BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END - SEMESTER EXAMINATION)

CLASS : MSC | **SEMESTER: II**
BRANCH : MATHEMATICS | **SESSION: SP/2022**

SUBJECT: MA425 FLUID DYNAMICS

TIME : 2 HOURS | **FULL MARKS: 50**

INSTRUCTIONS :

1. The question paper contains 10 questions each of 5 marks. Students have to attempt all the questions, however, internal choices are given in Q. Nos. 5, 9 and 10.
2. The missing data, if any, may be assumed suitably.
3. Before attempting the question paper, be sure that you have got the correct question paper.

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1. Using material derivative, obtain the acceleration of a fluid particle at the point $(1,1,1)$ at the time $t = 1$second, if the velocity field $\overline{q} = (yz+t)\hat{i} + (xz-t)\hat{j} + xy\hat{k}.$

2. The velocity components (u, v, w) in a three - dimensional flow field for an incompressible fluid are:

$$u = x,\ v = y, w = -z$$

Determine the equation of streamlines.

3. In a fluid motion free from sources or sinks, if ρ is the density and $\overline{q}$ is the fluid velocity, then prove that the equation of continuity reduces to $\nabla^2\phi = 0$ for incompressible fluid, which exhibits the flow of potential kind with velocity potential φ.

4. For an incompressible fluid, check whether the motion specified by the following velocity components (u, v, w):

$$u = \frac{2ax}{y^2}, v = -\frac{2ay}{z^2}, w = -\frac{2a}{z}$$

gives a possible motion or not, where a is a constant.

5. Differentiate between the following types of fluid flow:

i. steady and unsteady

ii. laminar and turbulent

OR

The velocity at a point in a fluid for a one - dimensional flow may be given in the Eulerian coordinates by $u = At,$ where A is constant. If the initial position of the fluid particle is x_0 at initial time $t_0 = 0.$ Find the displacement of the fluid particle in Lagrangian coordinates.

6. For a two dimensional irrotational flow of an incompressible fluid, the velocity potential $\varphi = A(x^2 - y^2),$ where A is constant. Obtain the stream function using Cauchy - Riemann equations.

7. Determine the arrangement and strengths of sources and sinks in a two - dimensional flow that gives rise to the following complex potential at any point z :

$$W = \log\left(z^2 - 25\right) - \log z^2$$

Also, identify the stagnations points of the fluid flow.

8. Applying method of images, derive the expression of the complex potential for the image of a doublet of strength μ placed at the point $z = c$ and inclined at an angle α with the $x-$ axis, with regard to a line along $y-$ axis as rigid boundary.

9. Define sources and sinks in a two - dimensional flow of an incompressible fluid. Why these sources and sinks are referred to be singularities of a flow field?

OR

A source S and a sink T of equal strengths m are situated within the space bounded by a circle whose centre is O. If S and T are at equal distances from O on opposite sides of it and on the same diameter AOB along $x-$axis. If OS = OT = c and radius of the circle is a, then, using Circle theorem, obtain the complex potential at any point z in Argand plane.

10. Explain Buchkingam $\pi-$ theorem to obtain the dimensionless numbers of the variables of physical problem in a dimensional analysis technique.

OR

Using Rayleigh technique, prove that the resistance (R) to the motion of a sphere of diameter (D) moving with a uniform velocity (v) through a real fluid having density (ρ) and viscosity (μ) is given by:

$$R = \rho D^2 v^2 f\left(\frac{\mu}{\rho VD}\right)$$

where f is an arbitrary function.

06/05/2022 E
