BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION SP2023)

CLASS:	**IMSc**	**SEMESTER : IV**
BRANCH:	**MATHEMATICS & COMPUTING**	**SESSION : SP2023**

SUBJECT: MA206 LINEAR ALGEBRA

TIME:	**02 Hours**	**FULL MARKS: 25**

INSTRUCTIONS:
1. **The question paper contains 5 questions each of 5 marks and total 25 marks.**
2. **Attempt all questions.**
3. **The missing data, if any, may be assumed suitably.**
4. **Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

			CO	BL
Q.1(a)	Find whether the following set is a subspace of $\mathbb{R}^3$ under the usual operations of addition and scalar multiplication defined on $\mathbb{R}^3$ $\{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 - 7a_2 + a_3 = 0\}$.	[2]	CO1	BL1
Q.1(b)	Let $W_1$and W_2 be subspaces of a vector space V. Show that $W_1 \cup W_2$ is a subspace of V iff $W_1 \subset W_2$ or $W_2 \subset W_1$.	[3]	CO1	BL3
Q.2(a)	Examine whether the following set is a basis for $\mathbb{R}^3$ $\{(-1,3,1), (2,-4,-3), (-3,8,2)\}$.	[2]	CO1	BL2
Q.2(b)	Compute bases for the following subspaces of $\mathbb{R}^5$: $W_1 = \{(a_1, a_2, a_3, a_4, a_5) \epsilon \mathbb{R}^5 : a_1 - a_3 - a_4 = 0\}$ and $W_2 = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 : a_2 = a_3 = a_4 \ and\ a_1 + a_5 = 0\}$. What are the dimensions of $W_1$and W_2?	[3]	CO1	BL3
Q.3(a)	Find whether $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear mapping or not, where $T(a_1, a_2) = (1, a_2)$.	[2]	CO1	BL1
Q.3(b)	Suppose that $T: \mathbb{R}^2 \to \mathbb{R}^2$is linear, $T(1,0) = (1,4)$ and $T(1,1) = (2,5)$. Compute $T(x, y)$ for any $(x, y) \in \mathbb{R}^2$.	[3]	CO1	BL3
Q.4(a)	Find if $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$ is an one-to-one or onto linear mapping.	[2]	CO1	BL1
Q.4(b)	Let β and γ be standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$respectively. For each linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(a_1, a_2) = (2a_1 - a_2, 3a_1 + 4a_2, a_1)$. Compute $[T]_\beta^\gamma$. Is T invertible?	[3]	CO1	BL3
Q.5(a)	Find an example of a linear operator on a finite dimensional vector space having no eigen value.	[2]	CO2	BL1
Q.5(b)	Compute eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}, F = \mathbb{R}$.	[3]	CO2	BL3

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