UG

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER : IV**
BRANCH: IMSC Maths & Comp **SESSION : SP/2024**

SUBJECT: MA209 INTEGRAL EQUATIONS AND GREEN'S FUNCTION
TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

	Question	Marks	CO	BL
Q.1(a)	Show that the function : $y(x) = 1$ is a solution of the integral equation: $y(x) = e^x - x + \int_0^1 x(1 - e^{xt})\, y(t)dt$	[5]	1	1
Q.1(b)	Convert the following initial value problem into the corresponding integral equation: $y''(x) + \lambda x y(x) = f(x)$ subject to the conditions: $y'(0) = 0,\ y(0) = 1$	[5]	1	2
Q.2(a)	Find the the equations in λ , the eigen value for which the following homogeneous Fredholm integral equations with degenerate kernels has nontrivial solutions: $$y(x) = \lambda \int_0^1 (6x - t) y(t) dt$$	[5]	2	1
Q.2(b)	Prove that the following non-homogeneous Fredholm Integral equation has no solution: $$y(x) = x + \frac{1}{\pi}\int_0^{2\pi} sin(x+t) y(t) dt$$	[5]	2	2
Q.3(a)	Prove that the resolvent kernel $R(x, t; \lambda)$ for the integral equation: $$y(x) = f(x) + \lambda \int_a^b k(x,t) y(t) dt$$ satisfies the integral equation: $$R(x,t;\lambda) = k(x,t) + \lambda \int_a^b k(x,z) R(z,t;\lambda) dz$$	[5]	3	2
Q.3(b)	Solve the following Voltera integral equation with the help of resolvent kernels: $$y(x) = \sin x + 2\int_0^x e^{x-t} y(t)\, dt$$	[5]	3	2
Q.4(a)	If the kernel $K(x,t)$ is real, symmetric, continuous, and identically not equal to zero, then all the eigenvalues are real.	[5]	4	2
Q.4(b)	Using Hilbert-Schmidt's theorem solve the following symmetric integral equation of second kind: $$y(x) = (x^2 + 1) + \frac{3}{2}\int_{-1}^{1} \{1 + xt\} y(t) dt$$	[5]	4	2
Q.5(a)	Find the adjoint equation of following differential equations: $x^2 \frac{d^2y}{dx^2} + (2x^3 + 1)\frac{dy}{dx} + y = 0$	[2]	5	2
Q.5(b)	Solve the following boundary-value problems using Green's functions : $y''(x) = x,\quad y(0) = y'(1),\quad y'(0) = y(1).$	[8]	5	2

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