{\Delta t} = \frac{\Delta \theta \hat{e}_r}{\Delta t} \quad \Delta \hat{e}_\theta = -(\Delta \theta) \hat{e}_r$$

$$\frac{d\hat{e}_\theta}{dt} = \lim_{\Delta t \rightarrow 0} \left(-\frac{\Delta \theta}{\Delta t} \hat{e}_r \right)$$

$$\dot{\hat{e}}_\theta = \frac{d\hat{e}_\theta}{dt} = -\dot{\theta} \hat{e}_r$$

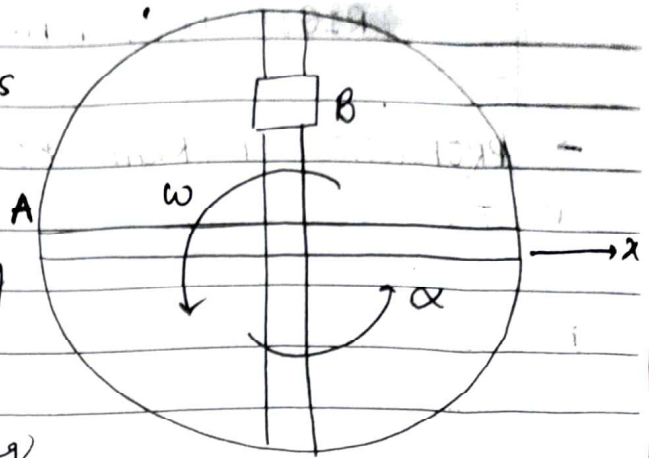
$$\vec{a} = \frac{d\vec{v}}{dt} = (\ddot{r} - r\dot{\theta}^2) \hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta}) \hat{e}_\theta$$

Teacher's Signature.....
Coriolis component of accⁿ

Q: A wheel A is rotating with angular velocity $\omega = 5 \text{ rad/s}$
 $\alpha = 2 \text{ rad/s}^2$.

A body B is moving towards centre with speed 3 m/s relative to spoke.

and its speed is increasing at the rate of 1.6 m/s^2 . The data is given at instant when B is 0.6 m from the centre.



WHAT IS THE VELOCITY & ACCELERATION REL OF BODY B relative to fixed reference frame at instant.

$\omega = 5 \text{ rad/s}$, $\alpha = 2 \text{ rad/s}^2$

$v = \omega r$

$r = 0.6 \text{ m}$

$v = 5 \times 0.6 = 3 \text{ m/s}$

$\dot{\theta} = 5 \text{ rad/s}$, $\dot{r} = -3 \text{ m/s}$

$\alpha = 2 \text{ rad/s}^2$

$\ddot{\theta} = 2 \text{ rad/s}^2$, $\ddot{r} = 1.6 \text{ m/s}^2$

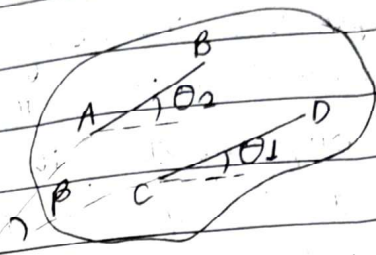
$a = \omega^2 r$

$= 25 \times 0.6$

$= 15 \text{ m/s}^2$

RIGID BODY KINEMATICS - PLANE MOTION

- PROPERTIES OF RIGID BODY
- 1) The distance b/w any two points in the rigid body remains constant
 - 2) The angle between two lines of a rigid body remain constant.



$$\theta_2 = \theta_1 + \beta$$

differentiating both sides w.r.t time

$$\frac{d\theta_2}{dt} = \frac{d\theta_1}{dt} + \frac{d\beta}{dt}$$

$$\dot{\theta}_2 = \dot{\theta}_1$$

$$\ddot{\theta}_2 = \ddot{\theta}_1$$

CONCEPT 1: Thus, all lines on a rigid body in its plane of motion have the same angular displacement, same angular velocity, same angular acceleration

→ Rotation of a body about a fixed point

$$\vec{r} = r \hat{e}_r$$

$$\frac{d\vec{r}}{dt} = \frac{dr}{dt} \hat{e}_r + r \frac{d\hat{e}_r}{dt}$$

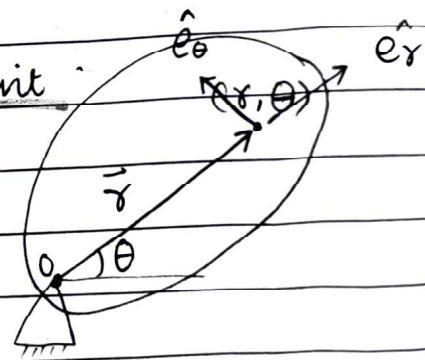
$$= r \dot{\hat{e}}_r$$

$$= r \dot{\theta} \hat{e}_\theta$$

$$= r \dot{\theta} (\hat{k} \times \hat{e}_r)$$

$$= (\dot{\theta} \hat{k}) \times (r \hat{e}_r) = \vec{\omega} \times \vec{r}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$



\hat{k} is perpendicular to the plane of motion

$\Delta\theta$ is a vector while θ is a scalar, a no.

$$\theta_x + \theta_x = \theta_x + \theta_x$$

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \quad \vec{A} + \vec{B} = \vec{R}$$

- ω is along axis of rotation
- The sense is determined by right hand screw rule
- \vec{r} is the radial vector. It heads from any point on the axis of rotation to the point whose velocity is required to be determined.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

$$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}$$

$$\vec{a} = \vec{\alpha} \times \vec{r} + (\vec{\omega} \cdot \vec{r})\vec{\omega} - (\vec{\omega} \cdot \vec{\omega})\vec{r}$$

for in plane motion, $\vec{\omega}$ is perpendicular to \vec{r}
 $\therefore \vec{\omega} \cdot \vec{r} = 0$

, $\vec{\alpha}$ is also \perp to plane

$$\vec{a} = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$$

Q[2.4]

Pg. 12. Q(2.4)

C(0.24, 0.36, 0)

$\vec{\omega}$ is along \vec{AC} .

