

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc
BRANCH: MATHEMATICS AND COMPUTING

SEMESTER : II
SESSION : SP/2025

SUBJECT: MA105R1 CALCULUS II

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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|---|-----|----|
| Q.1(a) Find the foot of the perpendicular from the point $P(1,2,3)$ to the plane $3x + 4y + 5z = 6$ | [5] | 1 |
| Q.1(b) For the sphere: $x^2 + y^2 + z^2 = 49$, find the equation of the tangent plane at the point $(7,0,0)$. | [5] | 1 |
| Q.2(a) Find the area inside the circle $x^2 + y^2 = 9$ and above the line $y = 2$. | [5] | 2 |
| Q.2(b) Change the order of integration of the following double integral and hence evaluate | [5] | 2 |
| $\int_0^1 \int_0^x \frac{x}{x^2+y^2} dy dx .$ | | |
| Q.3(a) Let $\vec{F}(x, y, z) = (x^2yz)\hat{i} + (y^2z)\hat{j} + (xyz^2)\hat{k}$. Check whether the vector function is solenoidal . | [5] | 3 |
| Q.3(b) Let $f(x, y, z) = x^2 + y^2 + z^2$. Find the maximum directional derivative of f at the point $(1,2,-2)$ and give the direction in which it occurs. | [5] | 3 |
| Q.4(a) For $\vec{F} = (y^2 + z)\hat{i} + (xz)\hat{j} + (xy)\hat{k}$. Evaluate the line integral $\oint_C \vec{F} \cdot d\vec{r}$ along C , where C is the curve parameterized by $r(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$, for $0 \leq t \leq 1$. | [5] | 4 |
| Q.4(b) Using the Gauss Divergence Theorem, for $\vec{F}(x, y, z) = (x^2y)\hat{i} + (y^2z)\hat{j} + (z^2x)\hat{k}$, compute $\iint \vec{F} \cdot d\vec{s}$ across the hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$. | [5] | 4 |
| Q.5(a) Derive $div \vec{\phi}$ for a vector point function $\vec{\phi}$ in curvilinear coordinate system. | [5] | 5 |
| Q.5(b) Find the unit vectors and scale factors in cylindrical coordinate system. Further, represent $\vec{F} = xz\hat{i} + yz\hat{j}$ in cylindrical coordinates and hence determine curl \vec{F} . | [5] | 5 |

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