SUBJECT: CL318 TRANSPORT PHENOMENA
TIME: $\quad 3$ Hours
FULL MARKS: 50
INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
Q. 1 (a) Using Navier -Stoke's equation, determine the velocity distribution in steady, laminar flow of an incompressible and viscosity fluid between two parallel plates placed vertical while the upper plate moves steadily in a direction parallel with the other plate kept fixed.
Q.1(b) liquid of constant density and viscosity is in a cylindrical container of radius $R$. The container is caused to rotate about its own axis at an angular velocity ( $\omega$ ). The cylinder axis is vertical, so that $g_{r}=0, g_{0}=0$, and $g_{z}=-g$, in which $g$ is the magnitude of the gravitational acceleration. Find the shape of the free surface of the liquid when steady state.
Q. 2 A Newtonian fluid is in laminar flow in a narrow slit formed by two parallel walls a distance $2 B$ apart. The flow is generated due to the motion of the wall in positive $z$ direction at $x=B$ and corresponding velocity of the wall is $U$. It is understood that $B \ll W$, so that edge effects are unimportant. Obtain the following quantities by making a shell momentum balance when $v_{x}=0, v_{y}=0$ and $p=p(z)$
a. Shear-stress and velocity distribution inside the slit
b. Mean fluid velocity
c. Maximum fluid velocity

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Q. 3 An incompressible, isothermal Newtonian fluid is held between two vertically placed co-axial cylinders. Determine the velocity distributions for the flow of the fluid when
(i) the outer cylinder is rotating at an angular velocity $\omega_{2}$ while the inner cylinder is stationary.
(ii) The inner and outer cylinders are rotating at angular velocities of $\omega_{1}$ and $\omega_{2}$, respectively.

Use Navier-Stoke's equation.
Q.4(a) Consider a copper rod of circular cross-section with radius R and electrical conductivity $\mathrm{K}_{\mathrm{e}}$ ohm $\mathrm{m}^{-1} \mathrm{~cm}^{-1}$ through where an electrical current with current density I amp $\mathrm{cm}^{-2}$ is flowing. The transmission of an electrical current is an irreversible process and some electrical energy is converted to thermal energy. The rate of heat production per unit volume $\left(\mathrm{S}_{\mathrm{e}}\right)$ is given by the expression $\mathrm{S}_{\mathrm{e}}=I^{2} / \mathrm{K}_{\mathrm{e}}$. The surface of the rod is maintained at temperature of $\mathrm{T}_{0}$. Make a shell balance and obtained the temperature profile in the rod. Also find out the expression for heat flow at the surface.
Q.4(b) A current of 50 A is passed through a steel wire 1 m long and 4 mm diameter submerged in a bath. The bulk temperature in the bath is $120{ }^{\circ} \mathrm{C}$. Calculate center-line temperature of the wire. Data: Specific resistance of steel $=1.5 \times 10^{-6}$ ohm-m; Thermal conductivity of steel $=20 \mathrm{~W} / \mathrm{m}^{2} . \mathrm{K}$; Convective heat transfer coefficient $=300 \mathrm{~W} / \mathrm{m}^{2} . \mathrm{K}$. $\left[\mathrm{T}_{\mathrm{c}}=\mathrm{T}_{\mathrm{w}}+\frac{q r^{2}}{4 K}\right.$ ] [All teams usual meaning]
Q. 5 Derive the concentration profile during the absorption of oxygen $\left(\mathrm{O}_{2}\right)$ into a falling water film.

## §B. 4 THE EQUATION OF CONTINUITY ${ }^{a}$

$$
[\partial \rho / \partial t+(\nabla \cdot \rho \mathbf{v})=0]
$$

Cartesian coordinates $(x, y, z)$ :

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho v_{x}\right)+\frac{\partial}{\partial y}\left(\rho v_{y}\right)+\frac{\partial}{\partial z}\left(\rho v_{z}\right)=0 \tag{B.4-1}
\end{equation*}
$$

Cylindrical coordinates ( $r, \theta, z$ ):

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\frac{1}{r} \frac{\partial}{\partial r}\left(\rho r v_{r}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\rho v_{\theta}\right)+\frac{\partial}{\partial z}\left(\rho v_{z}\right)=0 \tag{B.4-2}
\end{equation*}
$$

Spherical coordinates $(r, \theta, \phi)$ :

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(\rho r^{2} v_{r}\right)+\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}\left(\rho v_{\theta} \sin \theta\right)+\frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}\left(\rho v_{\phi}\right)=0 \tag{B.4-3}
\end{equation*}
$$

${ }^{a}$ When the fluid is assumed to have constant mass density $\rho$, the equation simplifies to $(\nabla \cdot v)=0$.

