BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC. **SEMESTER : VIII**
BRANCH: MATHS & COMP. **SESSION : SP/2023**

SUBJECT: MA412 TOPOLOGY

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Define a basis for a topology. Give an example.	[5]	CO1	1.10
Q.1(b)	Prove that every finite point sent in a Hausdroff space is closed.	[5]	CO1	1.11
Q.2(a)	Let *X* be a metric space with metric *d*. Define $\overline{d} = X \times X \to R$ by the equation $\overline{d}(x, y) = \min\{1, d(x, y)\}$. Then prove that $\overline{d}$ is a metric that induces the same topology a *d*.	[5]	CO2	1.12 1.21
Q.2(b)	Let $f : X \to Y$. If the function *f* is continuous, then prove that every convergent sequence $x_n \to x$ in *X* the sequence $f(x_n) \to f(x)$.	[5]	CO3	1.12 1.21 1.24
Q.3	Let *X* be a topological space. Let one-point set in *X* be closed. The prove that *X* is regular if and only if given a point *x* of *X* and a neighbourhood *U* of *X*, there is a neighbourhood *V* of *X* such that $\overline{V} \subset U$.	[10]	CO3	1.21 1.31
Q.4	Let *Y* be a subspace of *X*. Then prove that *Y* is compact if and only if every covering of *Y* by sets open in *X* contains a finite subcollection covering *Y*.	[10]	CO4	1.25 1.31
Q.5	Prove that a metric space (*X*,d) is compact if and only if *it* is complete and totally bounded	[10]	CO5	1.24 1.32

:::::25/04/2023 E:::::