BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC. **SEMESTER : IV**
BRANCH: MATHS & COMPUTING **SESSION : SP/2023**
SUBJECT: MA209 INTEGRAL EQUATIONS AND GREEN'S FUNCTION
TIME: 3 HOURS **FULL MARKS: 50**

INSTRUCTIONS:
1. **The question paper contains 5 questions each of 10 marks and total 50 marks.**
2. **Attempt all questions.**
3. **The missing data, if any, may be assumed suitably.**
4. **Before attempting the question paper, be sure that you have got the correct question paper.**
5. **Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.**

			CO	BL
Q.1(a)	Show that the function $y(x) = xe^{-x}$ is a solution of the integral equation: $$y(x) = (x-1)e^{-x} + 4\int_0^{\infty} e^{-(x+t)}\, y(t)dt$$	[5]	1	1
Q.1(b)	Convert the following IVP into the corresponding integral equation: $y''(x) - sinx\, y'(x) + e^x y(x) = x,$ subject to the conditions: $y'(0) = -1, y(0) = 1$	[5]	1	3
Q.2(a)	If exists, find the Eigen values and Eigen functions of the following homogeneous Fredholm integral equations with degenerate kernels: $$y(x) = \lambda\int_0^1 (2xt - 4x^2)y(t)dt$$	[5]	2	1
Q.2(b)	Prove that the following non-homogeneous Fredholm Integral equation: $y(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} sin(x+t)y(t)dt$ has infinitely many solutions when $f(x) = 1$. Hence determine all such solutions.	[5]	2	2
Q.3(a)	Solve the following Fredholm integral equations with the help of resolvent kernels $y(x) = x + \int_0^{1/2} y(t)dt$	[5]	3	3
Q.3(b)	Solve the following Voltera integral equations by the method of Successive approximation: $y(x) = x3^x - \int_0^x 3^{x-t}\, y(t)\, dt$	[5]	3	3
Q.4(a)	Prove that the eigenfunctions of a symmetric kernel, corresponding to different eigenvalues are orthogonal.	[5]	4	2
Q.4(b)	*Using Hilbert-Schmidt's theorem solve the following symmetric integral equation of second kind:* $y(x) = (x+1)^2 + \lambda\int_{-1}^{1}\{xt + x^2t^2\}y(t)dt$	[5]	4	3
Q.5(a)	Define Adjoint and Self-adjoint equation of 2nd order homogeneous linear differential equation. Transform the following differential equations into an equivalent self-adjoint equations: $y'' - (\tan x)y' + y = 0$	[5]	5	3
Q.5(b)	Construct the Green's function of the following boundary value problem: $$y''(x) + \mu^2 y(x) = 0, \quad y(0) = 0, \quad y(1) = 0,$$	[5]	5	3

:::::28/04/2023 M:::::