

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(MID SEMESTER EXAMINATION SP/2024)

CLASS: IMSc  
BRANCH: MATHEMATICS & COMPUTING

SEMESTER : II  
SESSION : SP/2024

SUBJECT: MA110R1 COMPLEX ANALYSIS

TIME: 02 Hours

FULL MARKS: 25

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates
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|--------|--|-----|-----|-----|
|        |  |     | CO  | BL  |
| Q.1(a) | For the complex number $z = -1 + i$ , find $r$ and $\theta$ such that $z = re^{i\theta}$ .   | [2] | CO1 | BT1 |
| Q.1(b) | Express the function $f(z) = z^3$ in the form $U(x, y) + iV(x, y)$ .   | [3] | CO1 | BT2 |
|        |  |     |     |     |
| Q.2(a) | Use Cauchy-Riemann equations to check the differentiability of the function $f(z) = xy + iy$   | [2] | CO1 | BT3 |
| Q.2(b) | Find a harmonic conjugate of the function $u(x, y) = e^x \cos y$   | [3] | CO1 | BT1 |
|        |  |     |     |     |
| Q.3(a) | Determine the integral $\int \frac{z^3+5}{z-i} dz$ along the positively oriented contour $C,  z  = 2$ .                                      | [2] | CO2 | BT4 |
| Q.3(b) | Let $C$ be the positively oriented unit circle $ z  = 1$ . Show that for any real constant $b$ , $\int_C \frac{e^{bz}}{z} dz = 2\pi i$ .     | [3] | CO2 | BT1 |
|        |  |     |     |     |
| Q.4    | Consider the positively oriented circle $C,  z  = \frac{1}{2}$ and $f(z) = e^{2z}$ , determine the integral $\int_C \frac{e^{2z}}{z^4} dz$ . | [5] | CO2 | BT4 |
| Q.5    | Find the Laurent series that represents the function $f(z) = \frac{1}{z-2}$ in the domain $ z  < 2$ .  | [5] | CO3 | BT1 |

:23/02/2024:::E