BIRLA INSTITUTE OF TECHNOLOGY
MESRA – 835215, RANCHI, INDIA

UG

Name: ………………………………………………………. Roll No.: …………………………………………..

Branch: ………………………………………………………. Signature of Invigilator: ……………………

Semester: VIth Date: 30/04/2022 (MORNING)

Subject with Code: **MA428 NUMERICAL & STATISTICAL METHODS**

Marks Obtained	Section A (30)	Section B (20)	Total Marks (50)

INSTRUCTION TO CANDIDATE

1. The booklet (question paper cum answer sheet) consists of two sections. First section consists of MCQs of 30 marks. Candidates may mark the correct answer in the space provided / may also write answers in the answer sheet provided. The Second section of question paper consists of subjective questions of 20 marks. The candidates may write the answers for these questions in the answer sheets provided with the question booklet.
2. The booklet will be distributed to the candidates before 05 minutes of the examination. **Candidates should write their roll no. in each page of the booklet.**
3. Place the Student ID card, Registration Slip and No Dues Clearance (if applicable) on your desk. All the entries on the cover page must be filled at the specified space.
4. Carrying or using of mobile phone / any electronic gadgets (except regular scientific calculator)/chits are strictly prohibited inside the examination hall as it comes under the category of unfair means.
5. No candidate should be allowed to enter the examination hall later than 10 minutes after the commencement of examination. Candidates are not allowed to go out of the examination hall/room during the first 30 minutes and last 10 minutes of the examination.
6. Write on both side of the leaf and use pens with same ink.
7. The medium of examination is English. Answer book written in language other than English is liable to be rejected.
8. All attached sheets such as graph papers, drawing sheets etc. should be properly folded to the size of the answer book and tagged with the answer book by the candidate at least 05 minutes before the end of examination.
9. The door of examination hall will be closed 10 minutes before the end of examination. Do not leave the examination hall until the invigilators instruct you to do so.
10. Always maintain the highest level of integrity. Remember you are a BITian.
11. Candidates need to **submit the question paper cum answer sheets before leaving the examination hall**.

Department of Mathematics, BIT Mesra, Ranchi

MA 428: Numerical and Statistical Method- UG

Instructions: Attempt all questions. Time: 2 hour

Section-A

1. The method $x_{n+1} = px_n + \frac{qa}{x_n^2},\ n = 0, 1, \cdots$, is used to find an approximate value of $a^{1/3}, a > 0$. Which of the following values of p and q is true, so that the order of the method is as high as possible.

   (a) $p = 2/3,\ q = 1/3$ (b) $p = 1/3,\ q = 2/3$

   (c) $p = 1/2,\ q = 3/2$ (d) $p = 3/2,\ q = 1/2$

2. Using two iterations of the Newton-Raphson method, approximate value of $\sqrt{(}13)$ is

   (a) 5.6 (b) 5.5679 (c) 5.5678 (d) 5.5680

3. Which of the following is the best approximation to the number $\frac{2}{3}$

   (a) 0.6 (b) 0.66

   (c) 0.67 (d) None of the above

4. Simpson's $\frac{3}{8}$ rule for the evaluation of $\int_a^b f(x)dx$ requires the interval $[a,\ b]$ to be divided into

   (a) an even number of subintervals

   (b) an odd number of subintervals

   (c) any number of subintervals

   (d) number of the subintervals should be multiple of 3

5. If a quadrature formula $\frac{3}{2}f(-\frac{1}{3}) + Kf(\frac{1}{3}) + \frac{1}{2}f(1)$, that approximates $\int_{-1}^{1} f(x)dx$, is found to be exact for quadratic polynomials, then the value of K is

   (a) 2 (b) 1 (c) 0 (d) -1

6. The convergence properties of the iteration method to solve system of linear equation depends on the iteration matrix H. If $\rho(H)$ is the spectral radius of H, then a necessary and sufficient condition for convergence is

   (a) $\rho(H) \leq 1$ (b) $\rho(H) < 1$ (c) $\rho(H) \geq 1$ (d) $\rho(H) > 1$

7. Consider the following statements:

   P: Gauss elimination method and Gauss Jacobi-method are direct methods.

   Q: Gauss-Seidel method and Gauss-Jordan method are iterative methods.

