BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC SEMESTER : VII
BRANCH: MATHS & COMP. SESSION : SP/22

SUBJECT: MA414-ADVANCED OPERATIONS RESEARCH

TIME: 2 Hrs. FULL MARKS: 50

Q1. Any two

a) A vessel is to be loaded with stocks of 3 items. Each unit of item i has a weight w_i and value r_i. The maximum cargo weight the vessel can lake is 5 and the details of the three items are as follows:

i	w_i	r_i
1	1	30
2	3	80
3	2	65

Develop the recursive equation for the above case and find the most valuable cargo load without exceeding the maximum cargo weight by using dynamic programming.

b) Use dynamic programming to solve the following LPP:

$$\begin{aligned} \text{Maximize } z &= 3x_1 + 5x_2 \\ \text{subject to } & x_1 \le 4 \\ & x_2 \le 6 \\ & 3x_1 + 2x_2 \le 18 \\ & x_1, x_2 \ge 0. \end{aligned}$$

c) Circle farms wants to develop a replacement policy for its 2-years tractor over the next 5 years. A tractor must be kept in service for at least 3 years, but must be disposed of after 5 years. The current purchase price of a tractor is $40,000 and increases by 10% a year. The salvage value of a 1-year-old tractor is $30,000 and decreases by 10% a year. The current annual operating cost of tractor is $1300 but is expected to increase by 10% a year. Determine the optimal replacement policy of the tractor over the next 5 years.

Q2. Any two

a) Find the optimal order quantity (q) for a product for which the price breaks are as follows:

q	Unit cost (Rs.)
$0 \le q < 50$	10
$50 \le q < 100$	9
$100 \le q$	8

The monthly demand for the product is 200 units, the cost of storage is 25% of the unit cost and the ordering cost is Rs. 20 per order.

b) An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the setup cost is Rs. 100 per setup and holding cost is Rs. 0.01per unit of item per day, find the economic lot size for one run, assuming that shortages are not permitted. Also find the time of cycle and minimum total cost for one run.

c) A company uses annually 50000 units of an item each costing Rs. 1.20. Each order costs Rs. 45 and inventory carrying cost 15% of the annual average inventory value.
   i. Find EOQ

ii. If the company operates 250 days a year, the procurement time is 10 days and safety stock is 500 units, find the reorder level, maximum, minimum and average inventory.

Q3. Attempt all

a) For the given pay-off matrix find the optimal strategies for the players and the value of the game.

	Player *B*		
Player *A*	B_1	B_2	B_3
A_1	3	−2	4
A_2	−1	4	2
A_3	2	2	6

b) Farmer can plant either corn or soybeans. The probabilities that the next harvest prices of these commodities will go up, stay the same, or go down are 0.25, 0.30 and 0.45, respectively. If the prices go up, the corn crop will net $30,000 and the soybeans will net $10,000. If the price remains unchanged, farmer will (barely) break even. But if the prices go down, the corn and soybean's crops will sustain losses of $35,000 and $5,000 respective1y.

i. Represent farmers problem as a decision tree.
ii. Which crop should the farmer plant?

Q4. Any two

a) A self-service store employs one cashier at its counter. Nine customers arrive on an average in 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival and exponertia1 distribution for service rate, find

i. the average number of customers in the system.
ii. average number of customers in queue
iii. average time a customer spends in the system.

b) A petrol station has 2 pumps. The service time follows exponertia1distribution with mean 4 minutes and cars arrive for service in a Poisson process at the rate of 10 cars per hour.

i. Find the probability that a customer has to wait for service.
ii. What proportion of time the pump remains idle?

c) The local one-person barbershop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting haircut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour (exponential service time).

i. What fraction of the potential customers arc turned away?
ii. How much time can a customer expect to spend in the barbershop?

Q5. Attempt all

a) Solve the following NLPP, using the method of Lagrange's multiplier.

Maximize Z = $6x_1 + 8x_2 - x_1^2 - x_2^2$
Subject to the constraints
$4x_1 + 3x_2 = 16, 3x_1 + 5x_2 = 15$

b) Use Kuhn-Tucker conditions to solve the given NLPP

Maximize Z = $2x_1^2 + 12x_1x_2 - 7x_2^2$
Subject to the constraints
$2x_1 + 5x_2 \leq 98$