

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)**

**CLASS: I MSc
BRANCH: MATHEMATICS & COMPUTING**

**SEMESTER : II
SESSION : SP/2023**

SUBJECT: MA110-COMPLEX ANALYSIS

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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		CO	BL
Q.1(a) Find whether the following function is differentiable at $z = 0$? Is it analytic at $z = 0$?	[5]	1	1
$f(z) = z ^2 + z$			
Q.1(b) Find whether the function $U = x^3 - 3xy^2$ is harmonic in \mathbb{R}^2 . If so, find its Harmonic conjugate.	[5]	1,3	1
Q.2(a) Calculate (i) $\int_C \frac{dz}{(z^2+9)}$ where C is the circle $x^2 + (y - 1)^2 = 1$ oriented anti-clockwise and (ii) $\int_C \frac{dz}{ z ^2}$, where C is the unit circle oriented anti-clockwise.	[5]	3	3
Q.2(b) Calculate $\int_{ z-i =2} \frac{z^3+2z+1}{(z+2)z^2} dz$	[5]	3	3
Q.3(a) Compute the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n^n}{3^n n!} z^n$. Does the series converge at $z = 1$?	[5]	3	3
Q.3(b) Find Taylor's series expansion of $\frac{z+1}{z-4}$ about $z = 3$ and also compute its radius of convergence.	[5]	3	3
Q.4(a) Classify singularities of the functions at mentioned points. (i) $\frac{\sin z}{z^2}$ at $z = 0$ (ii) $\frac{z}{z^2-1}$ at $z = 1$ (iii) $z^3 e^{\frac{1}{z}}$ at $z = 0$.	[5]	3	3
Q.4(b) Compute residues of the function $f(z) = \frac{z+1}{z(z-2)}$ at $z = 0, 2$ and then find $\int_{ z =3} \frac{z+1}{z(z-2)} dz$.	[5]	3	3
Q.5(a) Show that $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\pi}{6}$	[5]	4	3
Q.5(b) Let $f(z) = \frac{i-z}{i+z}$. Find the image of unit disk i.e. of the set $\{z \in \mathbb{C} : z < 1\}$ under f.	[5]	1	1

:::::20/07/2023:::::