BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER : II**
BRANCH: PHY & CHE. **SESSION : SP/2023**

SUBJECT: MA108R1 MATHEMATICS III

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Determine whether the sequence $\{\boldsymbol{a_n = 1 + (1/2)^n}\}$ is monotonic, bounded and convergent	[5]	1	2
Q.1(b)	Test the behaviour of the infinite series: $\sum_{n=1}^{\infty}\left(\frac{n^n}{n!}\right)$	[5]	1	2
Q.2(a)	Find the value of k for which the following three vectors $v_1 = \begin{pmatrix}1\\2\\1\end{pmatrix}, \quad v_2 = \begin{pmatrix}3\\-1\\k\end{pmatrix}, \quad v_3 = \begin{pmatrix}-2\\4\\14\end{pmatrix}$ are linearly dependent.	[5]	2	2
Q.2(b)	Test the consistency of the following system of equations $2x + 5y + 7z = 52$ $x + y + z = 9$ $2x + y - z = 0$ using elementary transformations. Also, find the solution, if exists.	[5]	2	3
Q.3(a)	Are the functions $f(x,y) = \frac{x+y}{1-xy}$ and $g(x,y) = \tan^{-1}(x) + \tan^{-1}(y)$ functionally related? If so, find the functional relation.	[5]	3	2
Q.3(b)	Examine the function $f(x,y) = x^3 + y^3 - 3xy$ for maxima and minima.	[5]	3	3
Q.4(a)	Evaluate $\iint e^{2x+3y}\,dxdy$ over the triangle bounded by $x = 0$, $y = 0$ and $x + y = 1$.	[5]	4	2
Q.4(b)	Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.	[5]	4	3
Q.5(a)	Find the directional derivative of $\phi(x,y,z) = xy^2z + 4x^2z$ at $(-1,1,2)$ in the direction of $2\hat{i} + \hat{j} - 2\hat{k}$.	[5]	5	3
Q.5(b)	Show that the vector field $\vec{F} = \left(y^2\cos x + z^3\right)\hat{i} + \left(2y\sin x - 4\right)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$ is conservative. Also, find the scalar function $\phi(x,y,z)$ satisfying $\vec{F} = \vec{\nabla}\phi$.	[5]	5	2

:::::20/07/2023:::::