# DEPARTMENT OF MATHEMATICS

**MA1201** **Engineering Mathematics** **(3-1-0-4)**

**Tutorial Sheet No. -- 1** **2016 – 17**

**MODULE I**

**1.** State the Cauchy's general principle of convergence of a sequence.

**2.** Show that the sequence $\{a_n\}$ cannot converge, where

(i) $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + .......... + \frac{1}{n}$ (ii) $a_n = 1 + \frac{1}{4} + \frac{1}{7} + .......... + \frac{1}{3n-2}$

**3.** If $a_n = \left(1 + \frac{1}{n}\right)^n, \forall\, n \in N,$ prove that $2 < a_n < 3,$ , and that the sequence $\{a_n\}$ is convergent.

**4.** If $a_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + .......... + \frac{1}{n!}$ , $\forall n \in N$, prove that $0 < a_n < 2$ , and that the sequence $\{a_n\}$ is convergent.

**5.** State Cauchy's first and second theorem on limits. Hence prove the followings:

(i) $\lim_{n \to \infty} \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + .......... + \frac{1}{(2n)^2}\right] = 0$

(ii) $\lim_{n \to \infty} \left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + .......... + \frac{1}{\sqrt{n+n}}\right] = \infty$

(iii) $\lim_{n \to \infty} \frac{1}{n}\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + .......... + \frac{1}{n}\right] = 0$

(iv) $\lim_{n \to \infty} \frac{1}{n}\left[1 + \sqrt{2} + \sqrt[3]{3} + \sqrt[4]{4} + .......... + \sqrt[n]{n}\right] = 1$

(v) $\lim_{n \to \infty} \left[\left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3 .......... \left(\frac{n+1}{n}\right)^n\right]^{1/n} = e$

**6.** Find the limit of the sequences $\{(a_n)^{1/n}\}$, converge and find their limits when

(i) $a_n = \frac{(3n)!}{(n!)^3}$ (ii) $a_n = \frac{[(m+1)(m+2).....(m+n)]^{1/n}}{n}$

**7.** Show that the sequence $\{S_n\}$ of positive real nos. such that $S_n = \frac{1}{2}(S_{n-1} + S_{n-2}),\ \forall\ n > 2,$ converges.

**8.** Show that the sequence $\{S_n\}$, defined by the recursion formula $S_{n+1} = \sqrt{3S_n},\ S_1 = 1,$ converges to 3.

**MODULE II**

**9.** Discuss about the convergence and divergence of the Geometric series.

**10.** Prove that the series $\sum \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \le 1$

**11.** For positive term series, describe:

(i) Comparison tests (ii) D'Alembert's ratio test (iii) Cauchy's root test

(iv) Raabe's test (v) Logarithmic test (vi) Higher Logarithmic test

(vii) Gauss test

**12.** State and prove Leibnitz test for an alternating series.

**13.** Define Absolute and Conditional convergence of a series of arbitrary term.

**14.** Test the behaviour of the following infinite series:

(i) $\sum \left( (n^3 + 1)^{1/3} - n \right)$

(ii) $\sum \left( \frac{n^2}{n^3 + 1} \right) x^{n-1}$

(iii) $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + .........$

(iv) $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} .........$

v) $\sum \frac{\log n}{n^{3/2}}$

(vi) $\sum \left( \frac{n}{n+1} \right)^{n^2}$

(vii) $\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + ......$

(viii) $\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \frac{1}{(\log 4)^p} + .....$

(ix) $\sum \frac{3.6.9.............3n}{7.10.13.........(3n+4)} x^n,\ \ x > 0$

(x) $1^p + \left( \frac{1}{2} \right)^p + \left( \frac{1.3}{2.4} \right)^p + \left( \frac{1.3.5}{2.4.6} \right)^p + ....$

