BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION SP2023)

CLASS: IMSc. **SEMESTER: II**
BRANCH: QEDS **SESSION: SP 2023**

SUBJECT: ED117 LINEAR ALGEBRA, VECTORS AND MATRICES

TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**
5. All the notations used in the question paper have usual meanings.

		Marks	CO	BL
Q.1(a)	Define basis of a vector space. Determine whether the set $S=\left\{\begin{pmatrix}1&1\\1&0\end{pmatrix},\begin{pmatrix}1&1\\0&1\end{pmatrix},\begin{pmatrix}1&0\\1&1\end{pmatrix},\begin{pmatrix}0&1\\1&1\end{pmatrix}\right\}$ is a basis of $\mathbb{R}^{2\times 2}$.	[3]	CO1	
Q.1(b)	Consider the subspaces $S=\{(x_1,x_2,x_3,x_4): x_2-2x_3+x_4=0\}$ and $T=\{(x_1,x_2,x_3,x_4): x_1=x_4, x_2=2x_3\}$ of $\mathbb{R}^4$. Find the dimension of $S\cap T$.	[2]	CO1	
Q.2(a)	Prove or give a counter example of the statement: Union of two subspaces of a vector space V is a subspace of V.	[3]	CO1	
Q.2(b)	Let $V\subset\mathbb{R}$ be a set of all ordered pair of all real numbers with standard operation of addition and the scalar multiplication defined by $a(x_1,x_2)=(ax_1,0)$. Show that V is NOT a vector space.	[2]	CO1	
Q.3(a)	Consider a linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x_1,x_2,x_3)=(x_1+x_2+x_3, 2x_1+x_2+2x_3, x_1+2x_2+x_3), (x_1,x_2,x_2)\in\mathbb{R}^3$. Find $Ker(T)$ and $Nullity(T)$.	[3]	CO1	
Q.3(b)	A linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x_1,x_2,x_3)=(2x_1+x_2-x_3, x_2+4x_3, x_1-x_2+3x_3), (x_1,x_2,x_2)\in\mathbb{R}^3$. Find the matrix of T relative to the standard basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ of $\mathbb{R}^3$.	[2]	CO1	
Q.4	Determine the values of a and b for which the system of linear equations, given by $x+2y+3z=6, x+3y+5z=9, 2x+5y+az=b$ has (i) no solution (ii) Unique solution (iii) infinitely many solutions.	[5]	CO2	
Q.5(a)	Decompose the matrix $A=\begin{bmatrix}2&-1&0&0\\-1&2&-1&0\\0&-2&2&-1\\0&0&-1&2\end{bmatrix}$ in form of $A=LU$, where L is a lower triangular matrix and U is an upper triangular matrix.	[3]	CO2	
Q.5(b)	Find the inverse of the matrix $A=\begin{bmatrix}1&3&3\\1&4&3\\1&3&4\end{bmatrix}$ by using elementary row operations.	[2]	CO2	

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