A(0, 0, 0.2)

radial vector starts from A or R and goes till point of interest $\rightarrow \vec{C}$ to \vec{B} is \vec{r}

$$\vec{r} = -0.36\hat{j}, = \vec{CB}$$

$$\vec{\omega} = 2 \left(\frac{0.24\hat{i} + 0.36\hat{j} - 0.2\hat{k}}{\sqrt{0.24^2 + 0.36^2 + 0.2^2}} \right) = 2 \hat{AC}$$

$$\vec{\omega} = \frac{2}{0.476} (0.24\hat{i} + 0.36\hat{j} - 0.2\hat{k})$$

$$= \hat{i} + 1.5\hat{j} - 0.84\hat{k}$$

Teacher's Signature..... 4.2

2.9, $\boxed{2.5 \& 2.8}$
(w x AB)

2.1 to 2.3

$$\vec{\omega} \times \vec{r} = (\hat{i} + 1.5\hat{j} - 0.84\hat{k}) \times (-0.36\hat{j})$$

$$= -0.36\hat{k} - 0.3\hat{i}$$

$$\vec{v} = -0.3\hat{i} - 0.36\hat{k}$$

$$\vec{\alpha} = 7 \left(\frac{0.24\hat{i} + 0.36\hat{j} - 0.2\hat{k}}{\sqrt{0.24^2 + 0.36^2 + 0.2^2}} \right)$$

$$= \frac{7}{0.476} = 14.7$$

$$\vec{\alpha} = 3.53\hat{i} + 5.3\hat{j} - 2.9\hat{k}$$

$$\vec{a} = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$$

$$\vec{\alpha} \times \vec{r} = (3.53\hat{i} + 5.3\hat{j} - 2.9\hat{k}) \times (-0.36\hat{j})$$

$$= -1.27\hat{k} - \hat{i}$$

$$\vec{\alpha} \times \vec{r} = -\hat{i} - 1.27\hat{k}$$

$$\vec{a} = (-\hat{i} - 1.27\hat{k}) - 3.9(-0.36\hat{j})$$

$$= -\hat{i} - 1.27\hat{k} + 1.4\hat{j}$$

$$\vec{a} = -\hat{i} + 1.4\hat{j} - 1.27\hat{k}$$

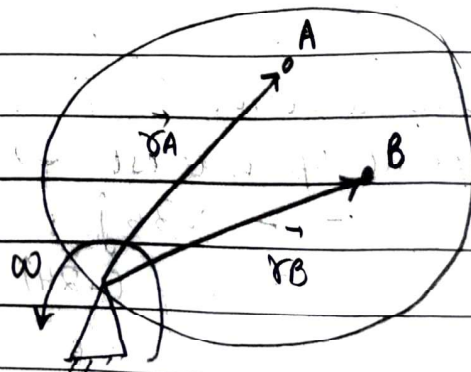
$$\vec{v}_A = \vec{\omega} \times \vec{r}_A$$

$$\vec{v}_B = \vec{\omega} \times \vec{r}_B$$

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$$= \vec{\omega} \times (\vec{r}_B - \vec{r}_A)$$

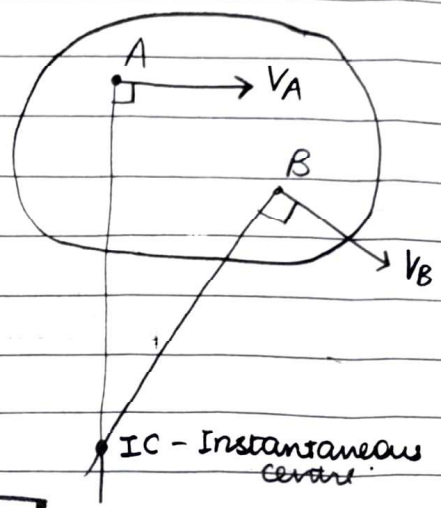
$$\vec{v}_{BA} = \vec{\omega} \times \vec{AB}$$



INSTANTANEOUS AXIS OF ROTATION

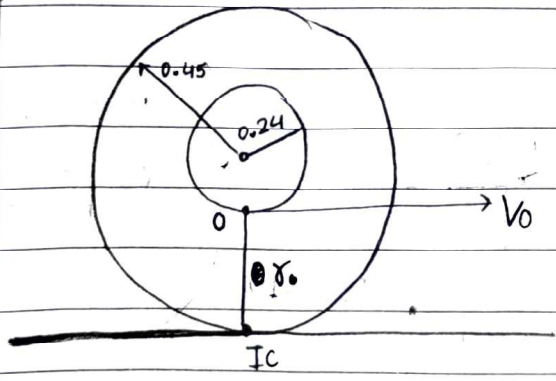
In this topic, we will solve problems choosing a unique reference point, which momentarily has zero velocity. As far as velocities are concerned, a body may be considered to be in pure rotation about an axis normal to the plane of motion passing through this point on the plane, called instantaneous centre of rotation, and the axis is called instantaneous axis of rotation.

if V_A & V_B are parallel,
 intersection
 no rotation, hence no
 rotation and \therefore no IC.



$$\omega = \frac{V_A}{r_A} = \frac{V_B}{r_B}$$

Q:
 [2.11]



no slipping \rightarrow rolling
 only possible b/w surface & wheel
 \rightarrow no relative velocity b/w
 hub and wheel
 \hookrightarrow this is ~~not~~ rigid body

$$\omega = \frac{V_0}{r_0} = \frac{0.7}{0.45 - 0.24}$$

$$= \frac{0.7}{0.21}$$

$$\omega = 3.33 \text{ rad/s}$$

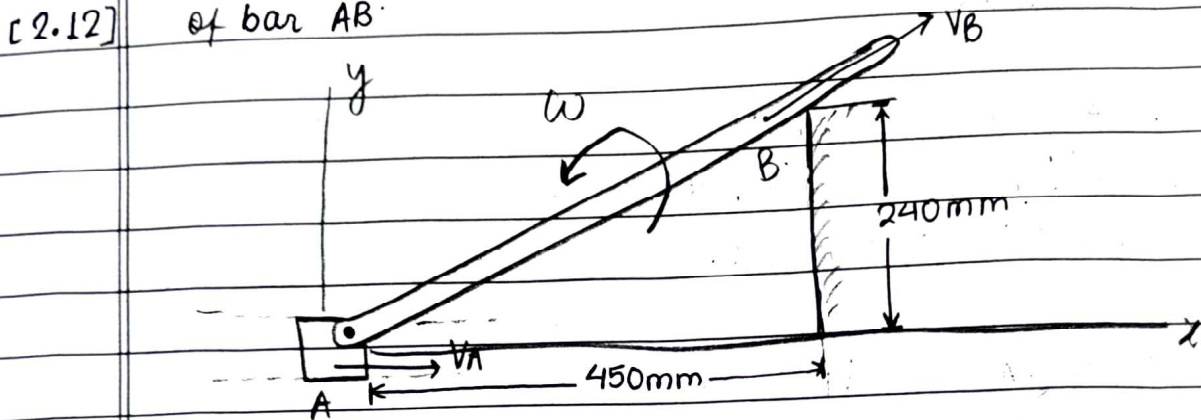
Teacher's Signature.....

