

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI**  
(END SEMESTER EXAMINATION)

CLASS: BTECH  
BRANCH: ALL

SEMESTER :II  
SESSION : SP/2023

SUBJECT: MA107 MATHEMATICS-II

TIME: 3 Hours

FULL MARKS: 50

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Before attempting the question paper, be sure that you have got the correct question paper.
  5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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|---|-----|----|-------|----------------|-----------------|---|------|---|----|----|----------------|-----------------|--|--|--|
|   |     | CO | BL    |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.1(a) Solve the differential equation  | [5] | 1  | 1,2   |                |                 |   |      |   |    |    |                |                 |  |  |  |
| $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x + x + 1$   |     |    |       |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.1(b) Solve Legendre's linear equation given as  | [5] |    |       |                |                 |   |      |   |    |    |                |                 |  |  |  |
| $(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a) \frac{dy}{dx} + 6y = x$   |     | 1  | 1,2   |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.2(a) Prove that $\int J_3(x) dx + J_2(x) + \frac{2}{x} J_1(x) = 0$  | [5] | 2  | 1,2,3 |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.2(b) Express in terms of Legendre polynomials: $x^3 + 2x^2 - x - 3$ .   | [5] | 2  | 1,2,3 |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.3(a) Obtain the half range sine series for $f(x)=2-x$ for $0 < x < 2$ and hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$ .   | [5] | 3  | 1,2,3 |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.3(b) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ , representing the vibration of a string of length $l$ , fixed at both ends, given that $y(0,t)=0$ , $y(l,t)=0$ , $y(x,0)=f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0, 0 < x < l$ .  | [5] | 3  | 1,2,3 |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.4(a) Prove that $u=y^3-3x^2y$ is a harmonic function. Find its harmonic conjugate and corresponding analytic function $f(z)=u+iv$ .   | [5] | 4  | 1,2,3 |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.4(b) Evaluate $\int_C \frac{5z-2}{z^2-z} dz$ where $C:  z =2$   | [5] | 4  | 1,2,3 |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.5(a) A random variable $X$ has the following probability distribution:  | [5] | 5  | 1,2,3 |                |                 |   |      |   |    |    |                |                 |  |  |  |
| <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 10px;">x</td> <td style="padding: 2px 10px;">0</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">2</td> <td style="padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">4</td> </tr> <tr> <td style="padding: 2px 10px;">P(x)</td> <td style="padding: 2px 10px;">k</td> <td style="padding: 2px 10px;">2k</td> <td style="padding: 2px 10px;">2k</td> <td style="padding: 2px 10px;">k<sup>2</sup></td> <td style="padding: 2px 10px;">5k<sup>2</sup></td> </tr> </table> | x   | 0  | 1     | 2              | 3               | 4 | P(x) | k | 2k | 2k | k <sup>2</sup> | 5k <sup>2</sup> |  |  |  |
| x   | 0   | 1  | 2     | 3              | 4               |   |      |   |    |    |                |                 |  |  |  |
| P(x)  | k   | 2k | 2k    | k <sup>2</sup> | 5k <sup>2</sup> |   |      |   |    |    |                |                 |  |  |  |
| Find the value of $k$ , mean and variance of $X$ .  |     |    |       |                |                 |   |      |   |    |    |                |                 |  |  |  |
| Q.5(b) The distribution of the number of road accidents per day in a city is Poisson with mean 4. Find the number of days out of 100 when there will be   | [5] | 5  | 1,2,3 |                |                 |   |      |   |    |    |                |                 |  |  |  |
| (i) no accidents    (ii) between 2 and 5 accidents.   |     |    |       |                |                 |   |      |   |    |    |                |                 |  |  |  |

:17/07/2023: