

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)**

CLASS: IMSC  
BRANCH: MATHEMATICS

SEMESTER : II  
SESSION : SP/2025

**SUBJECT: MA106R1 ORDINARY DIFFERENTIAL EQUATIONS**

TIME: 3 HOURS

FULL MARKS: 50

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Before attempting the question paper, be sure that you have got the correct question paper.
  5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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- Q.1(a) Check whether the given equation is self-adjoint. If not, write an equivalent self-adjoint form. [3+2] CO 4 BL 3

$$t^3 \frac{d^2x}{dt^2} - (t^3 + 2t^2 - t) \frac{dx}{dt} + (t^2 + t - 1)x = 0$$

Using Picard's iteration method find the first two approximation of solution.

$$y' = 1 + y^2, y(0) = 0$$

- Q.1(b)  $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(\pi) - y'(\pi) = 0$  [5] 5 5

Find the characteristic values and characteristic functions of the above Sturm-Liouville problem. Discuss all the cases.

- Q.2(a) Solve the initial value problem  $y''' - 2y'' - 5y' + 6y = 0, y(0) = 0, y'(0) = 0, y''(0) = 1$ . [3+2] 2 5

Solve the given differential equation

$$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$$

- Q.2(b) When  $M dx + N dy = 0$  is said to be an exact differential equation? [1+4] 2 5

Is the following equation exact? If not, convert the equation into an exact form and then solve.

$$(x^2 + y^2 + 1)dx + x(x - 2y)dy = 0.$$

- Q.3(a) Find the solution and the fundamental matrix of the corresponding homogeneous system. Then find the solution of the nonhomogeneous system. [5] 5 3

$$\frac{dx}{dt} = \begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix} x + (-2 \sin t, 6 \cos t)^T$$

- Q.3(b) Write the general solution of the differential equation [2+3] 2 1

$$x^2 y'' + xy' + \left(x^2 - \frac{9}{16}\right)y = 0$$

Determine

$$\int_{-1}^1 (P_3(x))^2 + P_2(x) dx,$$

where  $P_n(x)$  is the Legendre polynomial of degree n.

Q.4(a) Solve the following first-order differential equations, which are not of first degree. [5] 3 2  
$$x = 4(p + p^3)$$

Q.4(b) 
$$y = px + (1 + p + p^2)^{\frac{1}{3}}$$
 [5] 3 2  
Where  $p = \frac{dy}{dx}$ . Write the special name of this kind of equations.

Q.5(a) Does the following function satisfy the Lipschitz condition on the rectangle  $[1,2] \times [3,4]$ ? [5] 1 4  
Give a reason.

$$f(x, y) = \frac{y^2}{x^2}$$

Then, find the solution to the initial value problem

$$\frac{dy}{dx} = f(x, y), y(1) = 1.$$

Clearly mention the interval for the existence of the solution.

Q.5(b) Solve the differential equation using the method of undetermined coefficients. [5] 2 3  
$$y''' + 4y'' + y' - 6y = -18x^2 + 1$$

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