* NOT TO BE CONFUSED: A point ^{The points in the space} not on the rigid body ~~could~~ could be stationary, would that be instantaneous centre of rotation? NO! -

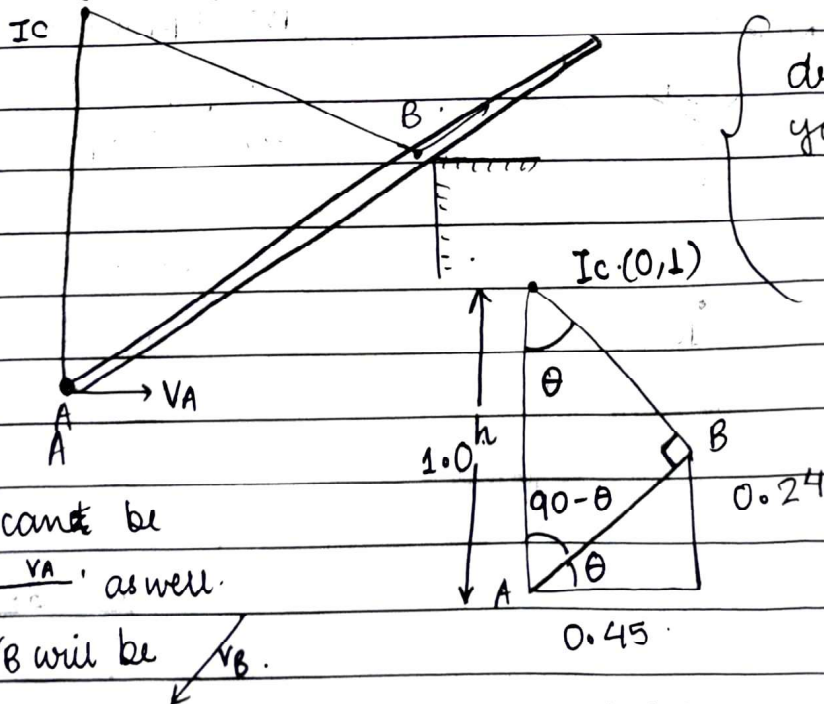
The space we are dealing with is an imaginary extension of the rigid body.

∴ Instantaneous centre must fall on the rigid body.

Q: Determine coordinates of instant centre for velocities of bar AB.



contacts must be maintained at all times
A is going along x-axis, B is going along the rod.



draw to scale you will get the answer

VA cannot be ← VA as well.

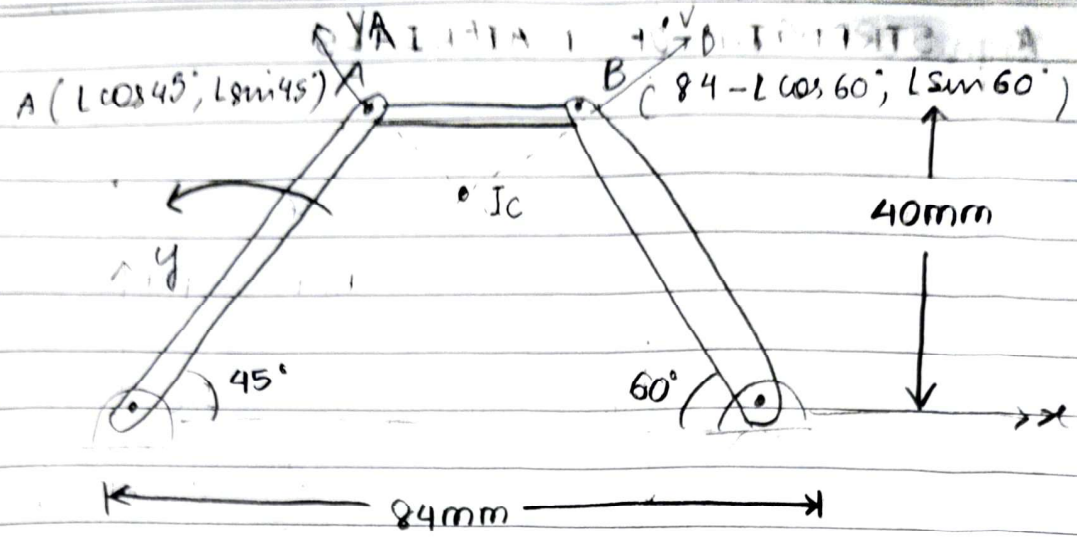
then VB will be ↗ VB.

$$AB = \sqrt{0.24^2 + 0.45^2} = 0.51$$

$$\sin \theta = \frac{AB}{h} \quad h = AB / \sin \theta = AB^2 / 0.24 = \frac{0.51^2}{0.24} = 1.08375$$

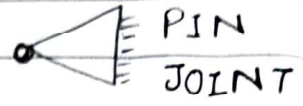
Teacher's Signature.....

Q:
 [2.13]

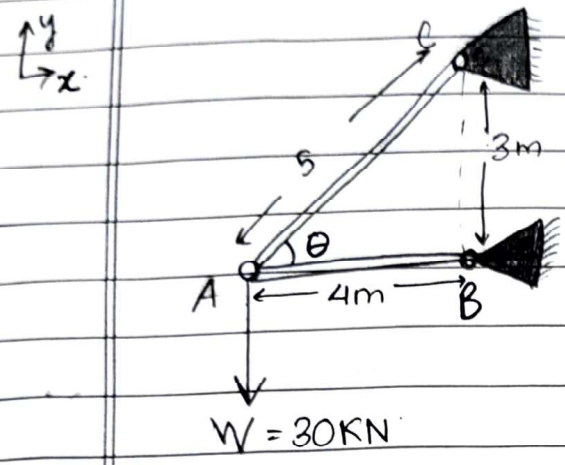


$$\left. \begin{aligned} y_A = y_B = 40 \text{ mm} \\ L = 75.3 \end{aligned} \right\}$$

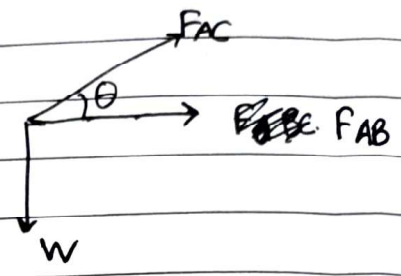
★ STRENGTH OF MATERIALS



considered the members are massless



FBD of pin at A



$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$F_{AC} \sin \theta = W$$

$$F_{AC} \cos \theta = -F_{AB}$$

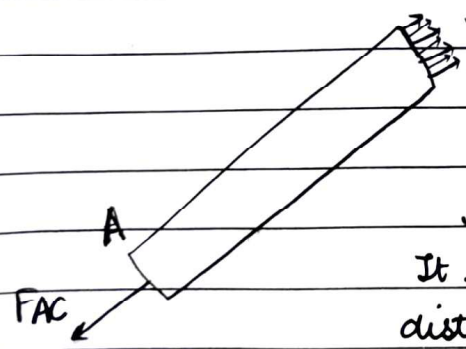
$$F_{AC} = \frac{30 \text{ kN} \times 5}{3} = 50 \text{ kN}$$

$$F_{AB} = -40 \text{ kN}$$

$$F_{AC} = 50 \text{ kN}$$

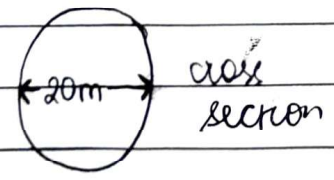
$$F_{AB} = -40 \text{ kN}$$

distributed resisting force



STRESS: qualitatively, stress is internal distributed resistive force. It is quantified as intensity of the distribution, that is ~~the total~~ Total distributed force, divided by area.

$$\sigma = \frac{F_{AC}}{A}$$

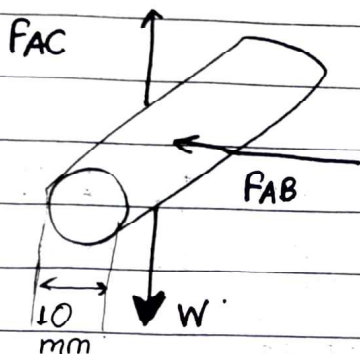


Teacher's Signature.....

