BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION SP2023)

CLASS: **IMSc** **SEMESTER : IV**
BRANCH: **PHYSICS & CHEMISTRY** **SESSION : SP2023**

SUBJECT: MA207R1 MATHEMATICS - IV

TIME: **02 Hours** **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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			CO	BL
Q.1(a)	Check whether the functions $f(x) = x^2$ and $g(x) = x^2 \log_e x$ are linearly independent or not for $x > 0$. Support your answer with proper reasoning.	[2]	1	2
Q.1(b)	Construct the general solution of the following differential equation: $$(D^2 - 4D + 3)y = e^{3x}$$ where $D \equiv \frac{d}{dx}$.	[3]	1	3
Q.2(a)	Using proper substitution, transform the following Cauchy - Euler differential equation: $$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0, \qquad x > 0$$ into the form $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = 0$ with t as the independent variable, where p and q are constants. Determine the values of p and q.	[2]	1	2
Q.2(b)	Develop the general solution of the following differential equation with the use of method of variation of parameters : $$y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$$	[3]	1	3
Q.3(a)	Identify and classify the singular point(s) of the following differential equation on the $x-$axis: $$(x-1)y'' + x^2y' + y = 0$$	[2]	2	1
Q.3(b)	For the differential equation: $$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$ Determine its general solution $y = \sum_{n=0}^{\infty} a_n x^n$ in the form $y = a_0 y_1(x) + a_1 y_1(x)$, where $y_1(x)$ and $y_2(x)$ are power series, a_0 and a_1 are arbitrary constants.	[3]	2	3
Q.4(a)	For the Legendre polynomial $P_n(x)$ of order n, prove that: i. $P_n(1) = 1$ ii. $P_n'(1) = \frac{n(n+1)}{2}$ where $P_n'(x) = \frac{d}{dx}\{P_n(x)\}$	[2]	2	2

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Q.4(b)	Define the expression of $J_n(x)$ in terms of gamma function. Hence, derive $$\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$$ where $J_n(x)$ denotes Bessel's function of first kind of order n.	[3]	2	3
Q.5(a)	State the definition to find the Laplace transform of any function $f(t)$ for all $t \geq 0$. Hence, apply it to compute the Laplace transform of $f(t) = e^{2t}$.	[2]	3	2
Q.5(b)	Using properties, determine the Laplace transform of the function $F(t) = (1 + te^{-t})^2$.	[3]	3	3

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