   (a) Only P is true (b) Only Q is true

   (c) Both P and Q are true (d) Both P and Q are false

8. Let X be a random variable such that $E(X^2) = E(X) = 1$. Then, $E(X^{100})$ is

   (a) 0 (b) 1 (c) 2^{100} (d) $2^{100} + 1$

9. Suppose that the time taken by a particle to move from one fixed point to another fixed point is distributed as $N(\mu, \sigma^2 = 4)$. A random sample of 9 readings has a mean of 50. By testing the hypothesis $H_0 : \mu = 52$ against $H_1 : \mu \neq 52$ at 1% level, we conclude that (Use $Z_\alpha = 2.58$ at 1%)

(a) $\mu \neq 52$
(b) $\mu = 52$
(c) $\mu = 50$
(d) None of the above

10. Which of the following is a third degree interpolating polynomial from the following data?$(0, 0), (1, 1), (2, 8), (3, 27), (4, 64)$.

(a) $P_3(x) = x + 3x(x-1) + x(x-1)(x-2)$
(b) $P_3(x) = 7(x-1) + 6(x-1)(x-2) + (x-1)(x-2)(x-3)$.
(c) $P_3(x) = 37(x-4) + 9(x-4)(x-3) + (x-4)(x-3)(x-2)$
(d) All of the above

11. Let X have a binomial distribution with parameters n and p, where n is an integer greater than 1 and $0 < p < 1$. If $P(X = 0) = P(X = 1)$, then the value of p is

(a) $\frac{1}{n-1}$
(b) $\frac{n}{n+1}$
(c) $\frac{1}{n+1}$
(d) $\frac{1}{n}$

12. One root of the equation $e^x - 3x^2 = 0$ lies in the interval (3, 4). The least number of the iterations of the bisection method, so that error $\leq 10^{-3}$ are

(a) 10
(b) 8
(c) 6
(d) 4

13. Let E and F be any two events with $P(E \cup F) = 0.8$, $P(E) = 0.4$ and $P(E/F) = 0.3$. Then, $P(F)$ is

(a) $\frac{3}{7}$
(b) $\frac{4}{7}$
(c) $\frac{3}{5}$
(d) $\frac{2}{5}$

14. Using Euler's method taking step size=0.1, the approximate value of y obtained corresponding to $x = 0.2$ for the initial value problem

$$\frac{dy}{dx} = x^2 + y^2$$

and $y(0) = 1$, is

(a) 1.322
(b) 1.122
(c) 1.222
(d) 1.110

15. A simple random sample of size 10 from $N(\mu, \sigma^2)$ gives 98% confidence interval (20.49, 23.51). Then, the null hypothesis $H_0 : \mu = 20.5$, against $H_A : \mu \neq 20.5$

(a) can be rejected at 2% level of significance.
(b) can not be rejected at 5% level of significance.
(c) can be rejected at 10% level of significance.
(d) can not be rejected at any level of significance.

Section-B

16. Find an interval of unit length which contains the smallest positive root of the equation $x^3 - 3x - 1 = 0$. Take the end points of this interval as initial approximations and perform two iterations of the secant method.

17. Let $f(x) = \ln(1 + x), x_0 = 1$ and $x_1 = 1.2$. Use Lagrange linear interpolating polynomial to obtain an approximate value of $\ln(1.1)$. Find the maximum absolute error at this point.

18. The values of the constants α, β, x_1 for which the quadrature formula $\int_0^1 f(x)dx = \alpha f(0) + \beta f(x_1)$ is exact for polynomials of degree as high as possible.

19. A box contains 7 white and 5 black balls. Three balls are drawn at random. Find the probability that they are not of the same colour

    1. when the balls are drawn at a time .
    2. when the balls are drawn one by one without replacement.
    3. when the balls are drawn one by one with replacement.

20. Let E be the permissible error in the means $\bar{x}$ and μ of a sample and population respectively. Let the standard deviation of the population be σ. Find the minimum sample size n if $P(|\bar{x} - \mu| < E) > 0.98$. Hence, find the minimum sample size to estimate the mean within 5 units of the true mean, if $\sigma = 30$ and with 98% confidence. (Use critical value $Z_\alpha = 2.33$.)