In member AC, $\sigma = \frac{50 \times 10^3}{\frac{\pi \times 20^2}{4}} = 159.23 \text{ N/mm}^2$
 $= 159.23 \times 10^6 \text{ N/m}^2$
 $\sigma = 159.23 \text{ MPa}$

Stress in AB = $\frac{-40 \times 10^3}{30 \times 40} = -33.33 \text{ N/mm}^2$ (compression)
 $= -33.33 \text{ N/mm}^2$

cutting \rightarrow shear failure. :-



double shear = $\frac{P/2}{\text{area}}$

shear force = $\frac{P}{\text{area}}$

shear stress in pin $\tau = \frac{FAC}{A_c}$

$A_c \rightarrow$ cross section area of the pin

Young's Modulus = $\frac{\sigma}{\epsilon} = \frac{\text{stress}}{\text{strain}}$

Hooke's law, $F = -Kx$

1.67 $A = 25^2 = 625 \text{ mm}^2$
 or $F = \frac{50 \times 10^3}{625} \text{ N/mm}^2 = 80 \text{ MPa}$
 1.57

Hooke's law $\Rightarrow \sigma = E \cdot \epsilon$

$$\epsilon = \frac{\sigma}{E} = \frac{80}{200 \times 10^3} = 400 \times 10^{-6}$$

$$\frac{\Delta L}{L} = \epsilon, \quad \Delta L = \epsilon L$$

ϵ \rightarrow strain.

1.58 $A = \pi (25^2 - 12^2) \text{ mm}^2 = 481 \pi / 4 = 1511.01 \text{ mm}^2 / 4$
 $F = \frac{40 \times 10^3}{A} = \frac{40 \times 10^3}{1511.01} = 26.47 \text{ MPa}$
 $A = 377.77$ $= 105.88 \text{ MPa}$

$A = 377.77$ $\sigma_{\text{max}} = 225 \text{ MPa} = \frac{P}{A}$

$$P_{\text{max}} = \sigma_{\text{max}} \cdot A$$

$$= 225 \times 377.77$$

$$= 84998.25 = 85 \text{ KN}$$

1.59 a) $\tau = \frac{F}{A}$ b) $\tau = \frac{F}{2A}$

$(\tau_{\text{max}})_{\text{Brass}} = 50 \text{ MPa}$

$$A = \frac{P}{\tau_{\text{max}}} = \frac{30 \times 10^3}{50 \times 10^5} \text{ m}^2$$

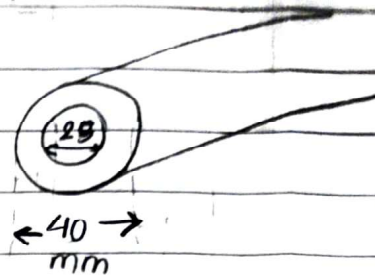
Teacher's Signature.....

$$1.6) \quad (\text{stress}) = (\sigma)_{\max} = 180 \text{ MPa}$$

$$\text{area Tube} = \frac{\pi (40^2 - 25^2)}{4}$$

$$= 765.763 \text{ MPa}$$

=



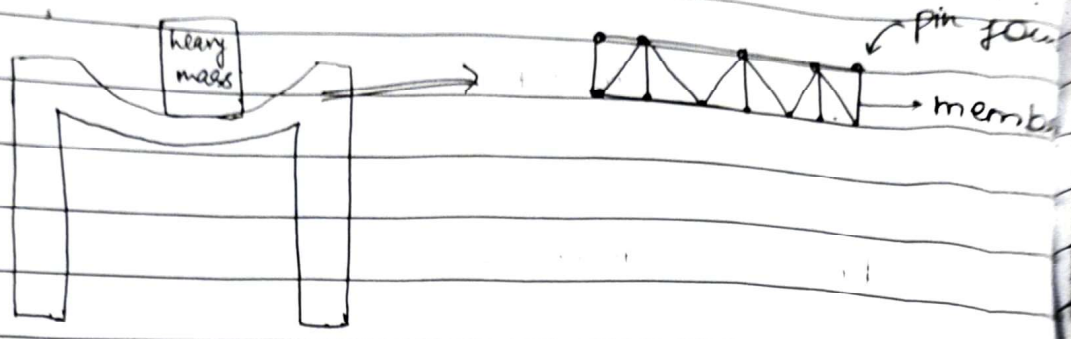
$$P_{\max} = \sigma_{\max} \cdot A$$

★ FRICTION

→ Revise 11th concepts

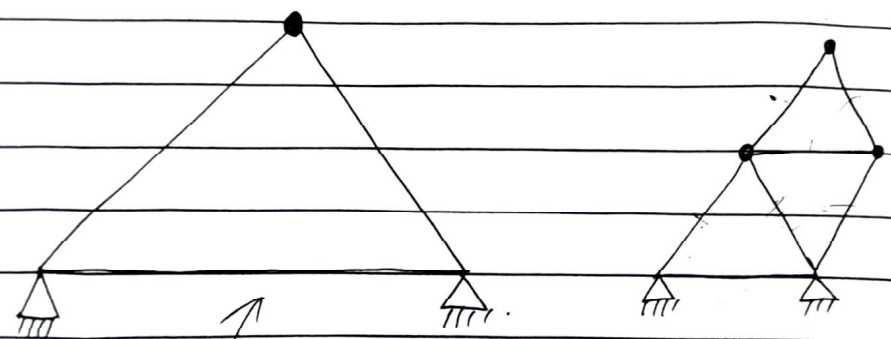
→ Solve 3.1 to 3.8

TRUSSES



A truss is considered to be a rigid structure. A truss is a system of members that are fastened together at their ends to support stationary and moving loads.

Q:

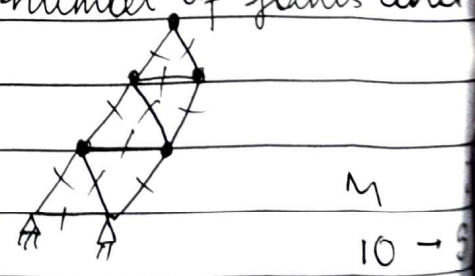


Smallest truss (coplanar)

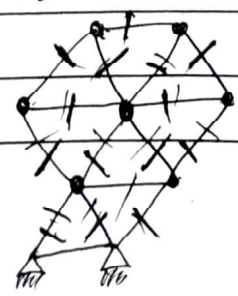
A larger rigid truss can be obtained by adding two new members attached to two existing joints and the new joint.

Q: find the relation between number of joints and no. of members.

