BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION SP2023)

CLASS: IMSc. **SEMESTER: II**
BRANCH: QEDS **SESSION: SP/2023**

SUBJECT: ED117 LINEAR ALGEBRA AND VECTORS & MATRICES

TIME: 03 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates
5. All the notations used in the question paper have usual meanings.

		Marks	CO	BL
Q.1(a)	Determine the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ which maps the basis vectors $(0,1,1),(1,0,1),(1,1,0)$ of $\mathbb{R}^3$ to the vectors $(2,0,0),(0,2,0),(0,0,2)$ respectively.	[5]	CO1	
Q.1(b)	Verify rank-nullity theorem for the linear transformation, defined in Q.2(a).	[5]	CO1	
Q.2(a)	Find the value(s) of λ, for which the system of linear equations $x+y+z=1, 2x+3y-z=\lambda+1, 2x+y+5z=\lambda^2+1$ is consistent.	[4]	CO2	
Q.2(b)	Use Cayley-Hamilton theorem to compute A^{100}, where $A=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$.	[6]	CO3	
Q.3(a)	Define norm of a vector in a Euclidean space. For $\alpha=(0,3,4)\in\mathbb{R}^3$ find $\lVert\alpha\rVert$ with standard inner product defined in $\mathbb{R}^3$.	[2+1]	CO3	
Q.3(b)	For any two vectors α,β in a Euclidean space V, prove the Schwarz's inequality $\lvert\langle\alpha,\beta\rangle\rvert \le \lVert\alpha\rVert\lVert\beta\rVert$, the equality holds when α,β are linearly dependent.	[7]	CO3	
Q.4(a)	Using Gram-Schmidt orthogonalization process convert the basis $B=\{(1,2,-2),(2,0,1),(1,1,0)\}$ of the Euclidean space $\mathbb{R}^3$with standard inner product into an orthogonal basis.	[6]	CO4	
Q.4(b)	Consider the orthogonal basis $\{(2,3,-1),(1,-2,-4),(2,-1,1)\}$ of the Euclidean space $\mathbb{R}^3$ with standard inner product. Find the projections of the vector $\alpha=(1,1,1)$ along these basis vectors and verify that α is the sum of its projections along these basis vectors.	[3+1]	CO4	
Q.5(a)	Reduce the quadratic form $2x^2+3y^2+4z^2-4xy+4yz$ into its normal form. Hence, check the positive definiteness of the quadratic form.	[4+1]	CO5	
Q.5(b)	Consider the matrix $A=\begin{bmatrix}4&0&0\\-2&1&0\\5&3&4\end{bmatrix}$. Show that 4 is an eigenvalue of the matrix. Find the characteristic space corresponding to the eigenvalue 4.	[2+3]	CO5	

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