BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END - SEMESTER EXAMINATION)

CLASS : IMSC /MSC **SEMESTER: VIII/II**
BRANCH : MATHEMATICS & COMPUTING/MATHEMATICS **SESSION: SP/2022**

SUBJECT: MA419 MATHEMATICAL ECOLOGY

TIME : 2 HOURS **FULL MARKS: 50**

INSTRUCTIONS :

1. The question paper contains 10 questions each of 5 marks. Students have to attempt all the questions, however, internal choices are given in **Q. Nos. 8, 9 and 10.**
2. The missing data, if any, may be assumed suitably.
3. Before attempting the question paper, be sure that you have got the correct question paper.

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1. Using suitable transformations, convert the following third order differential equation into a system of first order differential equations:

$$\frac{d^3x}{dt^3} - 4\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = 0$$

2. Obtain the general solution of the following linear system of first order differential equations:

$$\frac{dx_1}{dt} - x_1 - 2x_2 = 0; \frac{dx_2}{dt} = 3x_2$$

3. Check whether origin is an equilibrium state of the following system or not:

$$\frac{dx}{dt} = \cos y - \sin x - 1; \frac{dy}{dt} = x - y - y^2$$

If it is so, then investigate the stability of the system around origin.

4. Using $V(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2$ as Lyapunov function, check the stability of the following nonlinear system around the zero solution:

$$\frac{dx_1}{dt} = -2x_2 + x_2x_3 - x_1^3; \frac{dx_2}{dt} = x_1 - x_1x_3 - x_2^3; \frac{dx_3}{dt} = x_1x_2 - x_3^3$$

5. In the following single - species model with Allee effect,

$$\frac{dN}{dt} = rN\left(\frac{N}{K_0} - 1\right)\left(1 - \frac{N}{K}\right)$$

where $N(t)$ denotes the species population density at time t. Here, the parameter r is the intrinsic growth rate, K is the carrying capacity and K_0 is the Allee effect threshold, which are all positive. Discuss the possibility of occurrence of transcritical - bifurcation in the model.

6. Consider the following harvesting model :

$$\frac{dx}{dt} = x(1 - x) - \mu$$

where $x(t)$ denotes the population size of fish at time t. The fish population is being harvested at constant rate $\mu (> 0)$. Identify how many feasible steady states exist for the model and determine the range of μ under which they exist.

7. In a two - dimensional dynamical system, the characteristic equation from the Jacobian matrix at some of its equilibrium is obtained to be:

$$\lambda^2 + 2\alpha\lambda + (\alpha^2 + \beta^2) = 0$$

where α, β are parameters. Is there any possibility of occurrence of Hopf - bifurcation in the system? If it occurs, identify the Hopf - bifurcation parameter. Support your answers with proper reasoning.

8. Consider the following predator - prey model:

$$\frac{dx}{dt} = r(1-y)x$$

$$\frac{dy}{dt} = m(x-1)y$$

where $x(t)$ denotes the population size of prey and $y(t)$ denotes the population size of predator with r, m as positive parameters. Locate the equilibrium states of the model and classify them.

OR

Differentiate between the following:

i. Supercritical and subcritical Hopf bifurcation

ii. Holling type - I and Holling type - II functional responses

9. Develop a two -dimensional model for competing species A and B with $n_A(t)$ and $n_B(t)$ as population sizes at time t. The assumptions are:

i. The species A exhibits exponential growth in the absence of species with intrinsic growth rate r_A, whereas the species B exhibits logistic growth in the absence of species A with intrinsic growth rate r_B and carrying capacity K_B.

ii. The two species A and B compete with each other, where γ_{AB} and γ_{BA} are the interspecific rate coefficients of B on A and A on B respectively.

OR

Under what ecological situations, competition models can be formulated? How these models differ from mutualism ones? Discuss it with the help of some real life examples.

10. Using reaction - diffusion equation, develop a spatial one - dimensional mathematical model for some species, which exhibits logistic growth and the flux of movement of species from one place to another is given by simple Fickian diffusion.

OR

Discuss the following flux choices that can be incorporated in the spatially - structured population models to study the movement pattern of species from one place to another:

i. Advection and Convection'

ii. Diffusion
