BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION SP2025)

CLASS: IMSc. **SEMESTER: II**
BRANCH: QEDS **SESSION: SP/2025**

SUBJECT: ED24117 LINEAR ALGEBRA, VECTORS AND MATRICES

TIME: 03 Hours **FULL MARKS: 50**

INSTRUCTIONS:

1. **The question paper contains 5 questions each of 10 marks and total 50 marks.**
2. **Attempt all questions.**
3. **The missing data, if any, may be assumed suitably.**
4. **Before attempting the question paper, be sure that you have got the correct question paper.**
5. **Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**
6. **All the notations used in the question paper have usual meanings.**

		Marks	CO	BL
Q.1(a)	Consider the set of all 3× 3 real matrices, $V = \{A \in \mathbb{R}^{3\times 3} : A + A^T = O\}$. Find the basis and dimension of V.	[5]	CO1	1,2
Q.1(b)	Let $\{\alpha_1, \alpha_2, \alpha_3\}$ and $\{\beta_1, \beta_2, \beta_3\}$ be ordered bases of the real vector spaces V and W respectively. A linear mapping $T: V \to W$ maps the basis vector as $T(\alpha_1) = \beta_1, T(\alpha_2) = \beta_1 + \beta_2, T(\alpha_3) = \beta_1 + \beta_2 + \beta_3$. Find the matrix related to the ordered bases $\{\alpha_1, \alpha_2, \alpha_3\}$ of V and $\{\beta_1, \beta_2, \beta_3\}$ of W. Hence verify that T as well as the matrix $m(T)$ is non-singular.	[5]	CO1	2
Q.2(a)	For which values of $x \in \mathbb{R}$ the matrix $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 3 \\ 2 & 5 & 4 \end{bmatrix}$ is invertible? Using row elimination compute A^{-1} for $x = 1$.	[2+3]	CO2	3
Q.2(b)	Consider the matrix $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$. Find a matrix P for which $P^{-1}AP = D$, where D is a diagonal matrix. Hence compute A^5.	[5]	CO2	4
Q.3(a)	Consider the vector space $\mathbb{R}^{2\times 3}$ of all 2×3 matrices with inner product defined by $\langle A, B\rangle = trace(B^T A)$. If $A = \begin{bmatrix} 2 & a & 3 \\ 1 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 3 & a \\ 5 & 7 & 2 \end{bmatrix}$ such that A is orthogonal to B, find the value of a.	[5]	CO3	4
Q.3(b)	Let S be the orthogonal set of vectors $v_1 = (1,2,1)$, $v_2 = (2,1,-4)$, $v_3 = (3,-2,1)$ in the inner product space $\mathbb{R}^3$ with standard inner product. If $u = (7,1,9) \in span\{v_1, v_2, v_3\}$, represent u in terms of v_1, v_2 and v_3.	[5]	CO3	4
Q.4(a)	Consider the inner product $\langle , \rangle$ in $\mathbb{R}^2$ defined by $\langle \alpha, \beta\rangle = 4\alpha_1\beta_1 + \alpha_2\beta_2$, where $\alpha = (\alpha_1, \alpha_2)$ and $\beta = (\beta_1, \beta_2)$. If $\alpha = (-2,3), \beta = (3,-4)$, find $\|\alpha\|, \|\beta\|$ and $\|\alpha - \beta\|$.	[4]	CO4	4,5
Q.4(b)	Let $P_3(t)$ be the vector space of all polynomials of degree less than or equal to 3 with inner product defined by $\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)dt$. Apply Gram-Schmidt orthogonalization process to find an orthogonal basis from the basis $\{1, t, t^2, t^3\}$.	[6]	CO4	5
Q.5	Reduce the quadratic form $Q(x) = 2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$ into its canonical form. Hence determine the index, signature and nature of the quadratic form.	[10]	CO5	5,6

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