BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/SP2023)

CLASS:	**IMSc.**	**SEMESTER: IV**
BRANCH:	**QEDS**	**SESSION: SP/2023**

SUBJECT: ED217 STOCHASTIC PROCESS

TIME:	**02 Hours**	**FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

			CO
Q.1(a)	Describe Markov chain with some real-life examples. Consider a Markov chain with the state space S = {1, 2, 3} and transition probability matrix	[2]	CO1

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.0 & 0.6 & 0.4 \\ 0.5 & 0.2 & 0.3 \end{bmatrix} \end{array}.$$

Calculate P(X3 = 1 | X0 = 1, X1 = 2, X2 = 3) and P(X0 = 1, X1 = 2, X2 = 3)

Q.1(b)	Describe Chapman-Kolmogorov Equations and its significance in handling Markov Chain problems. Calculate three step transition probability matrix of P given in 1(a).	[3]	CO1
Q.2(a)	Consider a Markov chain with the one-step transition probability matrix.	[2]	CO1

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0 & 0.4 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0.6 & 0.4 \end{bmatrix} \end{array}.$$

(i) Plot the diagram of the Markov chain.
(ii) Show that the chain is non-ergodic because there are two invariant probability measures. Which one of them is the stationary distribution?

Q.2(b)	Find all recurrent and transient classes and their periods. Are there any absorbing or reflecting states?	[3]	CO1
Q.3(a)	Describe Random Walk and its categorization based on the transitions of the process.	[2]	CO1
Q.3(b)	In a random walk the probability that the walk advances by one step is p and retreats by one step is q = 1−p. At step n let the position of the walker be the random variable X_n. If the walk starts at x = 0, enumerate all possible sample paths which lead to the value $X_4 = -2$. Verify that	[3]	CO1

$$\mathbf{P}[X_4 = -2] = \binom{4}{1} pq^3$$

Q.4(a)	Describe Poisson process & the conditions for a process to be a Poisson process.	[2]	CO2
Q.4(b)	Explain the following diagram in reference to Poisson Process.	[3]	CO2

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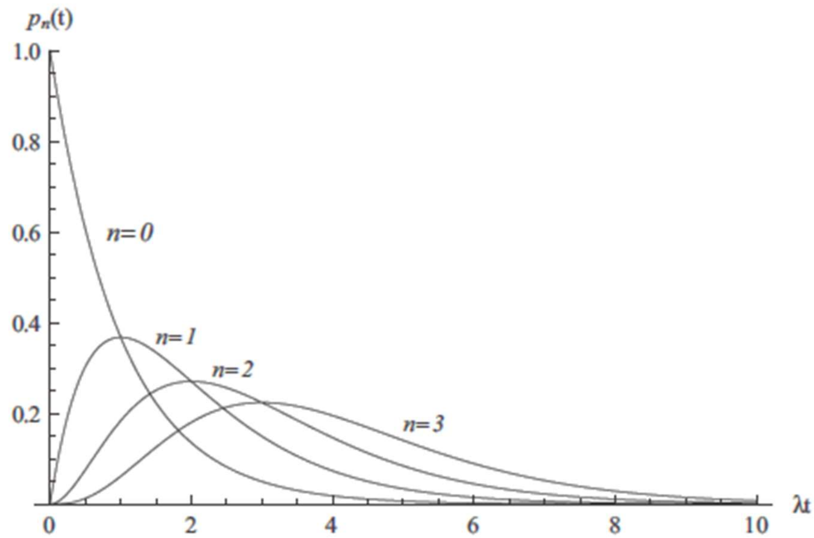
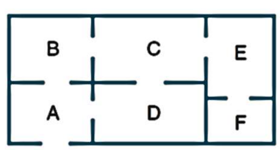


Q.5(a) A Himalayan view of winter might be described by the following transition matrix for a weather Markov chain, where r, s, and c denote rain, snow, and clear, respectively. [2] CO2

$$P = \begin{array}{c} \\ r \\ s \\ c \end{array} \begin{array}{c} \begin{array}{ccc} r & s & c \end{array} \\ \begin{pmatrix} 0.2 & 0.6 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.6 & 0.3 \end{pmatrix} \end{array}.$$

For tomorrow, the meteorologist predicts a 50% chance of snow and a 50% chance of rain. Evaluate the probability that it will snow 2 days later.

Q.5(b) A student visits an Ancient History Museum that is open between 9AM and 6PM. He enters the museum at 9AM and wanders the rooms in a random-walk fashion, spending 30 minutes in each room, and then choosing a door at random. The museum floor plan is given in the picture. [3] CO2



Describe the expected number of transitions between the rooms until he reaches the exit.

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