M	J	Member joints
2	1	6 → 3
		2 → 1

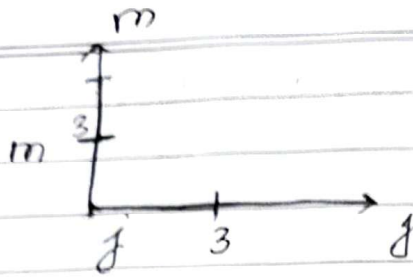


$2j = m$



M	J
	7

Teacher's Signature.....



$$\frac{dm}{dj} = 2$$

$$dm = 2j$$

$$m = 2j + C$$

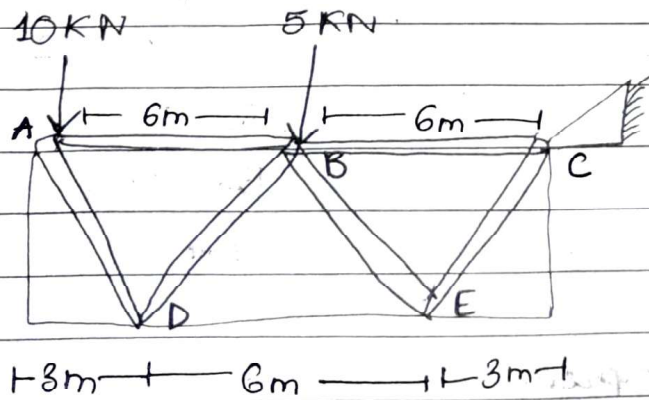
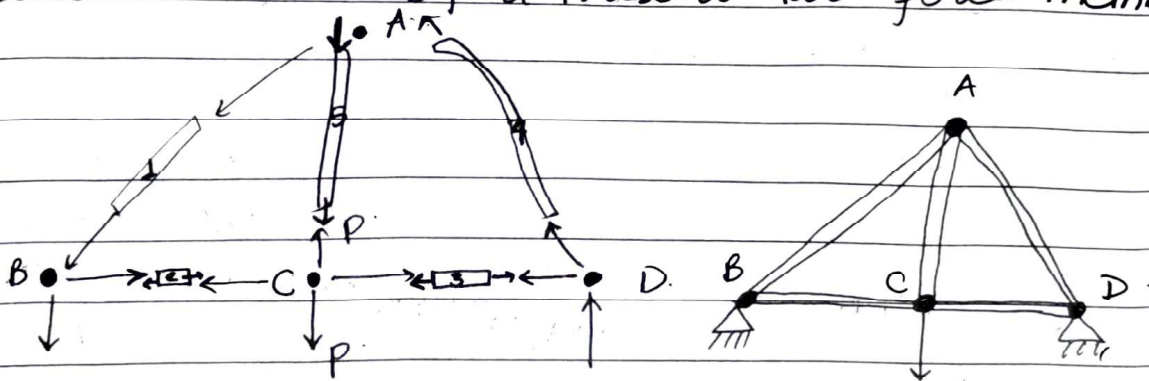
⇒ WHAT WE HAVE TO DO ?

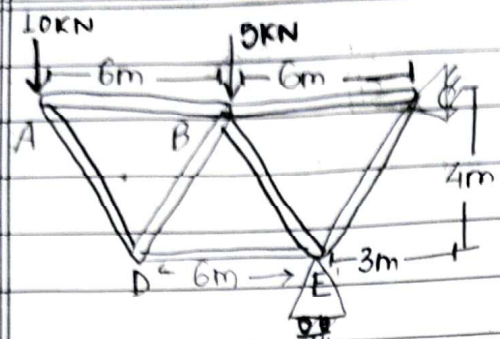
$$m = 2j - 3$$

In this chapter, we have to find forces in members of truss

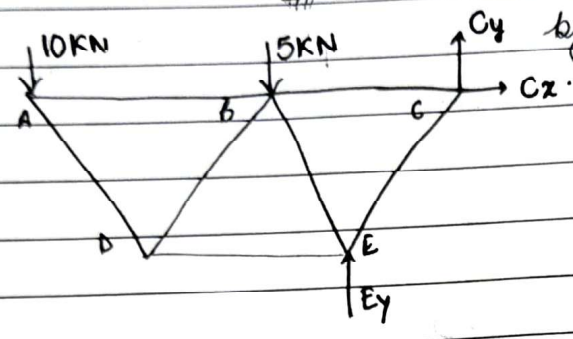
⇒ ASSUMPTIONS OF TRUSS :-

- 1) Truss members are connected only at their ends by frictionless pins.
- 2) Trusses are loaded only at their joints.
- 3) The weights of members are neglected.
- 4) each member of a truss is two force member.





Step 1: Draw FBD of whole truss that is, replace external supports by suitable external force.



Step 2: find reactions from support by applying equations of equilibrium.

$$\sum F_x = 0 \Rightarrow C_x = 0$$

$$\sum F_y = 0$$

$$\Rightarrow E_y + C_y - 10 - 5 = 0$$

$$C_y + E_y = 15 \quad \text{--- (1)}$$

Step 3: Begin with joint. The joint should not have more than ~~three~~ ^{two} unknown force members.

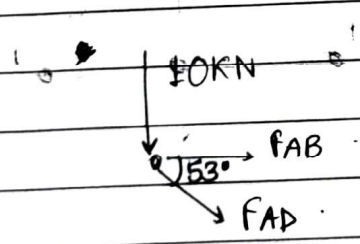
$$\sum M_c = 0$$

$$10 \times 12 + 5 \times 6 - E_y \times 3 = 0$$

$$E_y = 50 \text{ KN}$$

$$\downarrow C_y = -35 \text{ KN}$$

FBD of Joint A



FBD = 12.5 KN (T)

$F_{AD} = 12.5 \text{ KN (C)}$
$F_{AB} = 7.5 \text{ KN (T)}$
$F_{DE} = 15 \text{ KN (C)}$
$F_{BE} = -18.75 \text{ (C)}$
$F_{EC} = 43.75 \text{ KN (C)}$
$F_{CB} = 26.25 \text{ KN}$

$$\sum F_y = 0$$

$$F_{AD} \sin 53^\circ + 10 \text{ KN} = 0$$

$$F_{AD} = \frac{-10}{\sin 53^\circ} = -12.5 \text{ KN}$$

$$\sum F_x = 0$$

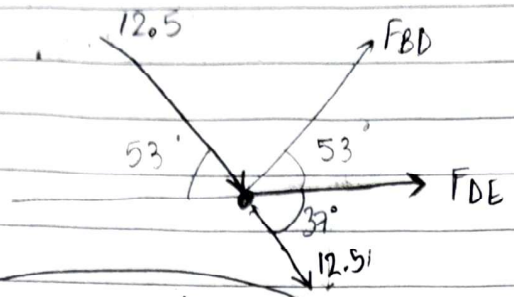
$$F_{AB} + F_{AD} \cos 53^\circ = 0$$

$$F_{AB} = 7.5 \text{ KN (T)}$$

$F_{AD} = 12.5 \text{ KN (Compressive)}$

Teacher's Signature.....

