BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc | **SEMESTER: IV**
BRANCH: MATHS & COMPUTING | **SESSION: SP/2025**

SUBJECT: MA206R1 LINEAR ALGEBRA

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Let V be a vector space of all 2×2 matrices over a field of reals. Then show that the set $W = \{\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}, x, y \in R\}$ is a subspace of V.	[5]	1	2
Q.1(b)	For what value of k, will the vector $v = (1, k, -4) \in R^3(R)$ is a linear combination of $v_1 = (1, -3, 2)$ and $v_2 = (2, -1, 1)$.	[5]	1	2
Q.2(a)	Let $V(R)$ be a vector space of integral function on R. Prove that a mapping $T: V \to R$ defined by $T(f) = \int_a^b f(x)dx, \quad f \in V, a, b \in R$ is a linear functional.	[5]	2	3
Q.2(b)	Let $T: V \to W$ be a linear transformation. Prove that the null space of T is a subspace of V.	[5]	2	3
Q.3(a)	Find a matrix representation of the linear transformation $T: \mathrm{R}^2 \to \mathrm{R}^3$ defined by $T(x, y) = (x - 2y, 3x + 4y, 5x - 6y)$ subject to the usual ordered basis for R^2 and R^3.	[5]	3	3
Q.3(b)	If λ is an eigenvalue of the linear operator T, then show that λ^n is an eigen value of T^n.	[5]	3	5
Q.4	If a matrix A is given by $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -6 \\ 2 & -2 & 3 \end{bmatrix}$. (i) Find the eigenvalues of A, (ii) Find a matrix P such $A = P^{-1} D P$.	[10]	4	5
Q.5(a)	(Cauchy-Schwarz Inequality): If V is an inner product space, then show that $\lvert < u, v > \rvert \leq \lVert u \rVert \lVert v \rVert$, for any vectors $u, v \in V$.	[5]	5	4
Q.5(b)	Use the Gram-Schmidt algorithm to find an orthonormal basis for the subspace W of $R^3(R)$ spanned by $v_1 = (1,1,1)$, $v_2 = (0,1,1)$, $v_3 = (0,0,1)$.	[5]	5	5

::::::29/04/2025::::::M