(xi) $\frac{x}{2} + \left( \frac{1.3}{2.4} \right) \frac{x^3}{6} + \left( \frac{1.3.5.7}{2.4.6.8} \right) \frac{x^5}{10} + ....$

(xii) $1 + \frac{2^2}{3^2} x + \frac{2^2.4^2}{3^2.5^2} x^2 + ...........$

(xiii) $x^2 + \frac{2^2}{3.4} x^4 + \frac{2^2 4^2}{3.4.5.6} x^6 + .........$

(xiv) $\sum \frac{2^2.4^2.6^2.............(2n-2)^2}{3.4.5.6..........(2n-2)(2n-1)} x^{2n},$

(xv) $1 + \frac{x}{1!} + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + ...., x > 0.$

(xvi) $\left(\frac{2^2}{1}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+....$ (xvii) $1+\frac{2x}{2!}+\frac{3^2x^2}{3!}+\frac{4^3x^3}{4!}+....,$.

(xviii) $1+\frac{x}{2}+\frac{2!\,x^2}{3^2}+\frac{3!\,x^3}{4^3}+..........$ (xix) $1+x^{1+\frac{1}{2}}+x^{1+\frac{1}{2}+\frac{1}{3}}+x^{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}..........$

(xx) $x^2(\log 2)^p+x^3(\log 3)^p+x^4(\log 4)^p+.....$ (xxi) $\sum\left(\frac{nx}{n+1}\right)^n, x>0$

(xxii) $1+\frac{\alpha.\beta}{1.\lambda}x+\frac{\alpha(1+\alpha)\beta(\beta+1)}{1.2.\gamma.(1+\gamma)}x^2+........., \; x>0.$

**15.** Show that the series $\frac{1}{1^p}-\frac{1}{2^p}+\frac{1}{3^p}-....................$ converges for $p>0$.

**16.** Prove that the series $\sum_{n=2}^{\infty}\frac{(-1)^{n+1}}{n^p}$ is absolutely convergent for $p>1$, but conditionally convergent for $0<p\le 1$.

**17.** Prove that the following series are absolutely convergent:

(i) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+....................$ (ii) $\sum\frac{\sin n\alpha}{n^2}$ (iii) $\sum\frac{(-1)^n(n+2)}{2^n+5}$

(iv) $\frac{1}{2}-\frac{1}{2.2^2}+\frac{1}{3.2^3}-....................$

**18.** Prove that the following series are conditionally convergent:

(i) $\sum\frac{(-1)^{n+1}}{n^{1/2}}$ (ii) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\log(1+n)}$

**MODULE III**

**19.** Expand the following function:

(i) $e^{\sin x}$ as far as the term containing $x^3$

(ii) log (sinx) in power of (x-3) (iii) $3x^3-2x^2+x-4$ in power of (x-2) .

**20.** Using $\delta-\varepsilon$ approach, show that

(i) $\lim_{(x,y)\to(1,2)}(x^2+2y)=5$ (ii) $\lim_{(x,y)\to(0,0)}\frac{xy(x^2-y^2)}{(x^2+y^2)}=0$

**21.** Show that the limit does not exist in each case:

(i) $\lim_{(x,y)\to(0,0)} \frac{xy^3}{(x^2+y^6)}$ (ii) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{(x-y)}$

**22.** Investigate the continuity at (0,0) of $f(x,y)=\begin{cases}\frac{(x^2-y^2)}{(x^2+y^2)}, (x,y)\neq(0,0)\\ 0, \quad (x,y)=(0,0)\end{cases}$

**23.** Show that the function $f(x,y)=\begin{cases}\frac{x^3+y^3}{(x-y)}, \quad x\neq y\\ 0, \quad x=y\end{cases}$

is discontinuous at origin but possesses partial derivatives $f_x$ and $f_y$ at every point, including origin.

