BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc. **SEMESTER : IV**
BRANCH: QEDS **SESSION : SP/2025**

SUBJECT: ED213 OPTIMIZATION TECHNIQUES

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
6. All the notations used in the question paper have usual meanings.

		Marks	CO	BL
Q.1	A logistics company must plan capital investments in a transportation network connecting five cities (City 1 to City 5). Due to infrastructure limitations, only specific direct routes are available for development. Each route represents an investment project with a fixed cost: $c_{12} = 20, c_{13} = 4, c_{14} = 10, c_{23} = 5. c_{25} = 10, c_{34} = 6, c_{35} = 6$ and $c_{45} = 20$, where $c_{ij} = c_{ji}$, and there is no route between the cities i and j, if a value of c_{ij} is not given. The company wants to select a subset of these routes to form a closed-loop tour that starts and ends at City 1 and visits each of the other four cities exactly once. Each selected route can either be fully invested in or not at all, and the objective is to minimize the total investment cost while ensuring that the selected routes form a connected tour visiting each city exactly once.	[10]	CO2	2,3

Q.2 Consider the following linear programming problem [8+2] CO1 1,2

$$\min z = 200x_1 + 250x_2 + 150x_3$$

subject to $x_1 + x_3 \geq 8;\ x_1 + x_2 \geq 12;\ x_2 + x_3 \geq 10;\ x_1, x_2, x_3 \geq 0.$

(a) Solve the dual of the given problem using an appropriate method.
(b) Using complementary slackness determine the primal solutions.

Q.3 Minimize the quadratic function $f(x) = \frac{1}{2}x^T A\, x - b^T x$, where $A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Perform two iterations using steepest descent method with exact line search starting from $x^{(0)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. (Use optimal step size $\alpha_k = \frac{\left(\nabla f(x^{(k)})\right)^T \nabla f(x^{(k)})}{\left(\nabla f(x^{(k)})\right)^T \mathrm{A}\, \nabla f(x^{(k)})}$ for k^{th} iteration). [10] CO3 3,4

Q.4 Consider the following optimal simplex table (relaxing the integer constraints) for a maximization type integer linear programming problem. Find the integer solutions of both the variables (x_1 and x_2) using Gomory cutting plane algorithm. [10] CO4 4,5

c_j			2	2	0	0
c_B	B	b	x_1	x_2	s_1	s_3
2	x_1	$\frac{4}{7}$	1	0	$\frac{2}{7}$	$-\frac{3}{7}$
2	x_2	$\frac{12}{7}$	0	1	$-\frac{1}{7}$	$\frac{5}{7}$
$z_j - c_j$			0	0	$\frac{2}{7}$	$\frac{4}{7}$

Q.5 Consider a production scheduling problem where each machine is represented as a node in a directed graph, and the processing times between the machines are represented by the weights of the edges. The processing times t_{ij} (in hours) from i^{th} to j^{th} machine are given as: $t_{12} = 3, t_{14} = 1, t_{23} = 2, t_{25} = 5, t_{35} = 1, t_{45} = 4, t_{56} = 2, t_{610} = 6, t_{57} = 2, t_{78} = 1, t_{810} = 3$. Find the shortest path from Machine 1 to Machine 10, minimizing the total processing time using dynamic programming. [10] CO5 5,6

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