

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION SP2023)

CLASS: IMSc
BRANCH: MATHEMATICS AND COMPUTING

SEMESTER : II
SESSION : SP/2023

SUBJECT: MA110 COMPLEX ANALYSIS

TIME: 02 Hours

FULL MARKS: 25

INSTRUCTIONS:

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

		CO	BL
Q.1(a) Find if $f(z) = z ^2$ is analytic at $z = 0$?	[2]	1	1
Q.1(b) Explain whether it is possible to have a nonconstant analytic function f whose $\text{Re} f$ is zero.	[3]	2	2
Q.2(a) Show that $u = x^3 - 3xy^2$ is harmonic in \mathbb{R}^2 .	[2]	3	3
Q.2(b) Construct harmonic conjugate of u in \mathbb{R}^2 .	[3]	3	3
Q.3(a) Calculate $\int_{ z-1 =1} x dz$.	[2]	3	3
Q.3(b) Explain if it is possible to have an analytic function f analytic on the closed disk $ z+1 \leq 5$, $f''(-1) = i$ and $\max_{ z+1 \leq 5} f(z) = 5$.	[3]	2	2
Q.4(a) Calculate $\int_C \frac{z dz}{(9-z^2)(z+i)}$, where $i^2 = -1$ and C is the circle $ z = 2$ described in the positive sense.	[2]	3	3
Q.4(b) Calculate the integral $\int_{ z =1} \frac{e^z}{z^3} dz$.	[3]	3	3
Q.5(a) If the series $\sum_{n=0}^{\infty} a_n z^n$ converges at $z = z_1$ then show that $\sum_{n=0}^{\infty} a_n z^n $ converges for all $ z < z_1 $.	[2]	2	1
Q.5(b) Compute the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ and interpret the result.	[3]	2	2,3

:29/05/2023:M