BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: Int. M.Sc. | **SEMESTER : VI**
BRANCH: PHYSICS | **SESSION : SP/2025**

SUBJECT: PH316 STATISTICAL MECHANICS

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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Q.1(a)	How do you define entropy **(S)** in statistical mechanics ? Calculate the entropy (**S**$_L$) for a solid lattice containing N atoms and ***n*** vacancies which are placed one by one on available lattice sites. What is the value of **S**$_L$ in the limit $n \to 0$?	[5]	1	1,2,3
Q.1(b)	Show that a system comprising of 2 non-degenerate energy levels can have negative absolute temperature (**T < 0**). Compute the partition function ($\mathcal{Z}$) of a paramagnetic solid of **N** magnetic atoms (each having magnetic moment μ) which is placed in a uniform magnetic field **B** along the z-axis. What is the entropy **(S)** of this system ?	[5]	1	2,3
Q.2(a)	Draw the plot of Spectral Intensity (**E**$_\lambda$) of Blackbody radiation as a function of wavelength (λ) for 2 different temperatures (**T**$_1$ > **T**$_2$). What are the two empirical laws regarding blackbody radiation that can be deduced from this plot?	[5]	2	1,2
Q.2(b)	Derive the Rayleigh Jeans law for Blackbody radiation and show that it leads to the ultraviolet catastrophe.	[5]	2	1,2
Q.3(a)	Derive the Planck's blackbody radiation formula for the spectral energy density (**u**$_\lambda$) assuming the radiation inside the blackbody to be in thermal equilibrium with the atomic oscillators in the walls of the container having quantized energies: ε = **nh**ν (where, **h** = Planck's constant, ν = radiation frequency, **n** = 0,1,2,…).	[5]	3	1,2,3
Q.3(b)	Show that the Planck's blackbody radiation formula derived above successfully explains both the empirical laws of blackbody radiation while avoiding the ultraviolet catastrophe.	[5]	3	1,2,3
Q.4(a)	Sketch and explain the specific heat of an ideal Bose gas as a function of the temperature parameter T/T_C.	[5]	4	1,2,3
Q.4(b)	The total number of particles in all the excited states of an ideal Bose system has an upper bound, that is, $N_e \leq V\frac{(2\pi mkT)^{3/2}}{h^3}\,\zeta\left(\frac{3}{2}\right)$. Considering liquid Helium as an ideal Bose system, find the temperature T_C at which Bose-Einstein condensation starts. Take $m = 6.65 \times 10^{-27}\text{kg}$ and $V = 27.6\ \times 10^{-6}\,\text{m}^3/\text{mole}$ for Helium. At what temperature half of the total number of particles will be in ground state?	[5]	4	1,2,3
Q.5(a)	Write the mean occupation numbers $\langle n_\varepsilon \rangle$ of the single-particle state of an ideal Fermi system. Sketch the plot of $\langle n_\varepsilon \rangle$ as a function ε in the limit $T \to 0$ and hence define Fermi energy.	[5]	5	1,2,3
Q.5(b)	The specific heat of an ideal Fermi system can be expressed in terms of Fermi-Dirac functions as $C_V = Nk\left\{\frac{15}{4}\frac{f_{5/2}(z)}{f_{3/2}(z)} - \frac{9}{4}\frac{f_{3/2}(z)}{f_{1/2}(z)}\right\}$. Show that the specific heat approaches the value $3Nk/2$ as temperature $T \to \infty$.	[5]	5	1,2,3

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