BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC — **SEMESTER : VIII**
BRANCH: MATHEMATICS & COMPUTING — **SESSION : SP/2023**

SUBJECT: MA419 MATHEMATICAL ECOLOGY

TIME: 3 Hours — **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Compute the general solution of the linear differential-equation system $\frac{d\overline{X}}{dt} = \overline{A}\overline{X}$, where $\overline{X} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $\overline{A} = \begin{pmatrix} 1 & 1 \\ 0 & 4 \end{pmatrix}$.	[5]	1	3
Q.1(b)	The following nonlinear system in three variables x_1, x_2, x_3 is given: $$\frac{dx_1}{dt} = -2x_2 + x_2x_3 - x_1^3;\ \frac{dx_2}{dt} = x_1 - x_1x_3 - x_2^3;\ \frac{dx_3}{dt} = x_1x_2 - x_3^3$$ Considering $V(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2$ as a Lyapunov function, check whether the zero equilibrium solution of the above system is stable or not.	[5]	1	3
Q.2(a)	A single species mathematical model is defined as: $\frac{dN}{dt} = \mu N - N^2$, where N is the population size and $\mu > 0$ is the parameter. Analyze the model for its stability at each of its equilibrium states. Also, investigate whether the model exhibits transcritical bifurcation or not.	[5]	2	2,3
Q.2(b)	The following single - species model for the fish population with Allee effect and harvesting is given: $$\frac{dN}{dt} = rN\left(\frac{N}{K_0} - 1\right)\left(1 - \frac{N}{K}\right) - qEN$$ where $N(t)$ denotes the fish population density at time t. Here, the parameter r is the intrinsic growth rate, K is the carrying capacity, K_0 is the Allee effect threshold, E represents harvesting effort, and q represents catchability coefficient, which are all positive. Obtain the steady states of the model and discuss the occurrence of saddle-node bifurcation.	[5]	2	2,3
Q.3(a)	Consider the following predator-prey model with $x(t)$ and $y(t)$ as population sizes of prey and predator, respectively at time t $$\frac{dx}{dt} = x(1 - x - y);\ \frac{dy}{dt} = by(x - a)$$ where a and b are positive parameters. Identify the coexistence equilibrium state of the model along with the condition of its feasibility. Also, explain the stability behavior and nature of the coexistence equilibrium. Is it possible for the model to exhibit Hopf-bifurcation around the coexistence equilibrium?	[5]	3	2,3
Q.3(b)	Define limit cycles. Explain how Hopf-bifurcation is associated with the limit cycles. Also, mention the difference between super-critical and sub-critical Hopf bifurcations.	[5]	3	1,2

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Q.4	If the interaction between honeybees and flowering plants is considered. The flowers of the plant provide nectar to the honeybee, which acts as a source of nutrients for the honeybee. The honeybees, in turn, provide a service of transferring pollen grains from one flower to another to aid the process of fertilization for flowering plants. Both the species (honeybee and flowering plants) can exist independently and exhibit logistic growth in the absence of the other. Then,			
	a) Identify the interacting population model that exists between honeybees and flowering plants. Hence, propose a two-dimensional nonlinear mathematical model as a system of ordinary differential equations that represents the interactive dynamics between them, considering $n_1(t)$ and $n_2(t)$ as population sizes of honeybees and flowering plants, respectively, at any time t. Support the proposed model with proper definition of chosen parameters and assumptions.	[5]	5	2
	b) Locate the coexistence equilibrium state of the proposed model, its feasibility (if any) and stability behavior.	[5]	5	3
Q.5(a)	Develop a spatial one-dimensional mathematical model through reaction-diffusion equation with $N(x,t)$ as the population size of some organism at any point x and time t. It is given that the organism exhibits unlimited growth with intrinsic growth rate r and the flux of movement from one place to another takes place through simple Fickian diffusion with diffusivity constant D. In addition to this, it is assumed that the organism population size at $t = 0$ is N_0 and the flux of organisms at the boundary points $x = 0$ and $x = L$ vanish identically.	[5]	5	2
Q.5(b)	Discuss Dirichlet, Neumann and Robin type boundary conditions that can be incorporated in one - dimensional spatially - structured population models. Also, interpret their ecological significance.	[5]	4	2

::::::28/04/2023::::::E