BIRLA INSTITUTE OF TECHNOLOGY
MESRA – 835215, RANCHI, INDIA

UG

Name: .... Roll No.: ....

Branch: .... Signature of Invigilator: ....

Semester: VIth Date: 07/05/2022 (MORNING)

Subject with Code: **MA523 COMPUTATIONAL MATHEMATICS**

Marks Obtained	Section A (30)	Section B (20)	Total Marks (50)

INSTRUCTION TO CANDIDATE

1. The booklet (question paper cum answer sheet) consists of two sections. First section consists of MCQs of 30 marks. Candidates may mark the correct answer in the space provided / may also write answers in the answer sheet provided. The Second section of question paper consists of subjective questions of 20 marks. The candidates may write the answers for these questions in the answer sheets provided with the question booklet.
2. The booklet will be distributed to the candidates before 05 minutes of the examination. **Candidates should write their roll no. in each page of the booklet.**
3. Place the Student ID card, Registration Slip and No Dues Clearance (if applicable) on your desk. All the entries on the cover page must be filled at the specified space.
4. Carrying or using of mobile phone / any electronic gadgets (except regular scientific calculator)/chits are strictly prohibited inside the examination hall as it comes under the category of unfair means.
5. No candidate should be allowed to enter the examination hall later than 10 minutes after the commencement of examination. Candidates are not allowed to go out of the examination hall/room during the first 30 minutes and last 10 minutes of the examination.
6. Write on both side of the leaf and use pens with same ink.
7. The medium of examination is English. Answer book written in language other than English is liable to be rejected.
8. All attached sheets such as graph papers, drawing sheets etc. should be properly folded to the size of the answer book and tagged with the answer book by the candidate at least 05 minutes before the end of examination.
9. The door of examination hall will be closed 10 minutes before the end of examination. Do not leave the examination hall until the invigilators instruct you to do so.
10. Always maintain the highest level of integrity. Remember you are a BITian.
11. Candidates need to **submit the question paper cum answer sheets before leaving the examination hall**.

Department of Mathematics, BIT Mesra, Ranchi

MA 428: Numerical and Statistical Method- UG

Instructions: Attempt all questions from Section A and any five questions from Section B. Time: 2 hour

Section-A

1. The functional $I(y(x)) = \int_a^b (y^2 + y'^2 - 2y\sin x)dx$ has the following extremal with c_1 and c_2 as arbitrary constants.

   (a) $y = c_1 e^{2x} + c_2 e^{-2x} + \frac{1}{2}\sin x$ (b) $y = c_1 e^{x} + c_2 e^{-x} - \frac{1}{2}\sin x$

   (c) $y = c_1 e^{2x} + c_2 e^{-2x} + \frac{1}{2}\cos x$ (d) $y = c_1 e^{x} + c_2 e^{-x} + \frac{1}{2}\sin x$

2. A necessary condition that the integral $I = \int_{x_1}^{x_2} F(x,\, y,\, y')dx$ will be stationary is

   (a) $\delta I = const$ (b) $\delta I = 0$ (c) $\delta I \neq 0$ (d) $\delta I \neq const$

3. In an integral $I = \int_{x_1}^{x_2} F(x,\, y,\, y')dx$ if F is explicitly independent of x, then the Euler Lagrange's equation is

   (a) $F - y'$ =const (b) $F - y'\frac{\partial F}{\partial y}$=const (c) $F - y'\frac{\partial F}{\partial y'}$=const (d) F=const

4. Find the central second difference of u in y-direction using the Taylor series expansion. (Note- i and j are in x and $y-$ direction respectively).

   (a) $\frac{u_{i,j+1}+2u_{i,j}+u_{i,j-1}}{(\Delta y)^2}$ (b) $\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{(\Delta y)^2}$ (c) $\frac{u_{i,j+1}-2u_{i,j}-u_{i,j-1}}{(\Delta y)^2}$ (d) $\frac{u_{i,j+1}+2u_{i,j}-u_{i,j-1}}{(\Delta y)^2}$

5. The truncation error in a finite difference expansion is $-\left(\frac{\partial^2 u}{\partial x^2}\right)_{ij}\frac{\Delta x}{2} - \left(\frac{\partial^3 u}{\partial x^3}\right)_{ij}\frac{(\Delta x)^3}{6}$. What is the order of accuracy of the finite difference equation?

