BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER : IV**
BRANCH: PHYSICS & CHEMISTRY **SESSION : SP/2023**

SUBJECT: MA207R1 MATHEMATICS - IV

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

CO BL

Q.1(a) Compute the general solution of the following differential equation using operator method: [5] 1 3

$$(D^2 - 5D + 6)y = e^{4x} + e^{-4x}$$

where $D \equiv \dfrac{d}{dx}$.

Q.1(b) Applying method of variation of parameters, solve the differential equation: [5] 1 3

$$y'' + y = \sec x$$

Q.2(a) Identify whether $x = 0$ is an ordinary or a singular point of the differential equation: [5] 2 1,3

$$(1 - x^2)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

Also, construct the series solution of the above differential equation about $x = 0$.

Q.2(b) Prove the following: [5] 2 2

i. $\int_{-1}^{1} P_m(x)P_n(x)dx = 0,$ when $m \neq n$

ii. $J_{-n}(x) = (-1)^n J_n(x),$ when n is any positive integer

where $P_n(x)$ **denotes Legendre polynomial of order** n **and** $J_n(x)$ denotes Bessel's function of first kind of order n.

Q.3(a) Develop the Laplace transforms of the following functions: [5] 3 3

i. $f(t) = e^{-3t}(\cos 4t + 3\sin 4t)$

ii. $g(t) = t^3 e^{5t}$

Q.3(b) Obtain the Fourier series of the function $f(x) = x^2, -\pi \leq x \leq \pi$. Hence, show that: [5] 3 3

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$$

Q.4(a) A function $f(z) = xy^2 + ix^2 y$ is given. Determine the points where [5] 4 2,3

i. $f(z)$ is continuous in the complex plane.

ii. Cauchy-Riemann equations get satisfied for the function $f(z)$.

iii. derivative $f'(z)$ exists

iv. $f(z)$ is analytic

Support your answers with proper reasoning.

PTO

Q.4(b)	State Cauchy's Integral Formula. Hence, using it, compute the value of the integral: $$I = \frac{1}{2\pi i}\oint_C \frac{z}{(z-1)(z-3)^2}dz$$ around the circle $C : \lvert z \rvert = 2.$	[5]	4	1,3
Q.5(a)	Construct the Laurent series expansions of the function $f(z) = \frac{1}{z(z-1)}$ that is valid in a i. deleted neighbourhood of $z = 0$ ii. deleted neighbourhood of $z = 1$ State the domains throughout which obtained series expansions are valid.	[5]	4	2,3
Q.5(b)	Identify the poles (with their orders) of the function $f(z) = \frac{50z}{(z+4)(z-1)^2}$. Also, obtain the residues at all the identified poles.	[5]	4	3

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