

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: ISc/MSc
BRANCH: PHYSICS

SEMESTER : X / IV
SESSION : SP/2023

SUBJECT: PH516 NONLINEAR DYNAMICS AND CHAOS

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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|--------|--|-------|----|
| Q.1(a) | Analyze the equation $\dot{x} = x^2 - 9$ graphically, sketch the vector fields on the real line and find all the fixed points. | [5] 1 | 4 |
| Q.1(b) | For the system given by $\dot{\theta} = \sin(2\theta)$, determine and classify the fixed points. Also sketch the vector fields on the circle. | [5] 1 | 3 |
| Q.2(a) | Determine the type of bifurcation and the critical value of r for a system given by $\dot{x} = 1 + rx + x^2$. | [5] 2 | 3 |
| Q.2(b) | Sketch the bifurcation diagram for a system defined by $\dot{x} = rx + x^2$. | [5] 2 | 3 |
| Q.3(a) | Find all the fixed points of the system $\dot{x} = -x + x^3$, $\dot{y} = -2y$ and use linearization to classify them. | [5] 3 | 4 |
| Q.3(b) | Show that the system $\dot{x} = y - y^3$, $\dot{y} = -x - y^2$ has a nonlinear center at the origin, and plot the phase portrait. | [5] 3 | 3 |
| Q.4(a) | Define limit cycles, stable limit cycles, unstable limit cycles and half-stable limit cycles. | [5] 4 | 1 |
| Q.4(b) | Prove that the van der Pol equation has a unique, stable limit cycle. | [5] 4 | 3 |
| Q.5(a) | Show that the deterministic one-predator-one-prey model proposed by Loteka and Volterra has neutral stability. | [5] 5 | 3 |
| Q.5(b) | Discuss two competitor species in a deterministic environment. | [5] 5 | 4 |

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