BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION SP2023)

CLASS: IMSc. **SEMESTER: II**
BRANCH: QEDS **SESSION: SP/2023**

SUBJECT: ED111 INTERMEDIATE ANALYSIS

TIME: 03 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**
5. All the notations used in the question paper have usual meanings.

		Marks	CO	BL
Q.1(a)	Investigate the uniform continuity for the series $\sum_{n=1}^{\infty}(-1)^n \frac{x^{2n}}{n^3(1+x^{2n})}, x \in \mathbb{R}$.	[5]	CO1	
Q.1(b)	Find the radius of convergence and the interval of convergence for the power series $$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}.$$	[5]	CO2	
Q.2(a)	Show that the function $[x]$, where $[x]$ is greatest integer not greater than x, is Riemann integrable in $[0,4]$ and find $\int_0^4 [x]\, dx$.	[5]	CO3	
Q.2(b)	Prove the following inequality. $$\frac{1}{3\sqrt{2}} < \int_0^1 \frac{x^2}{\sqrt{1+x}} dx < \frac{1}{3}.$$	[5]	CO3	
Q.3(a)	If $u = \sin^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$, then by Euler's theorem show that $xu_x + yu_y = \frac{5}{2}\tan u$.	[5]	CO4	
Q.3(b)	Evaluate the Jacobian $\frac{\partial(u,v,w)}{\partial(r,\theta,\phi)}$, where $u = \sqrt{yz}, v = \sqrt{zx}, w = \sqrt{xy}$, and $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$.	[5]	CO4	
Q.4(a)	Consider the function $f(x,y) = \frac{x^2y^2}{x^2+y^2}, (x,y) \neq (0,0)$ $= 0, (x,y) = (0,0)$. Check whether $f_{xy} = f_{yx}$ at $(0,0)$.	[5]	CO4	
Q.4(b)	Investigate the continuity of the function $f(x,y) = 2xy\frac{x^2-y^2}{x^2+y^2}, (x,y) \neq (0,0)$ $= 0, (x,y) = (0,0)$ at $(0,0)$.	[5]	CO4	
Q.5(a)	Evaluate the double integral $\int\int_R x^2 dx\, dy$, where R is the region bounded by the hyperbola $xy = 16$, and straight lines $y = x, y = 0, x = 8$.	[5]	CO5	
Q.5(b)	Find the integral $\int\int_R \sqrt{4a^2 - x^2 - y^2}\, dxdy$, where R is the region bounded by the upper half of the circle $x^2 + y^2 - 2ax = 0$, by transforming the integral into polar co-ordinates.	[5]	CO5	

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