END Semester Examinations, BIT, Mesra, Ranchi-835215, SP-2022 .

Subject: Topology-MA412, Time- 02 hours,

Full Mark: 50,

Answer all Questions.

1.a). Define a topology on a set X. Give an example. (02)

or

Give an example of a Topology on a set X.

1.b). Define a basis for a topology on a set X. (03)

or

Give an example of a basis on a space X.

1.c). If $\mathbb{B}$ is a basis for the topology X and $\mathbb{C}$ is a basis for the topology Y, then prove that the collection $\mathbb{D} = \{B \times C : B \in \mathbb{B} \textit{ and } C \in \mathbb{C}\}$ is a basis for the topolofy $X \times Y$. (05)

or

Prove that arbitrary intersections of closed sets are closed in a Topological space X.

2.a). Define a metric topology with an ϵ-Ball. (02)

or

Prove that the ϵ-Ball is an open set X.

2.b). Define Eucledean and standard metric on $\mathbb{R}^n$. (03)

or

Give examples of Eucledean and standard metric on $\mathbb{R}^n$.

2.c). Let $f : X \to Y$. If the function f is continuous, then for every convergent sequence $x_n \to x$, the sequence $f(x_n) \to f(x)$. Prove that the converse holds if X is metrizable. (05)

or

State and prove uniform limit theorem.

3.a). State Uryshon Lemma. (02)

or

Define second countability axiom of a space X.

3.b). Define a Regular space on a topological space X. Give an example. (03)

or

Define a Normal space on a topological space X. Give an example.

3.c). Let X be a topological space. Let one point set in X be closed. Then prove that X is Regular if and only if given a point $x \in X$ and a neighborhood U of x, there is a neighborhood V of x such that $\bar{V} \subset U$. (05)

or

Let X be a topological space. Let one point set in X be closed. Then prove that X is Normal if and only if given a closed set A and an open set U containing A, there is an open set V containing A such that $\bar{V} \subset U$.

4.a). Define a Compact space. (02)

or

Give an example of a Compact space X.

4.b). State and prove uniform continuity theorem. (03)

or

Prove that every closed subspace of a Compact space is compact.

4.c). State and prove Extreme value theorem. (05)

or

Prove that every Compact subspace of a Hausdroff space is closed.

5.a). Define a Cauchy sequence. (02)

or

Give an example of a Cauchy sequence.

5.b). Define the completeness in mteric space. (03)

or

Is the real set R is compact? Answer.

5.c). Prove that a metric space X is complete if every Cauchy sequence in X has a convergent subsequence. (05)

or

Prove that a metric space (X, d) is compact if and only if it is complete and totally bounded.

29.04.2022.

All The Best.