## §B. 5 THE EQUATION OF MOTION IN TERMS OF $\tau$

$$
[\rho D \mathbf{v} / D t=-\nabla p-[\nabla \cdot \boldsymbol{\tau}]+\rho \mathbf{g}]
$$

Cylindrical coordinates $(r, \theta, z)^{b}$

$$
\begin{align*}
& \rho\left(\frac{\partial v_{r}}{\partial t}+v_{r} \frac{\partial v_{r}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{r}}{\partial \theta}+v_{z} \frac{\partial v_{r}}{\partial z}-\frac{v_{\theta}^{2}}{r}\right)=-\frac{\partial p}{\partial r}-\left[\frac{1}{r} \frac{\partial}{\partial r}\left(\tau_{T H}\right)+\frac{1}{r} \frac{\partial}{\partial \theta} \tau_{\theta r}+\frac{\partial}{\partial z} \tau_{z t}-\frac{\tau_{\theta \theta}}{r}\right]+\rho q_{r}  \tag{B.5-4}\\
& \rho\left(\frac{\partial v_{\theta}}{\partial t}+v_{r} \frac{\partial v_{\theta}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{\theta}}{\partial \theta}+v_{z} \frac{\partial v_{\theta}}{\partial z}+\frac{v_{r} v_{\theta}}{r}\right)=-\frac{1}{r} \frac{\partial p}{\partial \theta}-\left[\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \tau_{\tau \theta}\right)+\frac{1}{r} \frac{\partial}{\partial \theta} \tau_{\theta \theta}+\frac{\partial}{\partial z} \tau_{z \theta}+\frac{\left.\tau_{\theta r}-\tau_{\theta \theta}\right]+\rho g_{\theta}}{r}\right.  \tag{B.5-5}\\
& \rho\left(\frac{\partial v_{z}}{\partial t}+v_{r} \frac{\partial v_{z}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{z}}{\partial \theta}+v_{z} \frac{\partial v_{z}}{\partial z}\right)=-\frac{\partial p}{\partial z}-\left[\frac{1}{r} \frac{\partial}{\partial r}\left(\tau_{n z}\right)+\frac{1}{r} \frac{\partial}{\partial \theta} \tau_{\theta z}+\frac{\partial}{\partial z} \tau_{z z}\right]+\rho g_{z} \tag{B.5-6}
\end{align*}
$$

${ }^{b}$ These equations have been written without making the assumption that $\tau$ is symmetric. This means, for example, that when the usual assumption is made that the stress tensor is symmetric, $\tau_{\text {th }}-\tau_{\theta t}=0$.

## §B. 6 EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT $\rho$ AND $\mu$

$$
\left\lceil\rho D \mathbf{v} / D t=-\nabla p+\mu \nabla^{2} \mathbf{v}+\rho \mathrm{g}\right\rfloor
$$

## Cylindrical coordinates $(r, \theta, z)$ :

$$
\begin{align*}
& \rho\left(\frac{\partial v_{r}}{\partial t}+v_{r} \frac{\partial v_{r}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{r}}{\partial \theta}+v_{z} \frac{\partial v_{r}}{\partial z}-\frac{v_{\theta}^{2}}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r v_{r}\right)\right)+\frac{1}{r^{2}} \frac{\partial^{2} v_{r}}{\partial \theta^{2}}+\frac{\partial^{2} v_{r}}{\partial z^{2}}-\frac{2}{r^{2}} \frac{\partial v_{\theta}}{\partial \theta}\right]+\rho g_{r}  \tag{6-4}\\
& \rho\left(\frac{\partial v_{\theta}}{\partial t}+v_{r} \frac{\partial v_{\theta}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{\theta}}{\partial \theta}+v_{z} \frac{\partial v_{\theta}}{\partial z}+\frac{v_{r} v_{\theta}}{r}\right)=-\frac{1}{r} \frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r v_{\theta}\right)\right)+\frac{1}{r^{2}} \frac{\partial^{2} v_{\theta}}{\partial \theta^{2}}+\frac{\partial^{2} v_{\theta}}{\partial z^{2}}+\frac{\partial}{r^{2}} \frac{\partial v_{r}}{\partial \theta}\right]+\rho g_{\theta}  \tag{B.6-5}\\
& \rho\left(\frac{\partial v_{z}}{\partial t}+v_{r} \frac{\partial v_{z}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{z}}{\partial \theta}+v_{z} \frac{\partial v_{z}}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_{z}}{\partial r}\right)+\frac{1}{r^{2}} \frac{\partial^{2} v_{z}}{\partial \theta^{2}}+\frac{\partial^{2} v_{z}}{\partial z^{2}}\right]+\rho g_{z} \tag{B.6-6}
\end{align*}
$$

