BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION SP2023)

CLASS: IMSc. **SEMESTER : II**
BRANCH: CQEDS **SESSION : SP2023**

SUBJECT: ED113 STATISTICAL METHODS II

TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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Q.1(a) One observation is taken on a discrete random variable with PMF $f(x;\theta)$; where $\theta \in [1,2,3]$. Find maximum likelihood estimate (MLE) of θ. $f(x;\theta)$ is give below: [2]

x	0	1	2	3	4
$f(x;1)$	1/3	1/3	0	1/6	1/6
$f(x;2)$	1/4	1/4	1/4	1/4	0
$f(x;3)$	0	0	1/4	1/2	1/4

Q.1(b) For a given random sample of size n from $Uniform(\theta - \frac{1}{2}, \theta + \frac{1}{2})$, obtain the maximum likelihood estimate (MLE) of θ. [3]

Q.2(a) Find minimum variance bound (MVB) estimator for the parameter θ of the distribution with PDF [2]

$$f(x;\theta) = \frac{e^{-\theta}\theta^x}{x!} \; ; \; x = 0,1,2,\ldots$$

Q.2(b) State and prove Cramer Rao Inequality. [3]

Q.3(a) Let $X_1, X_2, \& X_3$ be a random sample from $B(1,p)$. Prove/disprove that $T = X_1 + 2X_2 + X_3$ is sufficient for p. [2]

Q.3(b) Let $X_1, X_2, \ldots, X_n$ be IID random variables having discrete uniform distribution on $\{1,2,3,\ldots,N\}$, where N is unknown. Show that $Max(X_1, X_2, \ldots, X_n)$ is sufficient for N. [3]

Q.4(a) For Poisson distribution with parameter θ, find a consistent estimator for $\frac{1}{\theta}$. [2]

Q.4(b) Let $X_1, X_2, X_3 \;\&\; X_4$ be independent random variables such that $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$ for $i = 1,2,3 \;\&\; 4$. Consider $Y = \frac{X_1 + X_2 + X_3 + X_4}{4}$; $T = \frac{X_1 + 2X_2 + X_3 - X_4}{4}$ and $Z = \frac{X_1 + 2X_2 + X_3 + X_4}{5}$. Examine whether Y, T and Z are unbiased estimator for μ. Find the most efficient estimator among three of them. [3]

Q.5 Let $X_1, X_2, \ldots, X_n$ be a random sample from uniform distribution with PDF

$$f(x,\theta) = \frac{1}{\theta}; \quad 0 < x < \theta \quad \forall \quad \theta \in \Theta.$$

Find

Q.5(a) Sufficient estimator for θ [2]

Q.5(b) Unbiased estimator for θ [3]