BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc — **SEMESTER : IV**
BRANCH: PHYSICS and CHEMISTRY — **SESSION : SP/2024**

SUBJECT: MA207R1 MATHEMATICS IV

TIME: 3 Hours — **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Discuss order, degree, homogeneity of the following ODE, and solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$.	[5]	1	
Q.1(b)	Using method of variation of parameters, find the general solution to $ty'' - (t+1)y' + y = t^2$ given that $y_1(t) = e^t, y_2(t) = t+1$ form a fundamental set of solutions for the homogeneous differential equation.	[5]	1	
Q.2(a)	Let be the P_nLegendre polynomial of degree n. Prove that $P_n'(1) = \frac{n(n+1)}{2}$.	[5]	2	
Q.2(b)	Using power series method solve the following $y'' - 2xy' + y = 0$	[5]	2	
Q.3(a)	Solve the PDE $(mz - ny)p + (nx - lz)q = ly - mx$.	[5]	3	
Q.3(b)	Solve the equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, representing the vibration of a string of length l, fixed at both ends, given that $y(0,t) = 0, y(1,t) = 0, y(x,0) = f(x)$ and $\frac{\partial}{\partial t}y(x,0) = 0, 0 < x < l$.	[5]	3	
Q.4(a)	Let $u(x,y) = x^2 - y^2 - 2xy - 2x + 3y$. Find $v(x,y)$ such that $(u + iv)$ is an analytic function. Express $f(z) = u + iv$ in terms of z only, where $z = x + iy$.	[5]	4	
Q.4(b)	Evaluate $\oint_C \frac{2+3\sin z}{z(z-1)^2}dz$, where C is the square with vertices $3+3i, 3-3i, -3+3i, -3-3i$.	[5]	4	
Q.5(a)	Find the zeros and poles of $f(z) = \frac{z^2+4}{z^3+2z^2+2z}$. Determine the residue at each pole.	[5]	5	
Q.5(b)	Let $f(z) = \frac{z}{(z-1)(2-z)}$. Classify the singularities of $f(z)$. Expand $f(z)$ in a Laurent series for $1 < \|z\| < 2$.	[5]	5	

::::::23/04/2024::::::M