→ FBD of joint D.



$$\sum F_y = 0$$

$$FBD \sin 53 - 12.5 \sin 53 = 0$$

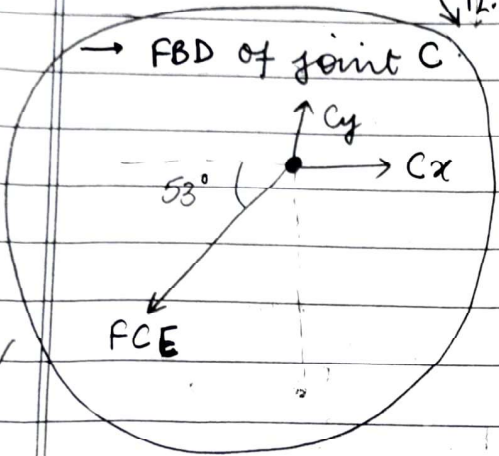
$$FBD = 12.5 \text{ KN (T)}$$

$$\sum F_x = 0$$

$$FDE + FBD \cos 53 - 12.5 \cos 53 = 0$$

$$FDE = 12.5 \cos 53 - 12.5 \cos 53 = 0$$

→ FBD of joint C.



$$FDE + FBD \cos 53 - 12.5 \cos 37 = 0$$

$$FDE = -12.5(0.6) + 12.5(0.8) = 12.5(0.2) = 2.5$$

$$12.5 \cos 53 + FDE + FBD \cos 53 = 0$$

$$-FDE = 2 \times 12.5 \times 0.6$$

$$= 2.5 \times 1.2 \times 12.5$$

$$FDE = -15 \text{ KN}$$

$$FDE = 15 \text{ KN (C)}$$

$$\sum F_x = 0$$

$$Cx = FCE$$

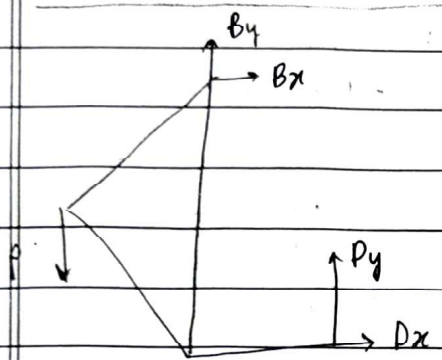
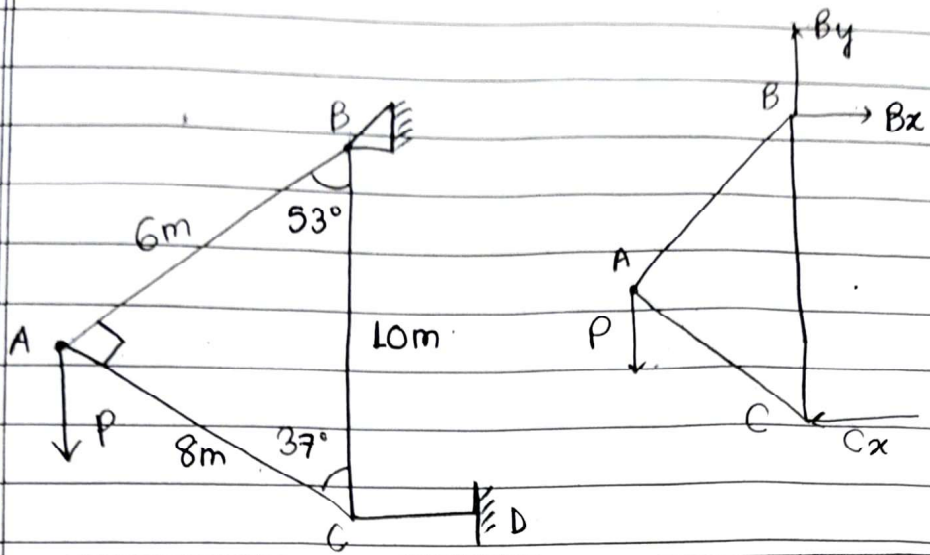
$$-FCE \sin 53 + Cy = 0$$

$$FCE = \frac{Cy}{\sin 53} = \frac{-35}{\sin 53} = -43.8 \text{ KN}$$

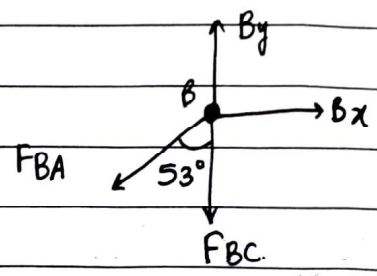
$$= 43.8 \text{ (C)}$$

→ FBD of Joint E.

Q:



Joint B



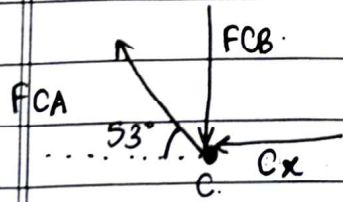
No. of eq^m eqⁿ
 $\sum F_x = 0, \sum F_y = 0, \sum M_o = 0$
 3 eqⁿ → 4 unknowns
 cannot be solved.

$$\sum F_x = 0 \quad B_x - F_{BA} \sin 53^\circ = 0$$

$$B_x = 1.25 C_x \cdot \sin 53^\circ$$

$$B_x = C_x$$

Joint C



$$\sum F_y = 0$$

$$-F_{BC} + B_y - F_{BA} \cos 53^\circ = 0$$

$$B_y = F_{BC} + F_{BA} \cos 53^\circ$$

$$= -1.32 C_x + 1.25 C_x$$

$$B_y = -0.07 C_x$$

$$B_y = 0.07 C_x \quad (C)$$

$$\sum F_x = 0$$

$$-F_{CA} \cos 53^\circ - C_x = 0$$

$$F_{CA} = \frac{-C_x}{\cos 53^\circ} = -1.66 C_x$$

$$\sum F_y = 0$$

$$-F_{CB} + F_{CA} \sin 53^\circ = 0$$

$$F_{CB} = F_{CA} \sin 53^\circ$$

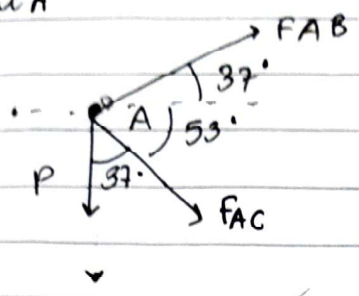
$$= -1.66 C_x \cdot \sin 53^\circ$$

$$F_{CA} = 1.66 C_x \quad (C)$$

$$F_{CB} = 1.32 C_x \quad (C)$$

Teacher's Signature.....