   (a) 2 (b) -2 (c) 1 (d) -1

6. The partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$; where $c \neq 0$ is known as

   (a) Heat equation (b) Wave equation

   (c) Poisson's equation (d) Laplace equation

7. The extremum of the functional $I = \int_0^1 ((\frac{dy}{dx})^2 + 12xy)dx$ satisfying the condition $y(0) = 0$, $y(1) = 1$ is attained on the curve

   (a) $y = \sin^2\frac{\pi x}{2}$ (b) $y = \sin\frac{\pi x}{2}$

   (c) $y = x^3$ (d) $y = \frac{1}{2}(x^3 + \sin\frac{\pi x}{2})$

8. The lowest eigen value of the 2×2 matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ is

   (a) 1 (b) 4 (c) 5 (d) 2

9. Matrix P to convert the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

into tridiagonal matrix using Given's method is

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{\sqrt{13}} & \frac{-3}{\sqrt{13}} \\ 0 & \frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{13} & \frac{-3}{13} \\ 0 & \frac{3}{13} & \frac{2}{13} \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{-3}{2} \\ 0 & \frac{3}{2} & \frac{2}{3} \end{bmatrix}$ (d) None

10. The general solution of the partial differential equation $\frac{\partial^2 z}{\partial x \partial y} = x + y$ is of the form

(a) $\frac{1}{2}xy(x+y) + F(x) + G(y)$ (b) $\frac{1}{2}xy(x-y) + F(x) + G(y)$

(c) $\frac{1}{2}xy(x-y) + F(x)G(y)$ (d) $\frac{1}{2}xy(x+y) + F(x)G(y)$

11. Find the first-order forward difference approximation of $(\frac{\partial u}{\partial x})_{i,j}$ using the Taylor series expansion (Note- i and j are in x and $y-$ direction respectively).

(a) $\frac{u_{i,j+1} - u_{i,j}}{2\Delta x}$ (b) $\frac{u_{i+1,j} - u_{i,j}}{2\Delta x}$ (c) $\frac{u_{i,j+1} - u_{i,j}}{\Delta x}$ (d) $\frac{u_{i+1,j} - u_{i,j}}{\Delta x}$

12. The set of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} = 0$ is

(a) $1, x, e^{-x}, xe^{-x}$ (b) $1, x, e^{x}, xe^{x}$ (c) $1, x, e^{x}, e^{-x}$ (d) $1, x, e^{x}, xe^{-x}$

13. What is the least order of accuracy for the second derivatives?

(a) first-order (b) third-order (c) fourth-order (d) second-order

14. If the partial differential equation $(x-1)^2 u_{xx} - (y-2)^2 u_{yy} + 2xu_x + 2yu_y + 2xyu = 0$ is parabolic in $S \in R^2$ but not in R^2/S, then S is

(a) $\{\ (x,\ y) \in R^2 : x = 1\, or\, y = 2\}$ (b) $\{\ (x,\ y) \in R^2 : x = 1\, and\, y = 2\}$

(c) $\{\ (x,\ y) \in R^2 : x = 1\}$ (d) $\{\ (x,\ y) \in R^2 : y = 2\}$

15. Absolute value of the largest eigen value using the power method for the matrix $\begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$ with the initial condition [1, 1] is

(a) 1 (b) 2.8 (c) 2 (d) None

Section-B

16. Find D'Alembert's solution of one-dimensional wave equation with the following initial conditions:

$$u(x,0) = \sin x, u_t(x,0) = \cos x.$$

17. A string is fixed at $x = 0$ and $x = L$ and lies initially along the x axis. If it is set in motion by giving all points $0 < x < L$ a constant transverse velocity $\frac{\partial u}{\partial t} = u_0$ at $t = 0$, then find the subsequent motion of the string.

18. Explain FTCS (forward time central space) scheme for heat equation $u_t = u_{xx}$.

19. Discuss the stability of the FTCS (forward time central space) scheme for heat equation using Von-Neumann analysis.

20. Find the weak form (variational form) of the following equations:

$$-\frac{d^2u}{dx^2} = \cos \pi x,\ \ 0 < x < 1;\ \ u(0) = u(1) = 0$$

21. Solve the problem described by the following equations using Finite Element Method (FEM)

$$-\frac{d^2u}{dx^2} = \cos \pi x,\ \ 0 < x < 1;\ \ u(0) = u(1) = 0$$

Use the uniform mesh of three linear elements to solve the problem.