**24.** Show that the function $f(x,y)=\begin{cases}\frac{(x^2+y^2)}{|x|+|y|}, (x,y)\neq(0,0)\\ 0, \quad (x,y)=(0,0)\end{cases}$

is continuous at origin but its partial derivatives $f_x$ and $f_y$ do not exist at origin.

**25.** If $z=e^{ax+by}f(ax-by)$, show that $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=2abz$

**26.** If $u=e^{xyz}$, find that $\frac{\partial^3 u}{\partial x\partial y\partial z}$

**27.** If $z=y\,f(x^2-y^2)$, show that $y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=\frac{xz}{y}$

**28.** If u = log($x^3 + y^3 + z^3$ - 3xyz), prove that

(i) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$ (ii) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{3}{(x+y+z)^2}$

(iii) $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$

**29.** State and prove Euler's theorem for a homogeneous function of x, y of degree n.

**30.** If $u=\sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$

**31.** If $u=\sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial x}+z\frac{\partial u}{\partial z}=2\tan u$

**32.** If $u = \sin^{-1}\left\{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right\}^{1/2}$, show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{12}\right)$

**33.** If $u = f(r)$, where $r^2 = x^2 + y^2$, show that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$$

**34.** If v is a function of r alone, where $r^2 = x^2 + y^2 + z^2$, show that

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=\frac{d^2 v}{dr^2}+\frac{2}{r}\frac{dv}{dr}$$

**35.** If $r^2 = x^2 + y^2 + z^2$, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\frac{1}{r}=0$.

**36.** If $u = f(y-z, x-y.z-x)$ , Show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}=0$

**37.** Transform $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}$ into polar coordinate system.

**38.** If $V(x,y,z)$ be a homogeneous function of x, y and z of degree n having continuous first order partial derivatives, show that $\frac{\partial V}{\partial x},\frac{\partial V}{\partial y},\frac{\partial V}{\partial z}$ are homogeneous function of x, y and z of degree n-1.

**39.** If $V(x,y,z)$ be a homogeneous function of x, y and z of degree n having continuous first order partial derivatives and if $V = f(X,Y,Z)$ where $X=\frac{\partial V}{\partial x}, Y=\frac{\partial V}{\partial y}, Z=\frac{\partial V}{\partial z}$ then prove that $X\frac{\partial V}{\partial X}+Y\frac{\partial V}{\partial Y}+Z\frac{\partial V}{\partial X}=\frac{n}{n-1}V$

**40.** If $x^2+y^2+z^2-2xyz=1$, show that $\frac{dx}{\sqrt{1-x^2}}+\frac{dy}{\sqrt{1-y^2}}+\frac{dz}{\sqrt{1-z^2}}=1$.

**41.** If $ax^2+by^2+cz^2=1$ and $lx+my+nz=0$, show that:

$$\frac{dx}{bny-cmz}=\frac{dy}{clz-anx}=\frac{dz}{amx-bly}.$$

**42.** Expand the following functions in powers of x and y up to the terms containing third degree:

(i) $f(x,y)=e^x\cos y$ (ii) $f(x,y)=e^x\sin y$

**43.** Expand the following functions by Taylor's theorem:

(i) $f(x,y)=\sin xy$ in powers of $(x-1)$ and $(y-\pi/2)$ up to the terms containing third degree:

(ii) $f(x,y)=x^2y+e^x+\sin y$ in powers of $(x-1)$ and $(y-\pi)$ up to the terms containing 2nd degree:

**44.** Examine the following functions for extreme values:

(i) $f(x,y)=xy-x^2-y^2-2x-2y+4$

(ii) $f(x,y)=\sin x+\sin y+\sin(x+y)$

**45.** Find the absolute maximum and minimum values of $f(x,y)=4x^2+9y^2-8x-12y+4$ over the rectangle in the first quadrant bounded by the lines $x=2,\ y=3$ and the coordinate axes.

