

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)**

CLASS: I MSc.
BRANCH: QEDS

SEMESTER : IV
SESSION : SP/2023

SUBJECT: ED213 OPTIMIZATION TECHNIQUES

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

- | | | | Marks | CO | BL | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-------------|--|----------|----------|----------|----------|--------|--------|--------|----------|-------|-------|---|-------|--------|-------|----------|-------|---|-------|--------|---|---|----------|---|------|--------|-------|---|---|----------|----|--------|---|---|-------|-----|----------|-----|---|-----|-----|-------------|--|--|---|---|---|---|-----|-----|--|-----|--|
| Q.1(a) | A salesman plans to visit five cities in such a way that he visits each city exactly once and return to the city from where he started . The distances between City i and City j ($i, j \in \{1, 2, 3, 4, 5\}$) are given in the following table. Find the shortest tour he can take using Hungarian method. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>City 1</th> <th>City 2</th> <th>City 3</th> <th>City 4</th> <th>City 5</th> </tr> </thead> <tbody> <tr> <th>City 1</th> <td>∞</td> <td>2</td> <td>5</td> <td>7</td> <td>1</td> </tr> <tr> <th>City 2</th> <td>6</td> <td>∞</td> <td>3</td> <td>8</td> <td>2</td> </tr> <tr> <th>City 3</th> <td>8</td> <td>7</td> <td>∞</td> <td>4</td> <td>7</td> </tr> <tr> <th>City 4</th> <td>12</td> <td>4</td> <td>6</td> <td>∞</td> <td>5</td> </tr> <tr> <th>City 5</th> <td>1</td> <td>3</td> <td>2</td> <td>8</td> <td>∞</td> </tr> </tbody> </table> | | City 1 | City 2 | City 3 | City 4 | City 5 | City 1 | ∞ | 2 | 5 | 7 | 1 | City 2 | 6 | ∞ | 3 | 8 | 2 | City 3 | 8 | 7 | ∞ | 4 | 7 | City 4 | 12 | 4 | 6 | ∞ | 5 | City 5 | 1 | 3 | 2 | 8 | ∞ | [7] | | CO2 | | | | | | | | | | | | | |
| | City 1 | City 2 | City 3 | City 4 | City 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| City 1 | ∞ | 2 | 5 | 7 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| City 2 | 6 | ∞ | 3 | 8 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| City 3 | 8 | 7 | ∞ | 4 | 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| City 4 | 12 | 4 | 6 | ∞ | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| City 5 | 1 | 3 | 2 | 8 | ∞ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q1.(b) | Find the dual of the following problem.
$\max x_1 + 2x_2 + x_3$
sub to $2x_1 + x_2 - x_3 \leq 2, -2x_1 + x_2 - 5x_3 \geq -6, 4x_1 + x_2 + x_3 \leq 6, x_1, x_2, x_3 \geq 0$. | | [3] | | CO2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q.2 | By using appropriate slack/surplus/artificial variables, solve the following linear programming problem.
$\max x_1 - x_2 + 3x_3$, sub to $x_1 + x_2 \leq 20, x_1 + x_3 - 5, x_2 + x_3 \geq 10, x_1, x_2, x_3 \geq 0$. | | [10] | | CO1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q.3(a) | Perform two iterations of steepest descent method to minimize the function $f(x_1, x_2) = 2(2x_1^2 - 2x_1x_2 + x_2^2)$ with initial starting point $x_0 = (2, 3)$. | | [7] | | CO3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q.3(b) | Find the number of experiments to be conducted in Fibonacci method to obtain a value of $\frac{L_n}{L_0} = 0.001$. | | [3] | | CO3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q.4(a) | Consider the following optimal simplex table (neglecting the integer constraints) for an integer linear programming problem. Find the integer solutions of both the variables (x_1 and x_2) using Gomory cutting plane method. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>c_j</th> <th></th> <th></th> <th>1</th> <th>2</th> <th>0</th> <th>0</th> <th>0</th> </tr> <tr> <th>c_B</th> <th>x_B</th> <th>B</th> <th>x_1</th> <th>x_2</th> <th>s_1</th> <th>s_2</th> <th>s_3</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>x_1</td> <td>7/2</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>-1/2</td> </tr> <tr> <td>0</td> <td>s_2</td> <td>4</td> <td>0</td> <td>0</td> <td>-2</td> <td>1</td> <td>1</td> </tr> <tr> <td>2</td> <td>x_2</td> <td>7/2</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1/2</td> </tr> <tr> <td>$s_j - c_j$</td> <td></td> <td></td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>1/2</td> </tr> </tbody> </table> | c_j | | | 1 | 2 | 0 | 0 | 0 | c_B | x_B | B | x_1 | x_2 | s_1 | s_2 | s_3 | 1 | x_1 | 7/2 | 1 | 0 | 1 | 0 | -1/2 | 0 | s_2 | 4 | 0 | 0 | -2 | 1 | 1 | 2 | x_2 | 7/2 | 0 | 1 | 0 | 0 | 1/2 | $s_j - c_j$ | | | 0 | 0 | 1 | 0 | 1/2 | [6] | | CO4 | |
| c_j | | | 1 | 2 | 0 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| c_B | x_B | B | x_1 | x_2 | s_1 | s_2 | s_3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | x_1 | 7/2 | 1 | 0 | 1 | 0 | -1/2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | s_2 | 4 | 0 | 0 | -2 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | x_2 | 7/2 | 0 | 1 | 0 | 0 | 1/2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| $s_j - c_j$ | | | 0 | 0 | 1 | 0 | 1/2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q.4(b) | Write down the steps to solve an integer linear programming problem using Branch-Bound techniques. | | [4] | | CO4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q.5 | Using dynamic programming technique, solve the following non-linear programming problem.
$\max x_1x_2x_3$, sub to $x_1 + x_2 + x_3 = 10, x_1, x_2, x_3 > 0$. | | [10] | | CO5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |