

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC
BRANCH: Mathematics

SEMESTER : II
SESSION : SP/2023

SUBJECT: MA106 ORDINARY DIFFERENTIAL EQUATIONS

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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| Q.1(a) Determine if the given equation is exact and hence solve: $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$. | [5] | CO
CO2 | BL
BT4 |
| Q.1(b) Find the orthogonal trajectories of the family of curves $y^2 = cx$. | [5] | CO1 | BT1 |
| Q.2(a) Solve the Cauchy-Euler equation
$x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 42/x^4$. | [5] | CO1 | BT3 |
| Q.2(b) Find the general solution of the non-homogeneous linear differential equation by the method of variation of parameters $\frac{d^2y}{dx^2} + y = 3x + 5 \tan x$. | [5] | CO1 | BT1 |
| Q.3(a) Solve the simultaneous differential equation
$\frac{dx}{dt} = 3x + 4y; \frac{dy}{dt} = 2x + y$. | [5] | CO1 | BT3 |
| Q.3(b) Find the solution of the total differential equation
$zydx + xzdy + xydz = 0$ by first verifying the condition of integrability. | [5] | CO1 | BT1 |
| Q.4 Find a Frobenius series solution of Bessel's equation of order zero $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + x^2y = 0$. | [10] | CO1 | BT1 |
| Q.5(a) Determine if the function $f(x, y) = y^{3/4}$ satisfies the Lipschitz condition on the rectangle $ x \leq 2, y \leq 3$. If yes, then find the Lipschitz constant. | [5] | CO2 | BT4 |
| Q.5(b) Find the eigen values and eigen functions of the Sturm-Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0, (y(0) = 0, y(L) = 0)$, where $L > 0$. | [5] | CO1 | BT1 |

:::::18/07/2023:::::