**46.** The temperature *T* on the surface of a sphere is given by $T=400\,x\,y\,z^2$. Find the highest temperature on the surface of a unit sphere $x^2+y^2+z^2=1$

**47.** Find the maximum and minimum of $x^2+y^2+z^2$ subject to the condition $xyz=a^3$.

**48.** Prove that the Jacobian of the transformation from the rectangular coordinates $(x,y,z)$ into the spherical coordinates $(r,\theta,\phi)$ is given by $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}=r^2\sin\theta$.

**49.** The roots of the equation $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ in $\lambda$ are *u, v, w.* Prove that the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}=-2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$.

**50.** .If $u=\frac{x+y}{1-xy}$ and $v=\frac{(x+y)(1-xy)}{(1+x^2)(1+y^2)}$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are they functionally related? If so, find the relationship.

**51.** Show that the functions $u = x + y + z$ , $v = xy + yz + zx$ and $w = x^3 + y^3 + z^3 - 3xyz$ are not independent but they are related by the equation $u^3 = 3uv + w$ .

**52.** If the variables $(x, y, u, v)$ are connected by the relation $u^2 + xv^2 - xy = 0$ and $u^2 + v^2 + xyv = 0$ , find $\frac{\partial u}{\partial x}$ .

**MA 1201 Engineering Mathematics (3-1-0-4)**

**Module I**

Sequences, bounded sequences, upper and lower bounds, monotonic sequences, limits of a sequence, convergence of sequence, Cauchy's general principle of convergence, Cauchy's theorems on limits (No proof). [3L]

**Module II**

Convergence of series of real numbers of positive terms. P series test, comparison tests, Cauchy's root test, D' Alembert's ratio test, Raabe's test. Gauss's Ratio Test, Logarithmic and Higher logarithmic Ratio Test, Absolute and conditional convergence, Leibnitz's Rule for Alternating series Test. [6L]

**Module III**

Generalized Mean Value Theorem, Maclaurin's series, Taylor's series of functions.

Functions of several variables, level curves, limits, continuity. Partial Derivatives. Euler's theorem on Homogeneous functions, chain Rule, transformation of independent variables, Total differential. Jacobians, Taylor's series in two or more variables. Maximum, minimum and saddle points of functions of two variables. Several independent variables Lagrange's method of Undetermined Multipliers. [8L]

**Module IV**

Beta and Gamma functions, Double integrals, area, change of order of integration, Evaluation of integrals by transforming into polar co-ordinates. Evaluation of Triple integrals. Volume and surface area by double and triple integration by transforming in to cylindrical and spherical polar co-ordinates. [5L]

**Module V**

Sketching polar equations of conic section, equation of tangent and normal line to a conic section equation of tangent and normal line to a conic section including chord of contact, director circle and asymptote. [4L]

**Model VI**

First order differential equations, linear and Bernoulli's equation, Reduction of order.

Curvature, normal vector, torsion and TNB frame, tangential and normal components of velocity and acceleration, radical and transverse acceleration. Motion in polar. And cylindrical coordinates. Directional derivatives, Gradient, Divergence and curl. Expansions, identities. Tangent plane and normal lines Gradient, divergence and curl in curvilinear co-ordinates. [5L]

**Module VII**

Line integrals, Work, Circulation, Flux, Paths independence, Potential function, Conservative field, Green's theorem in plane, surface and volume integrals Gauss's Divergence theorem, Stoke's theorem. Applications. [4L]

**Text Books:**

TB 1: M.D. Weir, J. Hass and F. R. Giordano: Thomas' Calculus, 11th edition, Pearson Educations, 2008.
TB 2: Dennis G. Zill, Warren S. Wright: Advanced Engineering Mathematics, 4th edition. Jones Nad Bartlertt Publishers, 2010

***Reference Books:***

(i) E. Kreyszig: Advanced Engineering Mathematics, 8th Edition John Wiley and sons 1999.
(ii) T.M. Apostol: Calculus Vols 1 and 11.2nd Edition. John Wiley and sons, 1967 and 1969.