BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION SP2023)

CLASS: IMSc. **SEMESTER: II**
BRANCH: QEDS **SESSION: SP 2023**

SUBJECT: ED111 INTERMEDIATE ANALYSIS

TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates
5. All the notations used in the question paper have usual meanings.

			CO
Q.1(a)	Show that the sequence of functions $\{f_n(x)\}$, where $f_n(x) = nxe^{-nx^2}, x \geq 0$ is pointwise convergent but NOT uniformly convergent on $[0,k], k > 0$.	[3]	CO1
Q.1(b)	Test the uniform convergency for the series of functions $\sum_{n=1}^{\infty} \frac{\sin(x^2+n^2x)}{n(n+1)}$ by Weierstrass M-test.	[2]	CO1
Q.2	Show that the series $\sum_{n=1}^{\infty} \frac{x}{n^p+x^2n^q}$ converges uniformly over any finite interval $[a,b]$ for $(i). p > 1, q \geq 0$ and $(ii). 0 < p \leq 1, p+q > 2$.	[5]	CO1
Q.3(a)	Determine the radius of convergence of the power series $$1 + \frac{\alpha.\beta}{1.\gamma}x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2.\gamma(\gamma+1)}x^2 + \cdots$$	[2]	CO2
Q.3(b)	Identify the exact interval of convergence of the following power series. $$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}x^{2n}.$$	[3]	CO2
Q.4	Represent the periodic function $f(x) = x - x^2$ in form of Fourier series in $-\pi$ to π.	[5]	CO2
Q.5(a)	Define refinement of a partition of a given interval $[a,b]$.	[1]	CO3
Q.5(b)	Suppose that $P, Q \in \mathbb{P}$, where $\mathbb{P}$ is the set of all the partitions of $[a,b]$, and Q is a refinement of P. Let $f: [a,b] \to \mathbb{R}$ be a bounded function and $\alpha: [a,b] \to \mathbb{R}$ be a monotonic increasing function. Then prove that $$U(P,f,\alpha) \geq U(Q,f,\alpha) \text{ and } L(P,f,\alpha) \leq L(Q,f,\alpha.)$$	[4]	CO3

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