

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION SP2023)**

CLASS: IMSC
BRANCH: Phy. & Che.

SEMESTER: II
SESSION: SP 2023

SUBJECT: MA108R1 MATHEMATICS III

TIME: 02 Hours

FULL MARKS: 25

INSTRUCTIONS:

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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|--|-----|----|----|
| Q.1(a) Determine whether the sequence $\{a_n = 4 + (3/4)^n\}$ is monotonic, bounded and convergent. | [2] | 2 | 2 |
| Q.1(b) Test the behaviour of the infinite series: $\sum_{n=1}^{\infty} ((n^3 + 1)^{1/3} - n)$ | [3] | 2 | 2 |
| Q.2(a) Apply integral test to test the convergence of the series $\sum_{n=2}^{\infty} \left[\frac{1}{n^2} \sin\left(\frac{\pi}{n}\right) \right]$ | [2] | 2 | 2 |
| Q.2(b) Discuss the convergence of the series $1 + \frac{x}{1!} + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \dots, x > 0.$ | [3] | 2 | 2 |
| Q.3(a) Is it possible to find two 2x2 matrices A and B such that $AB - BA = I_2$? Justify. | [2] | 2 | 2 |
| Q.3(b) Determine the values of b for which the rank of the matrix $\begin{pmatrix} b & 1 & 0 \\ 3 & b-2 & 1 \\ 3(1+b) & 0 & 1+b \end{pmatrix}$ is 2? | [3] | 2 | 3 |
| Q.4(a) Apply Cayley-Hamilton theorem to find the inverse of the matrix $\begin{pmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{pmatrix}.$ | [3] | 2 | 2 |
| Q.4(b) Check the diagonalizability of the matrix $\begin{pmatrix} 2 & 0 & -3 \\ 1 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix}.$ | [2] | 2 | 3 |
| Q.5(a) Evaluate whether following limit exists or not. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6}.$ | [2] | 3 | 1 |
| Q.5(b) Suppose for a two-variable function $f(x, y)$ both the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist. Is the function always continuous? Explain with an example. | [3] | 3 | 2 |

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