BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER: IV**
BRANCH: QEDS **SESSION: SP/2023**

SUBJECT: ED217 STOCHASTIC PROCESSES

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data handbook/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO
Q.1(a)	Two urns contain k balls each. Initially, the balls in the left urn are all red and the balls in the right urn are all blue. At each step, pick a ball at random from each urn and exchange them. Let Xn be the number of blue balls in the left urn. (Note that necessarily X0 = 0 and X1 = 1.) Argue that the process is a Markov chain. Evaluate the transition matrix.	[5]	CO1
Q.1(b)	Describe Markovian Queuing Models.	[5]	CO1
Q.2(a)	Google's PageRank search algorithm is based on the random surfer model, which is a random walk on the webgraph. For this graph, each vertex represents an internet page. A directed edge connects i to j if there is a hypertext link from page i to page j. When the random surfer is at page i, they move to a new page by choosing from the available links on i uniformly at random. A simplified network with 7 pages is described by the network matrix.	[5]	CO2

$$N = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 \\ 1/3 & 0 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1/4 & 0 & 0 & 1/4 & 0 & 1/4 & 1/4 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}.$$

For the problem of potentially getting stuck in small subgraphs of the webgraph, the solution proposed in the original paper by Brin and Page (1998) was to fix a damping factor $0 < p < 1$ for modifying the Q matrix. In their model, from a given page the random surfer, with probability $1 - p$, decides to not follow any links on the page and instead navigate to a new random page on the network. On the other hand, with probability p, they follow the links on the page as usual. Compute the stationary distribution of this network if the dumping factor is 0.85.

Q.2(b)	Describe Mover-Stayer Model.	[5]	CO2
Q.3(a)	Describe Branching Process. Evaluate the extinction probability for a branching process with offspring distribution a = (1/6, 1/2, 1/3).	[5]	CO3
Q.3(b)	Consider the male and female birth processes. Assume that births occur on a maternity ward at the average rate of 2 births per hour. (i) On an 8-hour shift, what is the expectation and standard deviation of the number of female births? (ii) Find the probability that only girls were born between 2 and 5 p.m. (iii) Assume that five babies were born on the ward yesterday. Calculate the probability that two are boys.	[5]	CO3
Q.4(a)	A spatial Poisson process in the plane has parameter $\lambda = 1/2$. Find the probability that a disk of radius 2 centered at (3, 4) contains exactly 5 points.	[5]	CO4
Q.4(b)	Occurrences of landfalling hurricanes during an El Niño event are modeled as a Poisson process in Bove et al. (1998). The authors assert that "During an El Niño year, the probability of two or more hurricanes making landfall in the United States is 28%.". Compute the rate of the Poisson process.	[5]	CO4

PTO

Q.5(a) Describe Martingales and Martingales Betting Strategy. Explain Filtration with an example of your choice. [5] CO5

Q.5(b) A Markov chain has following generator matrix. Exhibit the Kolmogorov backward equations. [5] CO5

$$Q = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -2 & 2 \\ 3 & 0 & -3 \end{pmatrix}$$

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