BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc **SEMESTER : VI**
BRANCH: CQEDS **SESSION : SP/2025**

SUBJECT: ED313 NONPARAMETRIC METHOD AND DECISION THEORY

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. **The question paper contains 5 questions each of 10 marks and total 50 marks.**
2. **Attempt all questions.**
3. **The missing data, if any, may be assumed suitably.**
4. **Before attempting the question paper, be sure that you have got the correct question paper.**
5. **Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.**

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Q.1(a) In a particular year, the Educational Testing Service (ETS) reported that the 75th percentile of scores on the quantitative section of the GRE was 693. A random sample of 15 first-year graduate students majoring in Statistics reported the following GRE quantitative Scores: 690, 750, 680, 700, 660, 710, 720, 730, 650, 670, 740, 730, 660, 750, and 690. Using this data, test whether the GRE quantitative scores of Statistics students are statistically consistent with the reported 75th percentile value for that year. Formulate and perform an appropriate hypothesis test using a two-sided alternative hypothesis. [5] 1 4

Q.1(b) Explains the importance of order statistics in constructing the test statistics. Further, suppose X1, X2,…, Xn be a random sample from a continuous distribution with probability density function f(x) and cumulative distribution function F(x). Let X(1) ≤ X(2) ≤ ⋯≤ X(n) be the order statistics from this sample. Obtain the joint PDF of two order statistics X(i) and X(j), where 1≤i<j≤n. [5] 1 3

Q.2(a) Briefly explain the Wilcoxon signed-rank test. [5] 2 3

Q.2(b) A researcher wants to test whether there is a difference in the two web applications' response times (in seconds). A random sample of response times is recorded for each application: [5] 2 3
Application A: 1.2, 1.5, 1.3, 1.6, 1.4;
Application B: 1.8, 1.7, 1.9, 2.0, 1.6;
At the 5% significance level, test whether there is a significant difference in the distributions of response times for the two applications using the Mann–Whitney U test. Assume the samples are independent and the response times are continuous. For a two-sided alternative hypothesis, the critical value is 2.

Q.3(a) Given a sample of data: x = 2.1, 2.5, 2.9, 3.0, 3.2, 3.8, 4.0. [10] 5 3
(a) Use a Gaussian kernel and bandwidth h=0.5 to estimate the density at x=3.
(b) Also, obtain the empirical cumulative distribution function (ecdf) at x=3

Q.4(a) Define the following terms: [3] 3 2
(a) Probability of concordance
(b) Probability of discordance
(c) p-th quantile point

Q.4(b) Consider the following dataset: [7] 3 4

Observation	1	2	3	4	5	6	7
X	2	4	6	8	10	12	14
Y	1	3	5	7	6	8	12

Compute the Kendall's tau coefficient for this dataset and the Pearson product-moment correlation coefficient. Also, interpret the differences (if any) between the two coefficients.

Q.5(a)	Define a Statistical game and its components. Explain the difference between decision theory and game theory with an example.	[5]	4	4
Q.5(b)	*A device is designed to detect whether a condition is safe or unsafe, where the true state of the system is denoted by* $\theta \in \{0,1\}$*, with:* $\theta=0$*: the system is safe;* $\theta=1$ *the system is unsafe.* *The system generates a signal* $X \in \{0,1\}$*, a noisy observation of the actual state. The conditional probability of observing X given* θ *is as follows:* $P(X=1/\theta=1) = 0.9$, $P(X=1/\theta=0) = 0.2$. *A decision must be made based on the observed value X: a=1: Raise an alarm, and a=0: Do not raise an alarm. The loss function is defined as follows (described in words):* • *If the actual state is unsafe (*$\theta=1$*) and no alarm is raised (a=0), the loss is 10.* • *If the actual state is safe (*$\theta=0$*) and an alarm is raised (a=1), the loss is 2.* • *In all other cases, the loss is zero.* *Write the loss function* $L(\theta, a)$ *in tabular form. List all possible deterministic decision rules* $d:\{0,1\} \rightarrow \{0,1\}$*, and for each decision rule, compute the risk* $R(\theta, d)$ *for both values of* θ*.*	[5]	4	4

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