Joint A



$$\sum F_x = 0$$

$$F_{AC} \cos 53^\circ + F_{AC} + F_{AB} \cos 37^\circ = 0$$

$$F_{AC} - 0.6 F_{AC} + 0.8 F_{AB} = 0$$

$$F_{AB} = \frac{0.6 F_{AC}}{0.8}$$

$$\sum F_y = 0$$

$$-P + F_{AC} \sin 53^\circ + F_{AB} \sin 37^\circ = 0$$

$$P = -1.66 C_x \times 0.8 + 1.25 C_x \times 0.8 = 0$$

$$P = -1.325 C_x + 1.0 C_x$$

$$P = 8.675 C_x \text{ (T)}$$

$$= +1.25 F_{AC} \cdot C_x$$

$$= 1.25 C_x \text{ (T)}$$

$$= F_{AB}$$

$$B_y - F_{BC} - F_{BA} \cos 53^\circ = 0$$

$$B_x - F_{BA} \sin 53^\circ = 0$$

$$\sum M_B = 0$$

$$P (8 \sin 37^\circ) - C_x \times 10 = 0$$

$$C_x = \frac{P (8 \sin 37^\circ)}{10} = 0.48 P$$

$$C_x = 0.48 P = \frac{2.4}{5}$$

$$\sum F_x = 0$$

$$B_x - C_x = 0$$

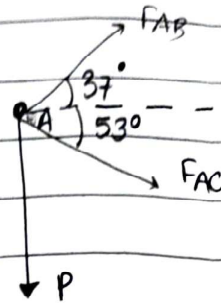
$$\sum F_y = 0$$

$$B_y - P = 0$$

$$B_y = P$$

Joint A

FBD of Joint (pin at A)



$$\sum F_x = 0$$

$$F_{AB} \cos 37^\circ + F_{AC} \cos 53^\circ = 0$$

$$\sum F_y = 0$$

$$F_{AB} \sin 37^\circ - P - F_{AC} \sin 53^\circ = 0$$

$$F_{AC} = -0.8P = 0.8P(C)$$

$$F_{AB} = 0.6P (T)$$

★ METHOD OF SECTIONS :-

SECTION METHOD

- To solve truss problems in which specific member forces are required.

In this method

- a section must divide truss into two parts.
- a section should cut not more than 3 members, preferably.

Q1: Classification and working of Boilers

Boilers are closed vessels where water is heated to produce steam. They are classified based on several criteria :

1 RELATIVE POSITION OF WATER AND HOT GASES.
Fire Tube and Water Tube

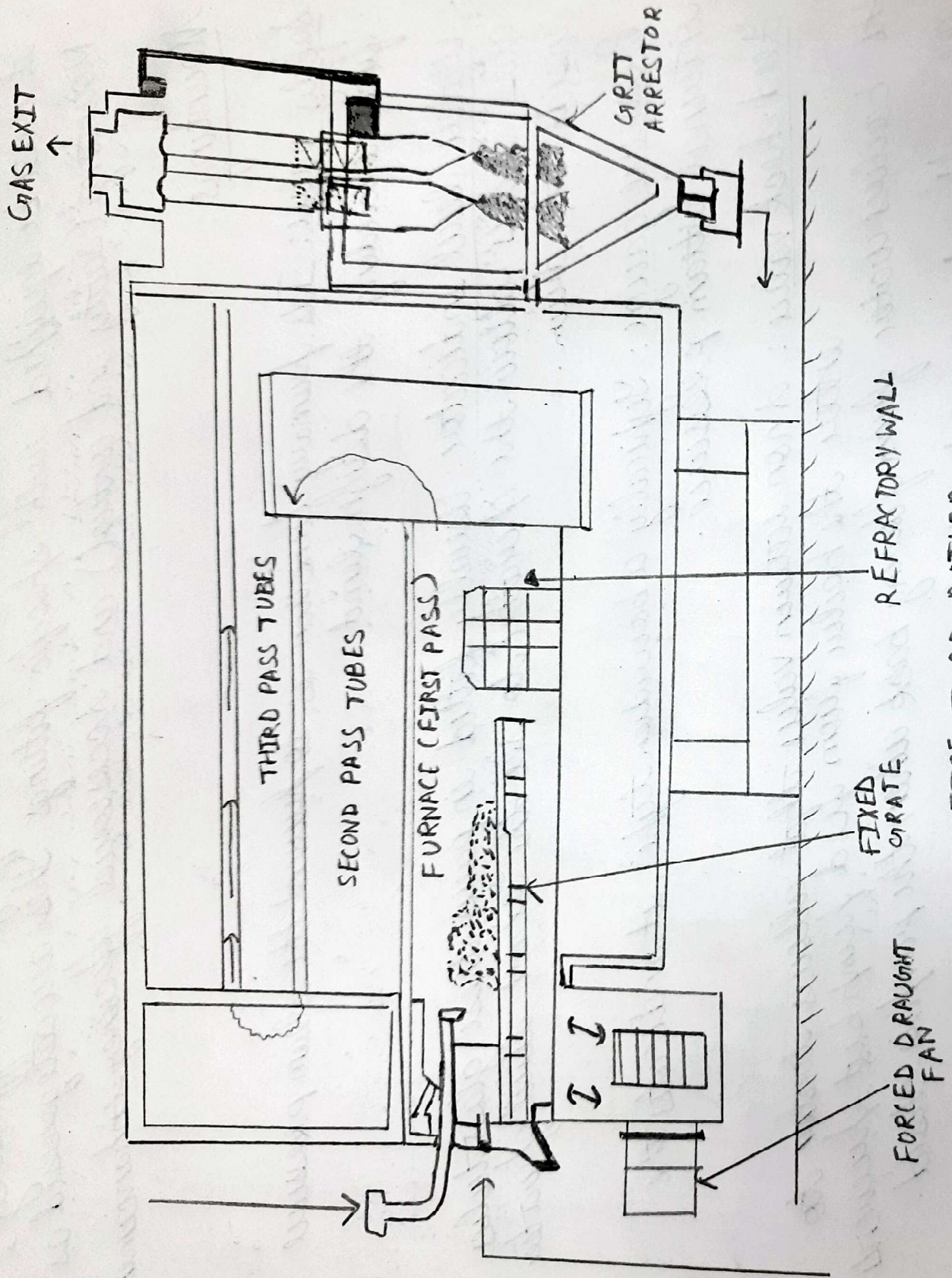
2 AXIS OF THE SHELL : Vertical and horizontal

