BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc | **SEMESTER : IV**
BRANCH: MATHEMATICS AND COMPUTING | **SESSION : SP/2023**

SUBJECT: MA206 LINEAR ALGEBRA

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Define 'basis' for a vector space? Explain if the the set $\{(2,-4,1),(0,3,-1),(6,0,-1)\}$ a basis for $\mathbb{R}^3$.	[5]	1	1+2
Q.1(b)	Compute bases for $W_1=\{(a_1,a_2,a_3,a_4,a_5)\epsilon\mathbb{R}^5: a_1-a_3-a_4=0\}$ and $W_2=\{(a_1,a_2,a_3,a_4,a_5)\in\mathbb{R}^5: a_2=a_3=a_4 \text{ and } a_1+a_5=0\}$. What are the dimensions of W_1 and W_2?	[5]	1	3
Q.2(a)	Let V and W be finite dimensional vector spaces of equal dimensions defined over the same field $\mathcal{F}$. Explain if it is possible to have a linear operator $T: V\longrightarrow W$ which is one-to-one but not onto and vice-versa	[5]	1	2
Q.2(b)	Let β be the standard ordered basis for $\mathbb{R}^3$ and $T:\mathbb{R}^3\longrightarrow\mathbb{R}^3$ defined by $T(a_1,a_2,a_3)=(2a_2+a_3,-a_1+4a_2+5a_3,a_1+a_3)$. Find matrix representation of T w.r.t. β.	[5]	2	2
Q.3(a)	For the following matrix $A\epsilon M_{3\times3}(\mathbb{R})$, compute eigen values and corresponding eigen vectors of A where A = $\begin{pmatrix}0 & 0 & 1\\ 1 & 0 & -1\\ 0 & 1 & 1\end{pmatrix}$	[5]	2	3
Q.3(b)	Let $V=\rho_2(\mathbb{R})$ and T be defined by $T(ax^2+bx+c)=cx^2+bx+a$. Test T for diagonalizability and if diagonalizable find a basis β for V such that $[T]_\beta$ is a diagonal matrix.	[5]	2	4
Q.4(a)	In $C[0,1]$, for $f,g\epsilon C[0,1]$, define $< f,g >= \int_0^1 f(t)g(t)dt$. For $f(t)=t$ and $g(t)=e^t$, compute $\parallel f\parallel$, $\parallel g\parallel$ and $\parallel f+g\parallel$.	[5]	1	3
Q.4(b)	Let T be the linear operator on $\rho_2(\mathbb{R})$ defined by $T\big(g(x)\big)=-g(x)-g'(x)$. Compute a Jordan canonical form of T	[5]	3	3
Q.5(a)	Apply the Gram-Schmidt process to the given subset S of the inner product space V to obtain an orthogonal basis for $Span\ S$. $V=\rho_3(\mathbb{R})$ with the inner product- $< f,g >= \int_{-1}^1 f(t)g(t)dx$, $S=\{1,x,x^2,x^3\}$.	[5]	1	3
Q.5(b)	Let $V=\rho_3(\mathbb{R})$ with the inner product $< f,g >= \int_{-1}^1 f(x)g(x)dx$, $\forall f,g\epsilon V$. Find the orthogonal projection of $f(x)=x^3$ on $\rho_2(\mathbb{R})$.	[5]	1	3

::::::27/04/2023::::::M