

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)

CLASS: BTECH/IMSC  
BRANCH: ALL/PHYSICS

SEMESTER : II/ADD  
SESSION : SP/2025

SUBJECT: MA24103/ MA107 MATHEMATICS-II

TIME: 3 Hours

FULL MARKS: 50

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Before attempting the question paper, be sure that you have got the correct question paper.
  5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
- 

- |  | CO  | BL    |
|--|-----|-------|
| Q.1(a) Solve: $x^2y'' + xy' + y = \sin(\log x^2)$ .  | [5] | CO1 2 |
| Q.1(b) Solve the following equation using the method of variation of parameters<br>$y'' - y = \frac{2}{1+e^x}$ .   | [5] | CO1 3 |
| Q.2(a) Find the power series solution of $(x^2 + 1)y'' + xy' - xy = 0$ in the power of $x$ .   | [5] | CO2 2 |
| Q.2(b) Use the definition of $J_n(x)$ to prove $\frac{d}{dx}(x^{-n}J_n(x)) = -x^{-n}J_{n+1}(x)$ .  | [5] | CO2 3 |
| Q.3(a) Find the Fourier series expansion of the following periodic function with period $2\pi$ :<br>$f(x) = x^2, \quad -\pi \leq x \leq \pi$ .<br>Hence, find the sum of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  | [5] | CO3 2 |
| Q.3(b) Use the method of separation of variables to solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions (i) $u = 0$ when $x = 0$ and $x = \pi$ , (ii) $\frac{\partial u}{\partial t} = 0$ when $t = 0$ and (iii) $u(x, 0) = x, 0 < x < \pi$ . | [5] | CO3 3 |
| Q.4(a) Expand the function $f(z) = \frac{1}{(z-1)(z-2)}$ for $1 <  z  < 2$ as a Laurent series expansion.  | [5] | CO4 2 |
| Q.4(b) Evaluate the integral $I = \oint_C \frac{dz}{z^2+z^3}, C:  z  = 2$ using Cauchy Residue theorem.  | [5] | CO4 3 |
| Q.5(a) If the probability density function is defined by $f(x) = \begin{cases} kx^3, & 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$ , then (i) find the value of $k$ , (ii) find the probability between $x = 1/2$ and $x = 3/2$ .  | [5] | CO5 3 |
| Q.5(b) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the proportion, of days on which (i) neither car is used (ii) some demand is refused.   | [5] | CO5 3 |

:25/04/2025:::E