3 METHOD OF FIRING : Externally fired and Internally fired

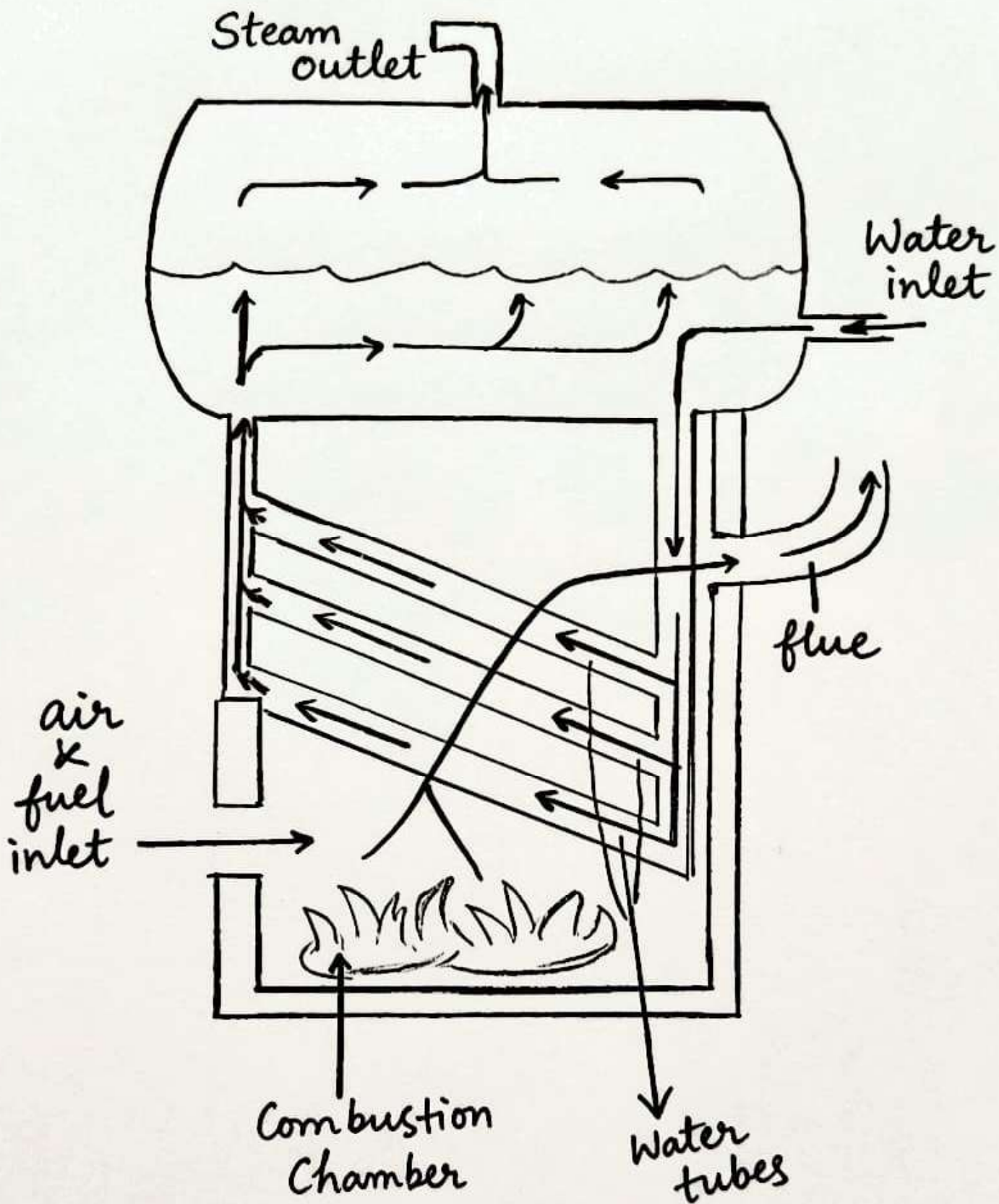
4 WATER CIRCULATION : Natural circulation and Forced circulation

5 PRESSURE OF STEAM : High pressure and low pressure.

FEATURES	FIRE TUBE BOILER	WATER TUBE BOILER
Working	Hot gases pass through tubes surrounded by water.	Water passes through tubes surrounded by hot gases.
PRESSURE	Limited to low pressure (upto 25 bar)	Can handle very high pressure (upto 165 bar)
Rate of Steam	Lower rate of steam production	High rate of steam production
Risk of Explosion	Less risk, less serious if it happens.	High risk due to high operating pressures.
Examples	Cochran, Lancashire, Locomotive boilers.	Babcock & Wilcox, Stirling Boilers.



FIRE TUBE BOILER



Q2: Boiler Mountings and Accessories.

→ **BOILER MOUNTINGS** : These are components fitted onto the boiler for safety and control. Without mountings, the boiler cannot operate safely.

- **SAFETY VALVE** : Automatically releases excess steam if pressure exceeds the limit.

- **WATER LEVEL INDICATOR** : Shows the level of water inside the boiler shell.

- **PRESSURE GAUGE** : Indicates the pressure of steam inside the boiler.

- **FUSIBLE PLUG** : Protects the boiler from overheating if the water level falls too low.

→ **BOILER ACCESSORIES** :

These are integral parts of the system used to increase the efficiency of the boiler.

- **ECONOMIZER** : heats the feed water using flue gases

- **AIR PRE-HEATER** : Heats the air used for combustion using waste flue gases.

- **SUPER-HEATER** : Increases the temperature of steam above its saturation point.

Q3: BOILER EFFICIENCY :

→ Definition : Boiler efficiency is the ratio of the heat actually utilized in generating steam to the heat released by the complete combustion of the fuel.

$$\text{Boiler Efficiency } (\eta) = \frac{m_s (h - h_{fl})}{m_f \times CV} \times 100$$

where :

- m_s = mass of steam generated
- h = Enthalpy of steam
- h_{fl} = Enthalpy of feed water
- m_f = mass of fuel used
- CV = calorific value of fuel

→ Factors affecting Efficiency :

- STACK LOSSES : Heat lost through flue gases.
- INCOMPLETE COMBUSTION : Fuel not burning fully.
- SCALE FORMATION : Deposit inside tubes reduces heat transfer.
- MOISTURE IN FUEL : Requires energy to evaporate, reducing net heat.

Q5: WORKING PRINCIPLE OF ENGINES.

→ FOUR - STROKE ENGINE

Completes one power cycle in four strokes of the piston (Suction, compression, Power and Exhaust). The crankshaft rotates twice (720°).

→ TWO - STROKE ENGINE

Completes one power cycle in two strokes. Suction and compression happen simultaneously, as do power and exhaust. The crankshaft rotates once (360°).

→ PETROL VS DIESEL ENGINES

Features	Petrol Engine (SI)	Diesel Engine (CI)
Fuel	Petrol (Gasoline)	Diesel
Ignition	spark plug used	Heat of compression used
Compression Ratio	Low (6 to 10)	High (15 to 25)
Efficiency	Lower Thermal Efficiency	Higher Thermal efficiency
Applications	light vehicles (Cars, Bikes)	Heavy vehicles (Truck, Buses)

Q6: NON-CONVENTIONAL ENERGY SOURCES.

→ Energy Resources.

- RENEWABLE: Sources that are replenished naturally (Solar, Wind, hydro)
- NON RENEWABLE: Sources that exist in finite amounts and take millions of years to form (Coal, Petroleum, Natural Gas).

→ Advantages of Renewable Energy.

- Environmentally friendly (low carbon footprint)
- Inexhaustible supply
- Reduces dependence on fossil fuels.

→ Working Principles

- SOLAR ENERGY: Uses photovoltaic (PV) cells to convert sunlight directly into electricity via the photoelectric effect.
- WIND ENERGY: Wind rotates turbine blades, converting kinetic energy into mechanical energy ~~via the photoelectric effect~~ which, a generator then converts into electricity.
- HYDRO ENERGY: Falling water turns a turbine connected to a generator. It converts the potential energy of water into electricity.